Interest rate and bank rescue policy

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Abstract

This paper presents a model in which the policy rate set by the central bank affects decisions about bank rescue policies when liquidity crises hit the banking system. We highlight a trade-off: maintaining an interest rate ensuring effective control over inflation escalates the costs of rescue interventions. We delve into this trade-off and determine the circumstances under which deviating from the target interest rate, thereby reducing intervention costs, enhances overall welfare. From a normative standpoint, our analysis indicates where liquidity risk is either low or high, the central bank should prioritize achieving the inflation target.

Keywords— Central Banking, Financial stability, Rescue Policies

JEL Code— G01, G21, G28

1 Introduction

The potential tension between price stability and bank stability as monetary policy targets has been explored extensively in the literature. Attention has been paid to the impact of interest rate changes – the primary monetary policy tool for maintaining inflation at desired levels – on the soundness of financial institutions and the probability of financial crises.

The common argument is that an increase in policy rates directly and indirectly leads to a depreciation of fixed income assets and an increase in non-performing loans that make the banking system more fragile (Goodfriend, 2002; Maddaloni and Peydró, 2011; Borio, 2014; Gomez *et al.*, 2021; Grimm *et al.*, 2023).

In this paper, we highlight a different and novel monetary policy trade-off between maintaining inflation at the target and safeguarding the stability of the banking system that arises from the links

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between the interest rate, the optimal policy for rescuing illiquid banks, and the optimal liquidity position of banks.

We consider a three-period banking system populated by commercial banks and a central bank. Commercial banks collect demand deposits and decide whether to invest in either a liquid, short-term asset or an illiquid, long-term asset. Short-term assets yield returns in each period equal to the risk-free rate set by the central bank and can be immediately liquidated at face value. Long-term assets also generate a fixed return, but only in the final period; however, in the intermediate period, they can be sold on the financial market to other banks to manage unexpectedly high deposit withdrawals *à la* Diamond and Dybvig (1983). The central bank plays a dual role: (i) it sets the economy's interest rate with the objective of maintaining inflation at the desired target, and (ii) it intervenes to rescue illiquid banks affected by a bank run.

In the interim period, the economy may experience an inflation shock. By raising the nominal interest rate to a level determined by a fixed inflation-targeting rule (hereinafter referred to as the inflation-target interest rate), the central bank can counteract the inflationary pressures. However, if the interest rate set by the central bank (hereinafter referred to as the policy interest rate) deviates from the inflation-target rate, the latter may incur losses proportional to the size of the imbalance.

At the same time, during the interim period, commercial banks face a liquidity risk due to possible withdrawals by depositors. If the market price at which banks undergoing a run can sell long-term assets is insufficient to meet depositors' demands, these banks face financial distress and, in the absence of central bank intervention, bankruptcy. The central bank can implement two costly rescue policies. First, it can provide emergency liquidity directly to illiquid banks, acting as a lender of last resort. Alternatively, it can inject liquidity into the financial market to support the demand and price of the long-term assets held by distressed banks. The costs of the central bank's interventions are proportional to the amount of new liquidity injected into the banking system (Lucas, 2019; Fahri and Tirole, 2023).[1]

The anti-inflationary interest rate rule followed by the central bank affects the stability of the banking system and the costs of bailout interventions. the policy rate implicitly sets the maximum price that liquid banks are willing to pay for the long-term assets of the distressed bank. When the policy rate in the interim period is high enough, the willingness to pay for long-term assets is so low that the price at which illiquid banks can sell their assets does not allow them to recover the liquidity needed to

[1]Intervention costs can be interpreted as the shadow price of creating and/or increasing liquidity required to implement the bailout policy, including operational costs for initiating and managing lending facilities, the costs of potential distortionary taxation, and the credit risk and reputational costs for the central bank

meet depositors' withdrawals. In this scenario, injecting liquidity into the market by extending credit to healthy banks is ineffective in preventing the default of distressed banks. The only possible rescue policy for the central bank is therefore to act as a lender of last resort, providing the illiquid bank with the liquidity needed to repay depositors and avoid bankruptcy.

When the central bank's policy rate is below this threshold rate, the bid price that liquid banks are willing to pay for long-term assets can potentially be high enough to prevent bank failures. However, the ability of a bank hit by a run to remain solvent by selling its long assets depends on the overall liquidity available to other banks. If this liquidity is low, cash-in-the-market pricing phenomena occur, and the market equilibrium price for illiquid assets does not allow illiquid banks to avoid insolvency (Shleifer and Vishny, 1992; Allen and Gale, 1994, 1998).

In such cases, the central bank can prevent the default of illiquid banks either by acting as a lender of last resort, providing illiquid banks with liquidity that they are unable to recover from the market, or by lending money to banks that hold short, liquid assets up to the point where overall liquidity in the market is sufficient to absorb the long assets of illiquid banks at a price that allows them to raise the resources needed to meet depositors' demands. These two alternatives are budget neutral as both require a liquidity injection equal to the difference between the overall liquidity needs and the overall liquidity available in the market. In both cases, since the central bank has to cover only a part of the liquidity that illiquid banks need to raise in order not to fail, the intervention costs are lower than in a regime of high interest rates.

Therefore, in setting its monetary policy rule, the central bank faces a trade-off between the costs of inflation and the costs of bailing out banks: keeping the policy rate at the target rate minimizes inflation costs but forces the central bank to implement expensive bailout policies. The alternative for the central bank is to keep the policy rate below the target when inflationary pressures are strong to save on rescue interventions, but this comes at the cost of higher inflation.

Our model explores this trade-off and identifies the conditions under which deviations from the target interest rate results in enhanced welfare. The model compares two monetary rules for the central bank. In the first case, the central bank has an "inflation targeting"mandate prioritizing the achievement of the inflation targeting. In the second case, the central bank's objective function is the minimization of a weighted sum of intervention costs and inflation costs.

The recent turmoil in the US banking sector provides anecdotal evidence of how central banks' anti-inflationary measures compel them to implement costly lender-of-last-resort interventions to mitigate potential destabilizing waves of banking crises. During the COVID-19 outbreak, many banks allocated a significant portion of their asset portfolios to Treasury securities and other low-yield long-term assets. Despite this, the low interest rates paid on deposits enabled banks to realize profitable interest margins (Zhou and Meng, 2023). However, when the Fed markedly increased interest rates to counter inflationary shocks and growing pressures on consumer prices, the market value of long-term assets plummeted sharply. This raised concerns about the financial stability of numerous banks, prompting withdrawals from uninsured depositors that compelled banks to liquidate their securities and recognize losses (Rajan and Acharya, 2023). The bank turmoil eventually led to the bankruptcy of Silicon Valley Bank (SVB), marking the second-largest default in the US banking system. To avert the SVB default from triggering a systemic crisis, the Fed initiated the Bank Term Funding Program (BTFP), a lender-of-last-resort facility designed to lend to banks with substantial unrealized losses on their long-term Treasury bonds. This program enables banks to exchange long-term assets (such as U.S. Treasury securities) for Federal funds at par value, irrespective of their current market value (Acharya *et al.*, 2023).

The BTFP has attracted criticism from those who highlight the effects of moral hazard on bank behavior and the potential fiscal costs to taxpayers. According to Buiter (2023), the BTFP offers too many advantages compared to market conditions. Consequently, financing through the BTFP becomes the preferred option for all banks, including those that could raise the liquidity they need under market conditions.[2]

In light of such criticism, our article interprets the Fed's actions as a consequence of its high interest rate policy, which necessitated the adoption of a bailout program like the BTFP that allows lending to all illiquid banks by valuing collateralizable securities above their market value. If the Fed had valued the long-term assets of borrowing banks at their market value, those banks unable to meet their liquidity needs under prevailing market conditions would have been at risk of default. Ultimately, by choosing to address inflationary pressures through sharp interest rate increases, the Fed has limited the tools at its disposal to maintain financial stability and has increased the costs of intervention. Had the interest rate increase been less drastic, the Fed could have upheld financial stability by permitting illiquid banks to face market discipline. However, this approach would have resulted in a weaker response to the inflationary shock.

Our paper relates to two relevant strands of literature. First, we contribute to the literature on the

[2]Precisely, in Buiter (2023)'s words, "*[...] this prudential response was not optimal, because the new Bank Term Funding Program created by the Fed, which offers one-year loans to banks with the collateral valued at par, should have been made available only on penalty terms. With market value well below par for many eligible debt instruments, the lender of last resort has become the lender of first resort – offering materially subsidized loans.*"

conflicts between price stability and financial stability, as well as the appropriate weight of financial stability in the central bank's mandate (Ferguson, 2002). A commonly accepted viewpoint is that central banks are charged with both financial and price stability duties; however, there is "the risk of financial dominance,"i.e., the risk that financial stability considerations undermine the credibility of the central bank's price stability mandate (Smets, 2018, p. 267). Therefore, financial and price stability duties should be separated, and monetary authorities should consider financial stability only insofar as it affects price stability (Bernanke and Gertler, 2000; Bernanke, 2012).

However, at the start of the new millennium, the global financial crisis made it evident that tracking the inflation rate was not sufficient to achieve financial stability (Rajan, 2006; Stiglitz, 2010; Bernanke, 2012). In the aftermath of the crisis, central banks were given powers and responsibilities directly related to financial stability (Borio, 2014). The idea is that the tasks of financial and price stability cannot be separated due to the involvement of banks in money creation and the effects of interest rates on the costs of refinancing distressed banks (Schwartz, 1998; Bordo and Wheelock, 1998; Mishkin, 2009; Fahri and Tirole, 2012; Brunnermeier and Sannikov, 2014). Additionally, a coordination between monetary policy and prudential regulation policy can help limit the impact of interest rates on banks' risk (Rajan, 2006; Paries *et al.*, 2011; Whelan, 2013; Cecchettia and Kohlerb, 2014). We contribute to this literature by examining the conditions under which it is optimal for the monetary authority to follow an interest rate rule that takes into account the costs of potential rescue interventions and allows for deviations from the inflation-target interest rate.

Another related strand of literature examines the effects of bank rescue policies on banks' incentives and investment behavior. The traditional approach focuses on lender-of-last-resort interventions, emphasizing their distorting effects on banks' risk-taking (Goodfriend and King, 1988; Freixas, 1999; Calomiris and Haber, 2015). More recently, new resolution mechanisms for distressed banks have been explored. First, a number of studies have considered bail-in policies consisting of a debt-equity swap mandated by the regulator. The use of bail-in resolution mechanisms allows for the recapitalization of banks by imposing losses on a small fraction of unsecured bank debt holders, thus avoiding the burden of distortionary taxes on taxpayers and eliminating the distortions introduced by bailout expectations. However, bail-ins increase the cost of debt for banks, introduce time inconsistency problems, and distort banks' risk-taking behavior(Chari and Kehoe, 2016; Walther and White, 2020; Pandolfi, 2022).

Second, a rescue policy similar to the one we consider in this article has been analyzed by Acharya and Yorulmazer (2007) and Acharya and Yorulmazer (2008). They pointed out that, anticipating that the regulator will find it efficient to bail out distressed institutions when the number of failures is high, banks respond by investing in common assets to increase the probability of joint distress. To overcome the conflict between crisis prevention and resolution, central banks can credibly commit to subsidizing non-distressed banks that have not invested in common assets. However, while the possibility of acquiring distressed banks creates an incentive for banks to differentiate themselves from others (Perotti and Suarez, 2002), it also pushes them to hold too much liquidity (Acharya *et al.*, 2011).

2 The model set-up

The economy lasts for three periods, initial ($t = 0$), interim ($t = 1$), and final ($t = 2$), and comprises a unitary mass of commercial banks and a central bank.

2.1 Banks

In the initial period, banks collect one unit of money in the form of demand deposits (d) and equities ($e = 1 - d$) and then invest it.

There are two investment opportunities: (i) holding liquid reserves in the central bank at the policy interest rate, and (ii) investing in an illiquid asset that yields $R > 1$ units of money in the final period ($t = 2$). For simplicity, following Acharya (2009), we assume that each bank can invest in one of the two assets but not both. From this point on, we will refer to banks that invest in central bank reserves as liquid banks and those that invest in long-term assets as illiquid banks.

In the interim period, with probability ω, a bank experiences a run, and all its depositors withdraw their deposits, while with probability $1 - \omega$, there are no deposit withdrawals. Assuming that the probability of a bank run is independent across banks, ω also represents the fraction of banks that suffer a run at $t = 1$. Liquid banks that experience a run are able to satisfy depositors' requests by operating with the central bank and withdrawing their reserves. Illiquid banks that experience a run on deposits can sell long-term assets in the financial market to other banks that have not experienced a run and have liquidity. If the liquidation proceeds are less than the demand deposits, these banks are financially distressed and unable to repay the depositors. In this case, the central bank intervenes to rescue distressed banks and ensure that depositors are paid. Commercial banks invest in reserves or long-term assets to maximize their expected value, given the central bank's interest rate and rescue policies.

2.2 The central bank: the policy interest rate

The central bank has a binding and publicly known mandate on the objectives of price stability and financial system stability that it can pursue by steering the policy interest rate and designing rescue interventions for banks in financial difficulty.

At $t = 0$, the policy interest rate is adjusted so that the inflation of the economy is at the target. We normalize to zero, $i_0^* = 0$, the interest rate that allows the central bank to reach the inflation target at $t = 0$.

In the interim period, an inflationary shock occurs with probability π. The central bank can counteract the inflation wave and maintain the inflation at the target by setting the interest rate at $i_\pi > 0$. With probability $1 - \pi$, there is no inflation shock and the interest rate allowing the central bank to reach the inflation target rate remains equal to zero:

$$i_T = \begin{cases} 0 & \text{with prob. } 1-\pi \\ i_\pi & \text{with prob. } \pi \end{cases} \tag{1}$$

If the policy rate deviates from the target rate, the central bank incurs losses proportionate to the absolute value of the deviation:

$$\Gamma = \gamma \left| i_1^* - i_T \right| \tag{2}$$

with $\gamma \in (0, \overline{\gamma})$. Hereafter, we will refer to Γ as inflation costs.

2.3 Financial market and distressed banks

At $t = 1$, a financial market opens in which banks can trade long-term assets. Following Wagner (2011), we assume that illiquid banks can sell the portfolio of long-term assets only as a whole.

Since the policy interest rate represents the opportunity cost of liquidity, the return for banks that demand long-term assets on the market, R/p, must be at least equal to the return on reserves $(1 + i_1^*)$. This means that the market cannot clear at a price higher than the present value of the fundamental return of the asset, $R/(1 + i_1^*)$.

When aggregate liquidity is not sufficient to absorb the total supply of illiquid bank assets at $R/(1+i_1^*)$, cash-in-the-market pricing prevails, and the financial market clears at a fire-sales price (Allen and Gale, 1994, 1998; Wagner, 2011).

The total liquidity in the financial market is given by the total amount of reserves held by the fraction

$(1-\omega)$ of liquid banks that do not suffer a run, while the long-term asset supply comes from illiquid banks that face deposit withdrawal. Let $\rho \in [0,1]$ be the mass of banks investing in long-term assets and $(1-\rho)$ be the mass of banks holding central bank reserves. The market liquidity and the supply of assets in the interim period are, respectively:

$$L = (1-\omega)(1-\rho) \tag{3}$$

$$S = \omega\rho \tag{4}$$

If $L/S \geq R/(1+i_1)$, illiquid banks can sell their long term assets at the fundamental value $R/(1+i_1)$, otherwise, the market clearing price is such that $L = pS$. That is,

$$p^{\star} = \min\left[\frac{R}{1+i_1^*}, \frac{(1-\omega)(1-\rho)}{\omega\rho}\right] \tag{5}$$

An illiquid bank experiencing a run is financially distressed if the total liquidity it can raise by selling long-term assets at the market-clearing price is lower than its total liabilities. Since illiquid banks invest all their funds in long-term assets, they are distressed if $p^* < d$. This can happen in two scenarios. First, when the central bank policy rate is high enough, such that $i_1^* > \tilde{i} = R/d-1$. In this case, the discounted fundamental value of the long-term asset is lower than d and any illiquid banks that experience a run on deposits would be in financial distress. Second, when the market liquidity is low. In this case, even if the central bank's policy rate is below $\tilde{i}$, illiquid banks can be forced to liquidate long-term assets at a fire sale price below d, at which they cannot absorb a potential bank run. From equations (3) and (4), it is immediately verified that this second scenario occurs when $\rho > \overline{\rho} = [1-\omega]/[1-\omega(1-d)]$.

2.4 Rescue interventions

In the absence of a capital injection from the central bank, commercial banks in financial distress will have no choice but to default. We assume that a bank default creates prohibitive (though not explicitly modeled) costs for the economy as a whole, so central bank interventions to prevent the default of distressed banks are always welfare-enhancing.

When the policy interest rate i_1^* is below the threshold $\tilde{i}$, the central bank can intervene in two ways to support the price of long-term assets at $p^* = d$ and avoid bank defaults. First, it can lend d units of money to a fraction q of distressed banks at a zero interest rate. In this way, the supply of long-term assets on the market decreases to $\widetilde{S} = S(1-q)$. Second, the central bank can lend an amount of liquidity Θ at a zero interest rate to (all or some) banks not affected by a run, thus increasing aggregate liquidity to $\widetilde{L} = L + \Theta$. Any combination of q and Θ such that $\widetilde{L} = d\widetilde{S}$ allows the supply of long-term assets in the financial market to be absorbed by the available liquidity at a market price equal to d.

We assume that the injection of liquidity by the central bank during the interim period generates costs (hereafter, rescue costs) that depend on the total amount of liquidity injected, regardless of the type of rescue intervention adopted.[3] In particular, for simplicity, we will assume that rescue costs increase linearly with liquidity injected by the central bank $\Lambda = \alpha(dq\omega\rho + \Theta)$, with $\alpha \in (0, \overline{\alpha})$.

From condition $\tilde{L} = d\tilde{S}$, we see that the amount of liquidity injected through the financial market, Θ, is inversely related to the amount of liquidity injected through lender of last resort interventions, q, and precisely that $\Theta = d\omega\rho(1-q) - (1-\omega)(1-\rho)$. Therefore, substituting this expression into Λ, we have that the rescue costs are equal to $\Lambda = \lambda\big(\rho - \overline{\rho}\big)$, where $\lambda = \alpha\big[1 - \omega(1-d)\big]$

When $i_1^* > \tilde{i}$, the fundamental value of the long-term assets of liquid banks in the interim period is lower than d. In this case, the only way to prevent the default of distressed banks is for the central bank to act as a lender of last resort by lending the necessary liquidity to all illiquid banks that experience a run, $d\omega\rho$. Therefore, rescue costs are:

$$\Lambda = \begin{cases} \max\big[0, \lambda(\rho - \overline{\rho})\big] & \text{if } \ i_1^* \leq \tilde{i} \\ \alpha d\omega\rho & \text{if } \ i_1^* > \tilde{i} \end{cases} \tag{6}$$

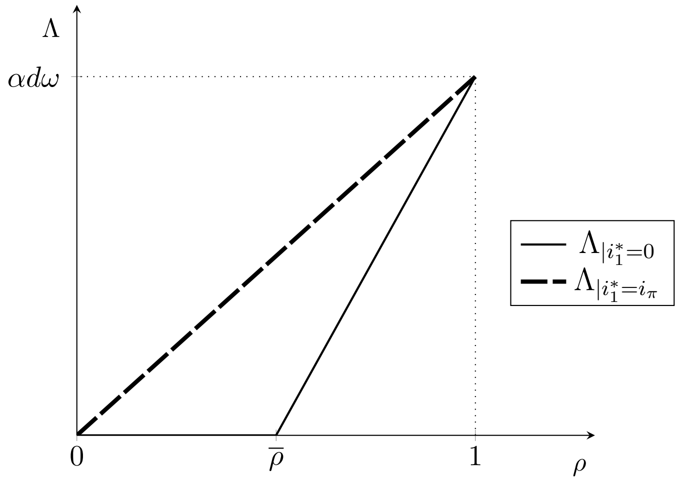
Figure 1 illustrates the rescue costs incurred by the central bank. It is interesting to note that rescue costs increase with the share of illiquid banks regardless of what the design of rescue interventions is (that is, what the combination of Θ and q is), even when a design is possible. However, for any $\rho < 1$, bailout costs are strictly higher when the policy interest rate exceeds $\tilde{i}$ (the dashed line) than when it is below that threshold (the continuous line). This happens because when $i_1^* < \tilde{i}$, the central bank can rely, in whole or in part, on the liquidity of other banks available to purchase the assets of banks in financial difficulty and therefore has to inject a smaller amount of additional liquidity into the market. From now on, to make the analysis interesting, we assume that the inflationary shock, when it occurs, is strong enough to require the central bank to set an interest rate higher than $\tilde{i}$ to counteract it:

Assumption 1. $i_\pi > \tilde{i}$.

[3]Rescue costs can include different type of costs for the central bank and the economy – fiscal costs, operational costs, reputation costs – that, for the sake of simplicity, we leave outside the scope of the model.

In this way, in the presence of an inflationary shock, a trade-off arises between inflation costs and rescue costs. If the central bank adjusts the policy interest rate at the inflation target i_π the rescue cost function is the dashed line in figure 1. Otherwise, if the central bank wants to reduce rescue costs (along the solid line in the figure 1) it must keep the policy rate below $\tilde{i}$ and bear inflation costs.

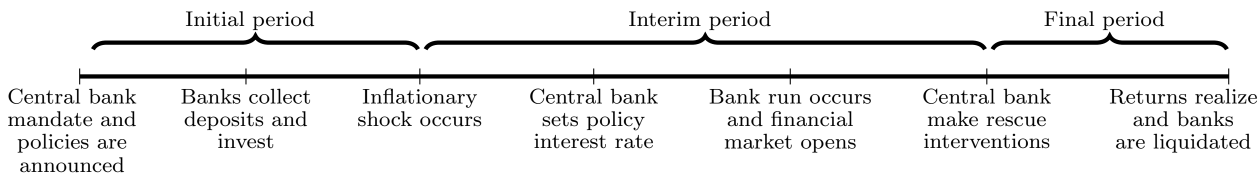
Figure 1: Rescue costs



2.5 Equilibrium

Figure 2 summarizes the sequence of model events. During the initial period, after that the mandate of the central bank is set and the monetary and rescue policies are announced, banks collect deposits and allocate them to one of the two investment opportunities, either central bank reserves or illiquid long-term assets. At the beginning of the interim period, a random inflationary shock occurs and the central bank sets the policy interest rate that will prevail between the interim and final periods. Then, possible bank runs materialize and, if necessary, the central bank implements rescue interventions. In the final period, banks are liquidated and the realized value is distributed to the depositors for consumption.

Figure 2: Model timeline



In the next section, we derive the time-consistent Nash equilibrium by backward induction under two possible mandates for the central bank. The "inflation targeting" mandate, or $\mathcal{I}$-mandate, establishes that the central bank prioritizes inflation costs over financial stability and rescue costs. In this case, the equilibrium is characterized by the triplet $\{i_1^*, \mathcal{B}^*, \rho^*\}$ for which:

E1. banks maximize their expected payoff conditional on their expectations about the policy rate, the rescue policy and the share of illiquid banks;

E2. the central bank chooses the policy rate so as to maintain price stability;

E3. the central bank chooses the rescue policy so as to minimize liquidity costs conditional on the optimal policy rate;

E4. banks' expectations are confirmed in equilibrium.

In the second mandate, which we label the "dual mandate" or $\mathcal{D}$-mandate, the central bank's monetary policy objectives are to promote both price stability and the stability of the financial system. In this case, equilibrium conditions **E1** and **E3** still apply. However, since optimal monetary policy both influences and is influenced by the state of (il)liquidity of the banking system, conditions **E2** and **E4** are replaced by:

E2'. the central bank chooses the policy rate so as to minimize a weighted sum of inflation and rescue costs;

E4'. bank's expectations are confirmed in equilibrium and consistent with the policy rate.

3 Inflation targeting mandate

The inflation targeting mandate requires that the policy rate set by the central bank during the interim period coincides with the realization of the target rate. Therefore, when an inflationary shock occurs, the policy interest rate is set at $i_{1\mathcal{I}}^* = i_\pi$. In this case, given Assumption 1, the only rescue policy for the central bank is to act as the lender of last resort by lending to all illiquid banks experiencing a bank run, that is, $\mathcal{B} = (0, 1)$.

In cases where inflationary shock does not occur, the policy interest rate is set at zero. If the share of banks investing in illiquid assets is greater than $\overline{\rho}$, the central bank is ex post indifferent between rescue policy designs $\mathcal{B} = (\Theta, q)$, such that $\tilde{L} = d\tilde{S}$. All of these policies ensure that the market price for long-term assets is equal to d and entail the same level of rescue costs on the solid line in figure 1, which increase with the share of illiquid banks.

Each bank invests in the asset, whether liquid or illiquid, that provides the highest payoff, conditional on the expectations about the central bank's monetary and rescue policies and the investment decisions of other banks. We assume that banks know the central bank's mandate and correctly anticipate its monetary and rescue policies. Therefore, they understand that, under the $\mathcal{I}$-mandate, the central bank, if faced with a wave of inflation, will have no choice but to act as a lender of last resort, supplying liquidity directly to banks in financial distress. Likewise, banks know that, in times of price stability, the central bank will either not intervene to rescue banks if market liquidity is greater than $1-\overline{\rho}$, or it will be indifferent between rescue policy designs that provide for an injection of liquidity in favor of healthy banks or banks in distress such that the market price of long-term assets settles at d. Thus, banks must conjecture the specific policy mix that the central bank will adopt to rescue banks in periods of monetary stability when $i_T = 0$. In summary, the banks' expectations about rescue interventions under the $\mathcal{I}$-mandate are:

$$\mathcal{B}^e_{\mathcal{I}} = \begin{cases} (0,0) & \text{if } i_T = 0 \text{ and } \rho^e \leq \overline{\rho} \\ (\Theta^e, q^e) & \text{if } i_T = 0 \text{ and } \rho^e > \overline{\rho} \\ (0,1) & \text{if } i_T = i_\pi, \end{cases} \tag{7}$$

where Θ^e and q^e are the shared expectations of the banks about the total liquidity that the central bank will extend to banks that do not experience a run and the fraction of distressed banks that will receive capital injection from the central bank during the interim period, and ρ^e is the expectation about the mass of illiquid banks. Finally, since banks that do not experience deposit withdrawals are identical except for the type of assets they hold, the share of liquidity they expect to receive from the central bank is the same, conditional on whether they hold liquid or illiquid assets. That is, $\Theta^e = (1-\omega)\left[\rho^e\theta^e_\rho + (1-\rho^e)\theta^e_{1-\rho}\right]$, where $\theta^e_{1-\rho}$ and θ^e_ρ are the expectations about the per capita liquidity that liquid and illiquid banks will receive from the central bank, respectively. The distribution of banks between the two investment opportunities is in equilibrium when no bank can increase its expected payoff by changing its investment decisions.

Therefore, in the inflation targeting mandate, we can easily derive the policy rate decision that the central bank will take and the rescue interventions expected by banks:

$$i^\star_{1,\mathcal{I}} = i_T \tag{8}$$

$$
\mathcal{B}_{\mathcal{D}}^{e} = \begin{cases} (0,0) & \text{If } i_T = 0 \text{ and } \rho^e \leq \overline{\rho} \quad (9a) \\ (d\omega\rho^e - (1-\omega)(1-\rho^e), 0) & \text{If } i_T = 0 \text{ and } \rho^e > \overline{\rho} \quad (9b) \\ (0,1) & \text{If } i_T = i_\pi \quad (9c) \end{cases}
$$

We move now to discuss banks' expected returns.

The expected individual payoffs from investing in long-term activities and liquid reserves, denoted by $\mathcal{A}_{\mathcal{I}}$ and $\mathcal{R}_{\mathcal{I}}$ respectively, depend on banks' expectations regarding the central bank's rescue policy in the event of a bank run. Since this policy is determined by the share of illiquid banks in the economy, banks' payoffs depend on their expectations about this share, specifically, from (7), on whether the expected degree of illiquidity of the banking system is greater or smaller than $\overline{\rho}$.

When banks believe that the banking system is sufficiently liquid (i.e., if $\rho \leq \overline{\rho}$), the payoffs are:

$$
\mathcal{A}_{\mathcal{I}}^{-} = \omega\left[\pi R + (1-\pi)p^*(\rho^e)\right] + (1-\omega)R \tag{10}
$$

for banks that invest in illiquid assets, and

$$
\mathcal{R}_{\mathcal{I}}^{-} = \omega + (1-\omega)\left[\pi(1+i_\pi) + (1-\pi)\frac{R}{p^*(\rho^e)}\right] \tag{11}
$$

for banks that invest in liquid reserves, where $p^*(\overline{\rho}) = d$.

With probability ω an illiquid bank suffers a run and its value depends on whether the economy as a whole is experiencing an inflationary wave or not. In the first case, with probability π, the illiquid bank is rescued by the central bank by receiving a loan equal to d that allows it to pay the depositors and not liquidate the long-term assets that yield R in the final period; with probability $1-\pi$ there is no inflationary pressure in the economy and the bank can sell the long-term assets on the market at a price $p^\star$ which depends on the share of banks expected to have invested in liquid reserves. With probability $1-\omega$, the bank does not face withdrawals and earns R in the final period.

For a bank that invests in liquid reserves, the returns are equal to 1 if a bank run occurs. In contrast, if a bank run does not occur, the returns are equal to $1+i_\pi$ during an inflation wave, and equal to $R/p^*(\rho^e)$, that is the return from the purchase of the long-term asset, if there are no inflationary pressures.

When banks expect the banking system to be illiquid, $\rho^e > \overline{\rho}$, the expected returns on investment in long-term assets or reserves are as follows:

$$\mathcal{A}_{\mathcal{I}}^{+} = \omega\big[\pi R + (1-\pi)(q^{e}R + (1-q^{e})d)\big] + (1-\omega)\left[R + (1-\pi)\left(\frac{R}{d} - 1\right)\theta_{\rho}^{e}\right] \tag{12}$$

$$\mathcal{R}_{\mathcal{I}}^{+} = \omega + (1-\omega)\left[\pi(1+i_{\pi}) + (1-\pi)\frac{R}{d} + (1-\pi)\left(\frac{R}{d} - 1\right)\theta_{1-\rho}^{e}\right] \tag{13}$$

For banks that choose to invest in long-term assets, the expected payoff in (12) differs in two respects from those reported in equation (10). First, if these banks experience a run and inflation pressure does not occur, they can benefit from central bank lending only with a probability of q^e, whereas with a probability of $1 - q^e$ the bank must liquidate its portfolio at a price of d. Second, with a probability of $(1-\omega)(1-\pi)$, illiquid banks do not face a deposit withdrawal, inflationary pressure does not materialize, and they can use the liquidity provided by the central bank, θ_{ρ}^{e}, to purchase the asset from other banks at a price of d, thereby realizing an additional return equal to $(R/d - 1)\theta_{\rho}^{e}$.

Liquid banks can always satisfy any withdrawal of deposits without entering financial distress. When withdrawals do not occur, in the absence of inflationary pressures and when $\rho^e > \overline{\rho}$, the payoff from investing in reserves includes the possibility that the central bank will allow liquid banks to borrow resources in the amount of $\theta_{1-\rho}^{e}$ to purchase long-term assets from distressed banks.

To avoid triviality, we assume that the return on long-term assets is larger than the expected returns on reserves when the latter are remunerated at the policy target rate. In fact, if this restriction was not in place, investment in reserves would always dominate investment in long-term assets and the only equilibrium distribution of banks over the two investment opportunities would be $\rho^* = 0$.

Assumption 2. $R - \pi(1 + i_{\pi}) - (1 - \pi) > 0$

By comparing Equations (12) and (13), it is straightforward to verify that investing in long-term assets becomes increasingly profitable for banks as the values of q^e and θ_{ρ}^{e} rise and the value of $\theta_{1-\rho}^{e}$ decreases. In economic terms, this implies that the greater the access to central bank credit for illiquid banks, whether or not they are experiencing financial distress, and the more restricted it is for liquid banks, the higher the profitability of long-term investments for banks. From (6), rescue costs increase with the number of illiquid banks in the economy, while they are unaffected by the mix of rescue interventions and how the injected liquidity is distributed among banks. Therefore, when $\rho^e > \overline{\rho}$, the optimal design of the central bank's rescue policy is to set $q^* = 0$ and $\theta_{\rho}^{*} = 0$. Since this policy design is time-consistent, banks anticipate it correctly and set their expectations consistently as $q^e = 0$ and

$\theta^e_\rho = 0$. Thus, for any $\rho^e > \overline{\rho}$, the expected payoff from investing in long-term assets is:

$$\mathcal{A}^+_{\mathcal{I}}(\rho^e) = \mathcal{A}^-_{\mathcal{I}}(\overline{\rho}) = \omega\big[\pi R + (1-\pi)d\big] + (1-\omega)R \tag{10'}$$

which coincides with the payoff derived in (10) in the case where $\rho^e = \overline{\rho}$, and $p^* = d$.

From $q^e = 0 = \theta^e_\rho = 0$ and (7) follows that the optimal (time-consistent) rescue policy involves $\theta^e_{1-\rho} = \frac{\Theta^e}{(1-\omega)(1-\rho^e)}$. Therefore, the expected payoff from investing in reserves is

$$\mathcal{R}^+_{\mathcal{I}} = \mathcal{R}^-_{\mathcal{I}}(\overline{\rho}) + (1-\omega)(1-\pi)\left(\frac{R}{d} - 1\right)\left(\frac{\omega d \rho^e}{(1-\rho^e)(1-\omega)} - 1\right) \tag{11'}$$

where, once again, $\mathcal{R}_{\mathcal{I}}(\overline{\rho})$ is the payoff derived in (11) when $\rho^e = \overline{\rho}$ and $p^* = d$.

Let $\Delta_{\mathcal{I}}$ denote the difference between the payoffs from investing in long-term assets and in liquid reserves for all ρ.

$$\Delta_{\mathcal{I}} = \begin{cases} \mathcal{A}^-_{\mathcal{I}} - \mathcal{R}^-_{\mathcal{I}} & \text{If } \rho^e \leq \overline{\rho} \\ \mathcal{A}^+_{\mathcal{I}} - \mathcal{R}^+_{\mathcal{I}} & \text{If } \rho^e \geq \overline{\rho} \end{cases} \tag{14}$$

Lemma 1. *$\Delta_{\mathcal{I}}$ is continuous at any point in $\rho^e \in [0,1)$, and strictly decreasing in $\rho^e \in [0,1)$. Moreover:*

$$\lim_{\rho^e \to 0} \Delta_{\mathcal{I}} > 0 \qquad \lim_{\rho^e \to 1} \Delta_{\mathcal{I}} < 0 \tag{15}$$

Proof. The continuity of payoffs $\mathcal{A}_{\mathcal{I}}$ and $\mathcal{R}_{\mathcal{I}}$ – and hence the continuity of $\Delta_{\mathcal{I}}$ – follows from the fact that in $\overline{\rho}$, the expressions in (10) and (10′) have the same value, as do those in (11) and (11′). Moreover, from (10) and (10′), $\mathcal{A}_{\mathcal{I}}$ is strictly decreasing as ρ^e increases between 0 and $\overline{\rho}$ and is constant for $\overline{\rho} < \rho^e \leq 1$, from (11) and (11′), $\mathcal{R}_{\mathcal{I}}$ is strictly increasing as ρ^e increases. Moreover, assumption (2) ensures that $\Delta_{\mathcal{I}} > 0$ for $\rho^e = 0$. From (11′), $\mathcal{R}_{\mathcal{I}}$ approaches infinity as ρ^e approaches 1. Thus, $\Delta_{\mathcal{I}}$ is strictly decreasing at any point $\rho^e \in [0,1)$ and limits satisfy (15). □

The reason why the difference between the expected return of the long-term asset and the expected return of reserves is decreasing is straightforward: the lower the liquidity in the banking system, the more liquidity the central bank must provide to solvent banks to maintain the market clearing price at a level that allows illiquid banks to meet withdrawal demands. The liquidity injection will boost the net worth of banks holding reserves.

The fact that $\Delta_{\mathcal{I}}$ is continuous and strictly decreasing provides a straightforward characterization of the equilibrium mass of banks investing in long-term assets. Specifically, the equilibrium value for ρ must be such that the two assets yield the same expected returns, which occurs when $\Delta_{\mathcal{I}} = 0$. Suppose, for instance, that $\Delta_{\mathcal{I}} > 0$. Given the continuity of $\Delta_{\mathcal{I}}$, a bank holding reserves can increase its expected value by adjusting its investment decision.

Before characterizing the equilibrium in the inflation targeting mandate, it is helpful to consider some reflections regarding the value of ρ at which $\Delta_{\mathcal{I}} = 0$. First, conditions (15) guarantee that such a value exists and is unique. Second, by comparing equations (10′) and (11′), it clearly emerges that the value for which $\Delta_{\mathcal{I}} = 0$ can be greater than $\overline{\rho}$ if and only if $\mathcal{A}^{-}_{\mathcal{I}}(\overline{\rho}) > \mathcal{R}^{-}_{\mathcal{I}}(\overline{\rho})$. It is possible to characterize this previous condition as follows:

$$\mathcal{A}^{-}_{\mathcal{I}}(\overline{\rho}) > \mathcal{R}^{-}_{\mathcal{I}}(\overline{\rho}) \iff i_{\pi} \leq \frac{1}{\overline{\rho}}\left[\tilde{i} - \frac{(1-d)\overline{\rho}}{\pi(1-\omega)}\right] - \frac{(1-d)\tilde{i}}{\pi} \equiv \iota \tag{16}$$

Therefore, depending on the value that i_{π} assumes, which we interpret as the strength of the inflationary wave (when it occurs), the value of ρ for which $\Delta_{\mathcal{I}} = 0$ can be greater than or less than $\overline{\rho}$. More precisely, the value of ρ such that $\Delta_{\mathcal{I}} = 0$ satisfies $\mathcal{A}^{-}_{\mathcal{I}} = \mathcal{R}^{-}_{\mathcal{I}}$ in the case of $i_{\pi} \geq \iota$, and satisfies $\mathcal{A}^{+}_{\mathcal{I}} = \mathcal{R}^{+}_{\mathcal{I}}$ in the case of $i_{\pi} < \iota$. For the rest of the paper, we will denote as $\rho^{-}_{\mathcal{I}}$ the value of ρ such that $\mathcal{A}^{-}_{\mathcal{I}} = \mathcal{R}^{-}_{\mathcal{I}}$, and as $\rho^{+}_{\mathcal{I}}$ the value of ρ such that $\mathcal{A}^{+}_{\mathcal{I}} = \mathcal{R}^{+}_{\mathcal{I}}$.

We can now characterize the equilibrium of the inflation targeting.

Proposition 1. *Under the $\mathcal{I}-$mandate, the sub-game perfect Nash equilibrium is characterized as follows:*

(i) If $i_{\pi} \geq \iota$, the policy rate satisfies (8)*, the rescue interventions satisfy* (9a) *and* (9c)*, the mass of banks investing in the long term asset is equal to $\rho^{-}_{\mathcal{I}} \leq \overline{\rho}$*

(ii) If $i_{\pi} < \iota$, the policy rate satisfies (8)*, the rescue interventions satisfy* (9b) *and* (9c)*, the mass of banks investing in the long term asset is equal to $\rho^{+}_{\mathcal{I}} > \overline{\rho}$*

Proof. The proof is straightforward. Since the bank's goal is to minimize inflation costs, the equilibrium policy rate is always equal to the target rate, i.e., (8). If $i_{\pi} \geq \iota$, we know from the above discussion that

$\Delta_{\mathcal{I}}$ equals zero for $\rho^{-}_{\mathcal{I}} < \overline{\rho}$. Therefore, the central bank does not inject liquidity in the absence of an inflation shock. Instead, if an inflation shock occurs, the central bank will lend to all illiquid banks.

Conversely, if $i_{\pi} < \iota$, we know from the above discussion that $\Delta_{\mathcal{I}}$ equals zero for $\rho^{+}_{\mathcal{I}} > \overline{\rho}$. Therefore, the central bank lends to liquid banks in the absence of an inflation shock. If an inflation shock occurs, the central bank will lend to all illiquid banks. □

The above result characterizes the equilibrium of the economy under the inflation targeting mandate. Depending on the model's parameters, the mass of banks investing in the long-term asset can be higher or lower than $\overline{\rho}$. When inflationary pressure is severe enough, i.e., i_{π} is larger than a critical threshold, the mass of banks investing in long-term assets is such that the central bank injects capital into the financial system only when inflationary pressures arise. The intuition is straightforward: a high i_{π} boosts the return of the liquid asset during inflationary pressure. Since both assets must have the same expected return in equilibrium, the market clearing price of the long-term asset, when inflationary pressure does not materialize, must be sufficiently high. Hence, ρ must be sufficiently low.

Instead, when inflationary pressure is not severe—i.e., when i_{π} is below a critical threshold—the mass of banks investing in long-term assets is such that the central bank is compelled to inject capital into the banking system, regardless of whether an inflation wave occurs. The economic reasoning mirrors what we stated previously. A low i_{π} decreases the return on liquid assets during inflationary pressure. In equilibrium, both assets must have the same expected return, so the market liquidity must be structured in a way that allows liquid banks to earn extra returns by leveraging capital lent by the central bank. This capital is used to purchase long-term assets at a discounted price, denoted by d, when inflationary pressure does not materialize.

In all instances, since the central bank always sets the interest rate equal to the target, the equilibrium arising under the inflation targeting mandate is characterized by positive expected rescue costs and null inflation costs.

We conclude this section by discussing how the equilibrium mass of banks investing in the long-term asset depends on the inflation risk (π) and the liquidity risk (ω).

Lemma 2. *The term $\rho^{+}_{\mathcal{I}}$ is increasing in π and decreasing in ω.*

Proof. See Appendix A. □

The above The comparative static results outlined in Lemma 2 state that, when inflation waves are mild and the mass of banks investing in the long-term asset is larger than $\overline{\rho}$, the equilibrium mass of banks investing in the long-term asset increases as inflation risk increases and decreases as liquidity risk increases. The economic reasoning is as a follows.

An increase in the probability of inflation waves raises the likelihood that illiquid banks will receive funds from the central bank. Banks holding reserves that are not affected by a liquidity shock lose the opportunity to take advantage of the capital lent by the central bank to acquire long-term assets in the market. As partial compensation, liquid banks receive the high interest rate on reserves, i_π. However, as $i_\pi < \iota$, the compensation for liquid banks is generally insufficiently high. Consequently, investing in long-term assets is becoming more attractive relative to holding reserves.

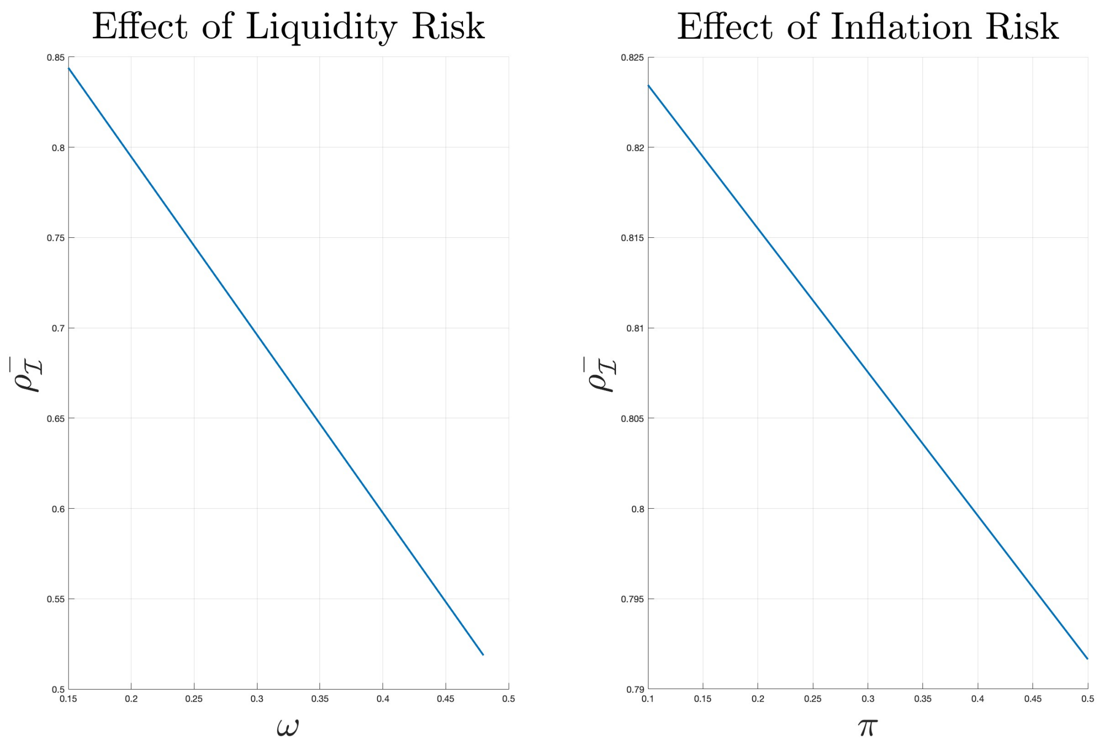
An increase in liquidity risk, instead, reduces the mass of banks investing in the long-term asset. The reason is as follows. Ceteris paribus, an increase in ω makes liquidation more likely and reduces the liquidity available in the market. For banks holding reserves, an increase in ω implies an increase in capital provided by the central bank when inflationary pressure is not realized. Consequently, investing in long-term assets is becoming less attractive relative to holding reserves.

For the case where the mass of banks investing in the long-term asset is equal to $\rho_{\mathcal{I}}^{-}$, the comparative static is performed using a numerical analysis. For the selection of different parameters, the results highlighted in Lemma 2 still hold (Figure 3).

The reason why an increase in the inflation risk decreases the mass of banks investing in the long-term asset mirrors the case we discussed above. An increase in the probability of inflation waves raises the likelihood that illiquid banks will receive funds from the central bank. Banks holding reserves that are not affected by a liquidity shock lose the opportunity to take advantage of the capital lent by the central bank to acquire long-term assets in the market. As partial compensation, liquid banks receive the high interest rate on reserves, i_π. As $i_\pi < \iota$, the compensation for liquid banks is high. Consequently, investing in long-term assets is becoming more attractive relative to holding reserves.

Regarding to the effect of liquidity risk, when the mass of banks investing in the long-term asset is lower then $\overline{\rho}$, the central bank does not provide capital to the liquid banks. Therefore, the argument we used above should be not valid. However, an increase in ω declines ceteris paribus the market clearing price, increasing liquidation losses for illiquid banks and increasing trade profit for liquid banks. As a result, investing in long-term assets is becoming less attractive relative to holding reserves.

Figure 3: Comparative static



The figure shows the first degree polynomial fit of the mass of banks investing in the long-term asset arising in equilibrium.

4 Dual mandate

In a dual mandate, the central bank's objective is to minimize a weighted sum of inflation and rescue costs:

$$\mathcal{C} = \beta\Lambda + \Gamma \tag{17}$$

where $\beta \geq 0$ is the relative importance of rescue costs versus inflation costs.

As Λ is a step function that shifts upwards when the policy rate is above $\tilde{i}$, in the absence of inflationary pressures, price stability and financial stability are not in conflict, and the central bank has no reason to set a policy rate different from the target rate. However, when the central bank is faced with an inflationary shock, a trade-off between inflation costs and rescue costs emerges, and the policy rate that minimizes the loss function (17) may be different from the target rate.

Consider now the case where inflationary pressure realizes. If the central bank sets a policy rate equal to the target rate i_π, it has no option but to act as lender of last resort for all illiquid banks and incurs a total loss equal to:

$$\mathcal{C}(i_\pi) = \beta\alpha d\omega\rho \tag{18}$$

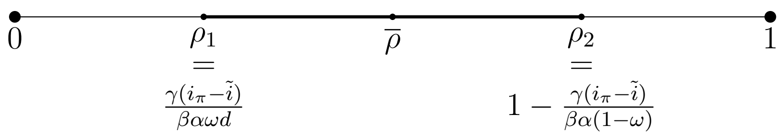
Suppose, instead, that the central bank decides on a policy rate different from the target i_π. In this case, the central bank sets the interest rate at $\tilde{i}$. In fact, policy rates lower than $\tilde{i}$ would increase inflation costs without affecting rescue costs, policy rates higher than $\tilde{i}$ but different from the target i_π would lead to inflation costs without allowing savings in intervention costs. Therefore, in case of deviation from the inflation target, the total loss for the central bank amounts to:

$$\mathcal{C}(\tilde{i}) = \beta \max\left[0, \lambda(\rho - \overline{\rho})\right] + \gamma\big(i_\pi - \tilde{i}\big) \tag{19}$$

By solving inequality $\mathcal{C}(i_\pi) > \mathcal{C}(\tilde{i})$ for ρ, we can characterize situations in which, given the general degree of illiquidity that prevails in the banking system, it is optimal for the central bank to deviate from the target rate during periods of inflationary pressure.

Specifically, it is possible to identify two thresholds $\rho_1 = \frac{\gamma(i_\pi - \tilde{i})}{\beta\alpha\omega d}$ and $\rho_2 = 1 - \frac{\gamma(i_\pi - \tilde{i})}{\beta\alpha(1-\omega)}$ such that if the illiquidity of the banking sector takes values in the interval $\mathcal{P} = [\rho_1, \rho_2]$, the total losses of the central bank with a policy rate at its inflation target $i^* = i_\pi$ are greater than those incurred when $i^* = \tilde{i}$ (see the figure 4).

Figure 4: The interval $\mathcal{P}$



Note: In bold the values of ρ for which $\mathcal{C}(i_\pi) > \mathcal{C}(\tilde{i})$ and the central bank, in the face of an inflationary shock, prefers to deviate from the inflation-target interest rate.

The economic intuition is as follows: If the share of illiquid banks is very high or very low, the benefits of setting a policy rate lower than the one that minimizes inflation costs are limited. This is because the savings in terms of rescue costs that the deviation from the target rate allows (i.e., the distance between the solid and dashed lines in figure 1) remain low when the mass of banks investing in the long-term asset is either very large or very small.

The existence of an interval $\mathcal{P}$ depends on the model parameters. In particular, deviations of the policy rate from the inflation target can be valuable if and only if the weight attached to financial stability and rescue costs in the $\mathcal{D}-$mandate is sufficiently large:

$$\beta > \frac{\gamma\big(i_\pi - \tilde{i}\big)\big[1-\omega(1-d)\big]}{\alpha\omega(1-\omega)d} \tag{20}$$

Moreover, the width of interval $\mathcal{P}$,

$$\mu = 1 - \frac{\gamma\big(i_\pi - \tilde{i}\big)\big[1-\omega(1-d)\big]}{\beta\alpha\omega(1-\omega)d}, \tag{21}$$

which represents how likely the central bank is to deviate from its inflation target during periods of high inflationary pressure, increases with the relative importance of the financial stability objective, the unit cost of liquidity injections, and the frequency of bank runs, while it decreases with the costs of inflation. Finally, it is easy to show that if there are situations in which the central bank finds it optimal to deviate from the target rate, then these certainly include the case in which the liquidity of the banking system is $\overline{\rho}$.

Lemma 3. *If $\mathcal{P} \neq \emptyset$, then $\overline{\rho} \in \mathcal{P}$.*

Proof. Let inequality (20) hold and let $\beta = b\frac{\gamma\left(i_\pi - \tilde{i}\right)\left[1-\omega(1-d)\right]}{\omega(1-\omega)\alpha d}$, with $b \geq 1$. As $\overline{\rho} = \frac{[1-\omega]}{[1-\omega(1-d)]}$, it is straightforward to verify that $\frac{\gamma\left(i_\pi - \tilde{i}\right)}{\omega\alpha d\beta} \leq \overline{\rho} \leq 1 - \frac{\gamma\left(i_\pi - \tilde{i}\right)}{\left(1-\omega\right)\alpha\beta}$. □

Intuition is simple. When $\rho = \overline{\rho}$, the savings in rescue costs that can be achieved by setting the policy rate below the target i_π are at their maximum value. Therefore, if in this case the central bank does not consider it worth deviating from the target rate, then $\mathcal{P} = \emptyset$ and, as in $\mathcal{I}$-mandate, setting a policy rate that minimizes the costs of inflation would always be the optimal choice, regardless of the degree of liquidity of the banking system.

Then, restricting the analysis to the case in which the set $\mathcal{P}$ exists,[4] under $\mathcal{D}$-mandate, the optimal policy rate for the central bank depends on the mass of banks investing in the long-term asset:

$$i^*_{1\mathcal{D}} = \begin{cases} i_T & \text{If } \rho \notin \mathcal{P} \quad \text{(22a)} \\ \begin{cases} 0 & \text{If } \rho \in \mathcal{P} \text{ and } i_T = 0 \\ \tilde{i} & \text{If } \rho \in \mathcal{P} \text{ and } i_T = i_\pi \end{cases} & \text{(22b)} \end{cases}$$

[4]If $\mathcal{P} = \emptyset$, the case of the mandate $\mathcal{D}$ is reduced to that of $\mathcal{I}$-mandate analyzed in section 3.

Turning to bailout policy, by departing from the inflation-targeting rule, the central bank expands the set of conditions under which it may refrain from injecting liquidity into the banking system to rescue illiquid banks affected by excessive deposit withdrawals or, if compelled to intervene, may abstain from acting as a lender of last resort in favor of illiquid banks. In particular, let $\mathcal{P}^{+}$ be the subset of $\mathcal{P}$ that contains values of ρ greater than $\overline{\rho}$ and $\mathcal{P}^{-}$ be the subset of those less than or equal to $\overline{\rho}$:

$$\begin{aligned}\mathcal{P}^{-} &= \left\{\rho \in \mathcal{P} : \rho \leq \overline{\rho}\right\} \\ \mathcal{P}^{+} &= \left\{\rho \in \mathcal{P} : \rho > \overline{\rho}\right\}\end{aligned} \tag{23}$$

In a similar manner, and for the sake of exposure, we denote as $\mathcal{P}^{C^{-}}$ the complement of the set $\mathcal{P}^{-}$ containing values of ρ lower than $\overline{\rho}$, and as $\mathcal{P}^{C^{+}}$ the complement of the set $\mathcal{P}^{+}$ containing values of ρ higher than $\overline{\rho}$.

$$\begin{aligned}\mathcal{P}^{C^{-}} &= \left\{\rho \notin \mathcal{P} : \rho < \overline{\rho}\right\} \\ \mathcal{P}^{C^{+}} &= \left\{\rho \notin \mathcal{P} : \rho > \overline{\rho}\right\}\end{aligned} \tag{24}$$

When the state of illiquidity in the banking system belongs to $\mathcal{P}^{-}$, setting the policy rate at $\tilde{i}$ allows the central bank to prevent a collapse in the prices of long-term assets and enables illiquid banks to find the necessary liquidity in the market to repay depositors. When the state of illiquidity is more severe and $\rho \in \mathcal{P}^{+}$, the central bank must design a rescue policy to avoid the failure of distressed banks by injecting the liquidity needed to absorb the illiquid banks' securities at a price of d. Since, as in the case of the inflation targeting mandate, the central bank has an interest in making investments in long-term assets relatively less attractive compared to investments in liquid assets, a policy design that limits access to credit solely to liquid banks—by setting $q = 0$ and $\theta_{1-\rho} = 0$—is optimal and intertemporally consistent.

Assuming that banks correctly anticipate central bank's interest rates and rescue policies, the expected rescue policy design is:

$$\mathcal{B}^{e}_{\mathcal{D}} = \begin{cases} (0,0) & \text{If } i \leq \tilde{i} \text{ and } \rho^{e} \leq \overline{\rho} \quad \text{(25a)} \\ (d\omega\rho^{e} - (1-\omega)(1-\rho^{e}), 0) & \text{If } i \leq \tilde{i} \text{ and } \rho^{e} > \overline{\rho} \quad \text{(25b)} \\ (0,1) & \text{If } i > \tilde{i} \quad \text{(25c)} \end{cases}$$

When $\rho^e \notin \mathcal{P}$, banks expect that even under a dual mandate, the central bank will set the policy rate at its inflation target. Consequently, the expected payoffs are the same as those obtained in the case of the $\mathcal{I}$-mandate. In particular, for any $\rho^e < \rho_1$, we have $\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^{C^-}} = \mathcal{A}_{\mathcal{I}}^-$ and $\mathcal{R}_{\mathcal{D}}^{\mathcal{P}^{C^-}} = \mathcal{R}_{\mathcal{I}}^-$, while for $\rho^e > \rho_2$, we have $\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^{C^+}} = \mathcal{A}_{\mathcal{I}}^+$ and $\mathcal{R}_{\mathcal{D}}^{\mathcal{P}^{C^+}} = \mathcal{R}_{\mathcal{I}}^+$.

Now, consider the cases in which $\rho^e \in \mathcal{P}$. When $\rho^e \in \mathcal{P}^-$, banks expect that, although the central bank's monetary policy may deviate from the inflation target when there is inflationary pressure, the overall liquidity in the banking system will be sufficiently high, preventing the need for additional liquidity injections by the central bank to rescue banks facing excessive deposit withdrawals. Therefore, the expected payoffs from investing in long-term assets and reserves are:

$$\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-} = \omega\big[(1-\pi)p^*(\rho^e) + \pi d\big] + (1-\omega)R \tag{26}$$

and

$$\mathcal{R}_{\mathcal{D}}^{\mathcal{P}^-} = \omega + (1-\omega)\left[(1-\pi)\frac{R}{p^*(\rho^e)} + \pi\frac{R}{d}\right] \tag{27}$$

With probability $\omega(1-\pi)$ banks investing in the long-term asset experience a run. If there are no inflationary pressures in the intermediate period, banks will be able to sell their assets on the financial market at $p^*(\rho^e) > d$. However, when inflationary pressures materialize and the policy rate is $\tilde{i}$, the central bank will inject just enough liquidity into the market to allow distressed banks to sell their assets at d. Finally, with probability $1-\omega$ there are no deposit withdrawals and banks investing in the long-term asset earn R. For banks that invest in liquid assets,

For banks holding reserves, in the event of deposit withdrawals, the returns are equal to 1. However, when they do not experience a bank run, with probability $1-\omega$, they will purchase long-term illiquid assets at a price of $p^*(\rho^e)$ or d, depending on whether the policy rate $i^*_{1\mathcal{D}}$ is 0 or $\tilde{i}$ with probability $1-\pi$ and π, respectively.

When $\rho^e \in \mathcal{P}^+$, regardless of the state of inflationary pressure, banks expect the central bank to intervene to prevent bank failures by providing additional liquidity to liquid banks, enabling them to purchase the long-term assets sold by distressed illiquid banks at the price d. Therefore, the expected payoff of investment in long-term assets is the weighted average of the return on the asset if held to maturity and the price at which it can be sold to meet deposit withdrawals,

$$\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^+} = \omega d + (1-\omega)R \tag{28}$$

On the other hand, banks that invest in reserves can leverage the capital lent by the central bank and obtain a payoff

$$\mathcal{R}_{\mathcal{D}}^{\mathcal{P}+} = \omega + (1-\omega)\left[\frac{R}{d} + \left(\frac{R}{d} - 1\right)\theta_{1-\rho}^{e}\right] \tag{29}$$

To sum up, the difference between the expected returns derived from investing in the two assets can be described as:

$$\Delta_{\mathcal{D}} = \begin{cases} \mathcal{A}_{\mathcal{I}}^{-} - \mathcal{R}_{\mathcal{I}}^{-} & \text{If } \rho^e < \rho_1 \\ \mathcal{A}_{\mathcal{D}}^{-} - \mathcal{R}_{\mathcal{D}}^{-} & \text{If } \rho^e \in \mathcal{P}^{-} \\ \mathcal{A}_{\mathcal{D}}^{+} - \mathcal{R}_{\mathcal{D}}^{+} & \text{If } \rho^e \in \mathcal{P}^{+} \\ \mathcal{A}_{\mathcal{I}}^{+} - \mathcal{R}_{\mathcal{I}}^{+} & \text{If } \rho^e > \rho_2 \end{cases} \tag{30}$$

Lemma 4. *The term $\Delta_{\mathcal{D}}$ is a piecewise function that strictly decreases in $\rho^e \in [0, \rho_1)$, strictly decreases in $\rho^e \in \mathcal{P}$, and strictly decreases in $\rho^e \in (\rho_1, 1)$. Moreover, it has a first-kind discontinuity at $\rho^e = \rho_1$ and $\rho^e = \rho_2$, and:*

$$\lim_{\rho^e \to 0} \Delta_{\mathcal{D}} > 0 \qquad \lim_{\rho^e \to 1} \Delta_{\mathcal{D}} < 0 \tag{31}$$

Proof. For $\rho^e \in (0, \rho_1)$, we have $\Delta_{\mathcal{D}} = \Delta_{\mathcal{I}}$. Therefore, Lemma 1 ensures that $\Delta_{\mathcal{D}}$ is decreasing and continuous over the interval $\rho^e \in (0, \rho_1)$. The same argument applies for the interval $\rho^e \in (\rho_2, 1)$. The discontinuity of $\Delta_{\mathcal{D}}$ at ρ_1 and ρ_2 clearly emerges by comparing (10) with (26) and (10′) with (28).

To demonstrate that Δ_D is decreasing over the interval $\mathcal{P}$, note that $\mathcal{A}_{\mathcal{D}}$ is strictly decreasing as ρ^e increases between $[\rho_1, \overline{\rho}]$ and it is constant for $[\overline{\rho}, \rho_2]$. Moreover, $\mathcal{R}_{\mathcal{D}}$ is strictly increasing over the interval $\mathcal{P}$. To demonstrate the behavior of $\Delta_{\mathcal{D}}$ as ρ^e approaches extremes, we can refer to Lemma 1. □

Lemma 4 tells us that, contrary to what happens in the $\mathcal{I}$-mandate (see Lemma 1), under the $\mathcal{D}$-mandate, the expected payoffs do not continuously decrease throughout the interval $\rho^e \in [0, 1)$. The reason is that in this case the central bank's monetary policy decision in the interim period depends not only on the realization of the inflationary shock but also on the liquidity of the banking system, which can make a deviation from the inflation target either advantageous or not. This occurs discontinuously at the threshold values ρ_1 and ρ_2.

Despite the discontinuity of the banks' expected payoff, as the next result states, it remains true that a necessary condition to meet our equilibrium definition is that the two assets provide the same expected returns. That is, $\rho^\star_{\mathcal{D}}$ must be such that $\Delta_{\mathcal{D}} = 0$.

Lemma 5. *If $\Delta_D \neq 0$ for all points $\rho^e \in [0,1]$, then there exists no equilibrium triplet $(i^\star_{1,\mathcal{D}}, B^\star_{\mathcal{D}}, \rho^\star_{\mathcal{D}})$ that meets our equilibrium definition.*

Proof. See Appendix B. $\square$

Before characterizing the equilibrium of the dual mandate, it is necessary to do some considerations about the values of ρ for which $\Delta_{\mathcal{D}} = 0$, $\rho^\star_{\mathcal{D}}$.

First, due to the discontinuity of $\Delta_{\mathcal{D}}$ such a value may not exist. Second, $\rho^\star_{\mathcal{D}}$ cannot belong to the interval $\mathcal{P}^+$. To understand this, it suffices to compare equations (28) and (29) and note that if banks expect both that the central bank targets inflation and that the liquidity level exceeds $\overline{\rho}$, all banks will prefer investing in reserves over long-term investments.

Third, since $\Delta_{\mathcal{D}}$ coincides with $\Delta_{\mathcal{I}}$ for $\rho^e \notin \mathcal{P}$, it is possible that $\rho^\star_{\mathcal{D}} = \rho^\star_{\mathcal{I}}$. This instances occurs whenever $\rho^\star_{\mathcal{I}} \notin \mathcal{P}$. Fourth, due to Assumption 2, a unique value for ρ such that $\mathcal{A}^-_{\mathcal{D}} - \mathcal{R}^-_{\mathcal{D}} = 0$ exists. We label this value $\rho^\diamond$. As $\Delta_{\mathcal{D}}$ coincides with $\mathcal{A}^-_{\mathcal{D}} - \mathcal{R}^-_{\mathcal{D}}$ for $\rho^e \in \mathcal{P}^-$, it is possible that $\rho^\star_{\mathcal{D}} = \rho^\diamond$. This instances occurs whenever $\rho^\diamond \in \mathcal{P}$.

In summary the equilibrium mass of banks investing in the long-term asset in the dual mandate can: (i) not exist, (ii) coincide with the one arising under the inflation targeting mandate, and (iii) coincide with the term $\rho^\diamond < \overline{\rho}$.

In the next result, we demonstrate that a sufficient condition to always have an equilibrium in the dual mandate is $\rho^\diamond > \rho^-_{\mathcal{I}}$.

Lemma 6. *If $\rho^\diamond \geq \rho^-_{\mathcal{I}}$ the dual mandate has at least one equilibrium. If d is sufficiently high, $\rho^\diamond > \rho^-_{\mathcal{I}}$ holds.*

Proof. Suppose that $\rho^\diamond < \rho^-_{\mathcal{I}}$. This means that it is possible to find a value for β such that $\rho^\diamond \notin \mathcal{P}$ and $\rho^-_{\mathcal{I}} \in \mathcal{P}$. This means that there are no points in $\mathcal{P}$ such that $\Delta_{\mathcal{D}} = 0$ and there are no points outside $\mathcal{P}$ such that $\Delta_{\mathcal{D}} = 0$.

From equations (26) and (27) it emerges that for $d \to 1$, $\rho^\diamond \to \overline{\rho}$. However, as long as $i_\pi \neq \iota$, $\rho^-_{\mathcal{I}} < \overline{\rho}$. Thus, for d sufficiently high the condition $\rho^\diamond \geq \rho^-_{\mathcal{I}}$ holds. $\square$

In the remaining of the paper, we assume that an equilibrium in the dual mandate always exists. That is, we assume that model's parameters are such that $\rho^\diamond \geq \rho^-_{\mathcal{I}}$ holds.

The next result characterizes the equilibrium in the dual mandate as a function of β.

Proposition 2. *Let:*

$$\underline{\beta} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho^\diamond} \qquad \overline{\beta} = \begin{cases} \dfrac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho_{\mathcal{I}}^-} & \text{If } i_\pi \geq \iota \\ \dfrac{\gamma(i_\pi - \tilde{i})}{\alpha(1-\omega)(1-\rho_{\mathcal{I}}^+)} & \text{If } i_\pi < \iota \end{cases} \tag{32}$$

Hence:

1. *If $\beta \geq \overline{\beta}$, the equilibrium mass of banks investing in the long-term asset is equal to $\rho_{\mathcal{D}}^\star = \rho^\diamond < \overline{\rho}$, the policy rate satisfies (22b) and the rescue intervention implemented by the central bank satisfies (25a).*

2. *If $\beta \in (\underline{\beta}, \overline{\beta}]$, the $\mathcal{D}-$mandate exhibits multiple equilibria. One equilibrium coincides with the one arising under the inflation targeting mandate. In the other equilibrium, the mass of banks investing in the long-term asset is equal to $\rho_{\mathcal{D}}^\star = \rho^\diamond < \overline{\rho}$, the policy rate satisfies (22b) and the rescue intervention implemented by the central bank satisfies (25a).*

3. *If $\beta < \underline{\beta}$, the equilibrium coincides with the one arising under the inflation targeting mandate.*

Proof. See Appendix C. □

Proposition 2 states that the equilibrium of the economy under the dual mandate depends on the measure of the set $\mathcal{P}$, which in turn depends on the weight attached to rescue costs in the central bank's mandate, β. Depending on β, three cases are possible. We illustrate these different cases through Figure 5. The figure illustrates the case for $i_\pi < \iota$.

When β is sufficiently large (Panel 5a), the economy has only one time-consistent equilibrium characterized by the absence of rescue interventions and rescue costs, but by positive inflation costs during inflationary pressure. Rescue costs are null because the mass of banks investing in the long-term asset must necessarily be in the set $\mathcal{P}^-$. This can be easily understood by comparing (28) and (29). At any $\rho^e > \mathcal{P}^+$, all banks will prefer reserves over the long-term asset. This follows because at any $\rho^e \in \mathcal{P}^+$, the central bank never raises the policy rate above $\tilde{i}$. Hence, there is no opportunity for illiquid banks to avoid liquidation losses by receiving capital from the central bank. Moreover, the liquidity in the market is poor enough to push the central bank to lend to liquid banks, which leverage this capital to make extra profits by purchasing the long-term asset in the market. Thus, if model's parameters are such that there is an equilibrium in $\mathcal{P}$, it must be in $\mathcal{P}^-$, where the mass of banks investing in the long-term asset allows the central bank to not inject liquidity during the interim period.

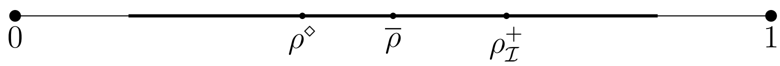
When β is neither too small or too large (panel 5b), two equilibria are possible. One is the same equilibrium we just discussed. The other is the equilibrium arising under the inflation targeting mandate. To understand why multiple equilibria arise, notice that for $\rho \notin \mathcal{P}$, the central bank behaves as it does under the inflation targeting mandate. Therefore, a given $\rho \notin \mathcal{P}$ is an equilibrium in the dual mandate if and only if it is an equilibrium in the inflation targeting mandate. Thus, if β is such that the set $\mathcal{P}$ does not contain $\rho^{\star}_{\mathcal{I}}$, the equilibrium arising under the inflation targeting mandate can also emerge in the dual mandate.

Finally, when β is sufficiently small, the dual mandate exhibits only one equilibrium, which coincides with the one arising under the inflation targeting mandate. To understand this result, we can again leverage the fact that for $\rho \notin \mathcal{P}$, the central bank behaves as it does under the inflation targeting mandate. Suppose $\mathcal{P}$ is so tiny that it involves just $\overline{\rho}$. From the comparison between (26) and (27) follows that, at $\overline{\rho}$, reserves dominate the investment in the long-term asset. Therefore, an equilibrium is not possible within $\mathcal{P}$. Consequently, the equilibrium must be outside $\mathcal{P}$, where the central bank behaves as if it has an inflation targeting mandate.

Note that the discussion for the case $i_\pi \geq \iota$ is specular to what we just stated. As the value of ρ that constitute an equilibrium for the inflation targeting mandate depends on i_π, the thresholds for β will be different.

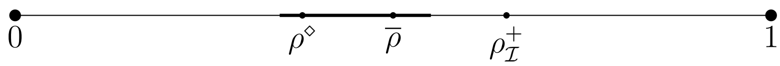
Figure 5: The interval $\mathcal{P}$

(a) High β



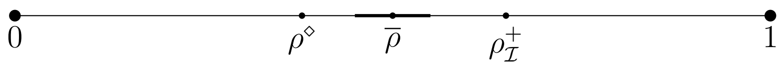
In bold the set $\mathcal{P}$. In this case, the dual mandate exhibits a unique equilibrium where the mass of banks investing in the long-term asset is equal to $\rho^{\diamond}$, the central bank deviates from the target rate during inflationary pressure, and there are no rescue interventions.

(b) Intermediate β



Note: In bold the set $\mathcal{P}$. In this case, the dual mandate exhibits multiple equilibria. Depending on banks expectations, we may have the equilibrium arising under the inflation targeting or the same equilibrium arising for high β.

(c) Low β



Note: In bold the set $\mathcal{P}$. In this case, the dual mandate exhibits the same equilibrium arising under the inflation targeting mandate.

We conclude this section by characterizing $\rho^{\diamond}$ in terms of inflation risk (π), and liquidity risk (ω). Although a closed solution for the term $\rho^{\diamond}$ is untractable, it has a smooth approximation around the value $\overline{\rho}$.

In fact, note that $\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-} = \mathcal{R}_{\mathcal{D}}^{\mathcal{P}^-}$ has a solution at $p^{\star} = d$ when $d = 1$. Therefore, a linear approximation of the market clearing price implicitly defined by the condition is $\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-} = \mathcal{R}_{\mathcal{D}}^{\mathcal{P}^-}$ around the point $d = 1$ and $p = d$ is:

$$p^{\star}(d) = 1 + \frac{\pi}{1-\pi}\Big[1 - d\Big]$$

From equation (5):

$$p^{\star}(d) = \min\left[R, \frac{L}{S}\right] = \frac{L}{S} \implies \rho^{\diamond} = \frac{1-\omega}{1+\omega\frac{\pi}{1-\pi}(1-d)} \tag{33}$$

From (33) emerges that the term $\rho^{\diamond}$ decreases as the inflation risk increases. That is, variations in the probability of an inflation waves have opposite effects compared to the equilibrium arising under the inflation targeting. The reason is that, when the mass of banks investing in the long-term asset is $\rho^{\diamond}$, the central bank will never raise the interest rate above $\tilde{i}$. Hence, an increase in π does not have the effect to increase the probability for illiquid banks to avoid liquidation losses. Contrary, it increases the probability of a reduction in the market clearing price. In fact, as $\rho^{\diamond} < \overline{\rho}$, the equilibrium market clearing price when the inflation waves will not materialize is higher than d. When inflation waves occur, the central bank sets the policy rate at $\tilde{i}$ and the market clearing price drops to 0. Therefore, liquidation losses for illiquid banks increase making the investment in reserves relative more attractive.

Finally, (33) also shows that $\rho^{\diamond}$ decreases in ω. The economic reasoning is specular to what we stated in the Section 3 to describe the effects of ω on $\rho_{\mathcal{I}}^{-}$. As $\rho^{\diamond} < \overline{\rho}$, illiquid banks never receive capital from the central bank. Therefore, as liquidity risk increases liquidation losses for illiquid banks increase making the investment in reserves relative more attractive.

5 Welfare comparison

In the previous section, we argue that, in cases where the central bank has a dual mandate, where the task of the central bank is to minimize a weighted sum between rescue and inflation costs, the equilibrium of the model depends on the relative costs' weights. If the relative weight attached to rescue costs is sufficiently high, the equilibrium of the economy is such that the central bank does not need to inject liquidity in the banking system to pacify liquidity crises.

On the other hand, if the relative weight attached to rescue costs is sufficiently low, the equilibrium of the economy is as if the central bank has an inflation targeting mandate. That is, cases where the central bank is forced to inject capital in the banking system to avoid the default of illiquid banks are possible, while inflation costs are always null.

Therefore, a natural question arises: Is it desirable to assign a weight to rescue costs in order to achieve an equilibrium where rescue costs are null? Or would it be better for the central bank to always minimize inflation costs? To answer this question, we compare, in terms of expected welfare, the equilibrium arising when the central bank has an inflation targeting mandate with the equilibrium arising when the central bank has a dual mandate and $\beta > \overline{\beta}$. That is, a case where the dual mandate has a unique equilibrium where the central bank finds it convenient to deviate from the inflation targeting mandate during inflationary pressure.

We assume that the welfare of the economy depends on the mass of banks investing in the long-term asset, mirroring the real output generated by the system, net of inflation and rescue expected costs. That is:

$$\mathcal{W} = \rho R - \Lambda^e - \Gamma^e \tag{34}$$

When β is sufficiently high, the mass of banks investing in the long-term asset is $\rho^{\diamond} < \overline{\rho}$ and the central bank does not raise the interest rate above $\tilde{i}$ during inflationary pressure. Therefore:

$$\begin{aligned} \Lambda^e_{\mathcal{D}} &= 0 \\ \Gamma^e_{\mathcal{D}} &= \pi\gamma\Big[i_\pi - \tilde{i}\Big] \end{aligned} \tag{35}$$

Therefore, the expected welfare arising in the dual mandate where the relative weight attached to rescue costs is sufficiently high is equal to:

$$\mathcal{W}_{\mathcal{D}} = \rho^{\diamond} R - \pi\gamma\Big[i_\pi - \tilde{i}\Big] \tag{36}$$

Let us now consider the welfare arising under the inflation targeting mandate. First, suppose that

$i_\pi \geq \iota$. In this case, the mass of banks investing in the long-term asset is equal to $\rho_{\mathcal{I}}^- < \overline{\rho}$. Therefore, the central bank does not need to inject capital in the financial system when the inflation shock does not realize. However, the central bank will lend to all illiquid banks during inflation waves. Therefore:

$$\begin{aligned} \Lambda_{\mathcal{I}}^e &= \pi\alpha d\omega\rho_{\mathcal{I}}^- \\ \Gamma_{\mathcal{I}}^e &= 0 \end{aligned} \tag{37}$$

Therefore, the expected welfare arising in the inflation targeting mandate in case of severe inflation waves ($i_\pi \geq \iota$) is equal to:

$$\mathcal{W}_{\mathcal{I}}^- = \rho_{\mathcal{I}}^- R - \pi\alpha d\omega\rho_{\mathcal{I}}^- \tag{38}$$

The case for $i_\pi < \iota$ differs because the mass of banks investing in the long-term asset is higher and the central bank is forced to inject liquidity in the financial system regardless of the realization of the inflation shock. Hence:

$$\begin{aligned} \mathcal{W}_{\mathcal{I}}^+ &= \rho_{\mathcal{I}}^+ R - \alpha\left[(1-\pi)\theta^\star(1-\omega)(1-\rho_{\mathcal{I}}^+) + \pi d\omega\rho_{\mathcal{I}}^-\right] \\ \mathcal{W}_{\mathcal{I}}^+ &= \rho_{\mathcal{I}}^+ R - \alpha\left[(1-\pi)(1-\omega(1-d))(\rho_{\mathcal{I}}^+ - \overline{\rho}) + \pi d\omega\rho_{\mathcal{I}}^+\right] \end{aligned} \tag{39}$$

Where in the second line we use the fact that, in equilibrium, the value of θ must be such that the market clearing price is equal to d.

From welfare equations immediately follows that:

$$\begin{aligned} \mathcal{W}_{\mathcal{D}} - \mathcal{W}_{\mathcal{I}}^+ > 0 \iff \alpha > \alpha_1 &= \frac{R\left(\rho_{\mathcal{I}}^+ - \rho^\diamond\right) + \pi\gamma\left(i_\pi - \tilde{i}\right)}{(1-\pi)(1-\omega(1-d))(\rho_{\mathcal{I}}^+ - \overline{\rho}) + \pi d\omega\rho_{\mathcal{I}}^+} \\ \mathcal{W}_{\mathcal{D}} - \mathcal{W}_{\mathcal{I}}^- > 0 \iff \alpha > \alpha_2 &= \frac{\pi\gamma\left(i_\pi - \tilde{i}\right) - R\left(\rho^\diamond - \rho_{\mathcal{I}}^-\right)}{\pi d\omega\rho_{\mathcal{I}}^-} \end{aligned} \tag{40}$$

The higher the two thresholds α_1 and α_2, the higher must be the unitary rescue cost to have an inflation targeting mandate dominated by the dual mandate. Trivially, α_1 and α_2 increase as γ, the parameter governing inflation costs, increases.

Understanding the effects of liquidity risk (ω) and inflation risk (π) is more complex. In both cases, there are two contrasting effects. An increase in liquidity risk (ω) reduces market liquidity, but it also decreases the proportion of banks investing in long-term assets. Consequently, the net effect on rescue costs is ambiguous. Variations in the inflation risk (π) generate similar contrasting effects and, in addition, boosts expected inflation losses.

Without making any further restrictions, we cannot understand which effects will prevail. However, if we assume that the mass of banks investing in the long-term asset (in all cases) is around the critical value $\overline{\rho}$, the thresholds assume a tractable form. Indeed, for $\rho_{\mathcal{I}}^{-}, \rho^{\diamond}, \rho_{\mathcal{I}}^{+} \to \overline{\rho}$:

$$\alpha_1 = \alpha_2 = \hat{\alpha} = \frac{\gamma(i_\pi - \tilde{i})}{d\omega\overline{\rho}} \tag{41}$$

Note that $\hat{\alpha}$ is a non-monotone, strictly convex function of ω, indicating that the inflation-targeting mandate is most likely to yield the highest welfare when liquidity risk is either small or large. Indeed, in these cases, rescue costs are sufficiently low to make deviations from the target rate not worthwhile.

6 Conclusion

This paper argues that conflicts between price stability and financial stability objectives arise from the influence of the interest rate on the rescue response when liquidity crises impact the banking system. As the interest rate determines the minimum return the market demands to purchase assets from illiquid banks, the minimum price required by these banks to avoid distress may exceed the maximum price liquidity holders are willing to pay if the central bank's policy rate is sufficiently high. In such situations, the central bank must intervene by providing emergency liquidity to distressed banks through lender-of-last-resort actions. The costs of intervention resulting from this rescue policy are higher compared to scenarios where the interest rate remains low.

Thus, we posit that during periods of high inflation, a trade-off exists. A monetary policy focused on combating inflation involves more costly bailout measures, as they necessitate higher liquidity provision. Our welfare analysis suggests that in economies where liquidity risk is either low or high, the central bank should prioritize achieving the inflation target.

Appendices

A Proof of Lemma 2

(i) From (10′) and (11′) we have:

$$\frac{d\rho^e}{1-\rho^e} = \frac{\Delta_{\mathcal{I}}(\overline{\rho})}{(1-\pi)\tilde{i}\omega} + \frac{1-\omega}{\omega} \tag{A.1}$$

Where:

$$\Delta_{\mathcal{I}}(\overline{\rho}) = \mathcal{A}_{\mathcal{I}}(\overline{\rho}) - \mathcal{R}_{\mathcal{I}}(\overline{\rho})$$

Since the L-H-S of (A.1) is increasing in ρ^e, a sufficient condition to have $\rho^\star_{\mathcal{I}}$ increasing in π is that the term $\Delta_{\mathcal{I}}(\overline{\rho})$ is not decreasing in π. Recall that:

$$\Delta_{\mathcal{I}}(\overline{\rho}) = \omega\Big[\pi R - 1 + (1-\pi)d\Big] + (1-\omega)\Big[R - \pi(1+i_\pi) - (1-\pi)\frac{R}{d}\Big] \tag{A.2}$$

By taking the derivative with respect to π, we have:

$$\omega\Big(R-d\Big) + (1-\omega)\Big(-(1+i_\pi) + \frac{R}{d}\Big) \geq 0 \tag{A.3}$$

Since $\tilde{i} = R/d - 1$, we can restate:

$$wd\tilde{i} + (1-\omega)(\tilde{i} - i_\pi +) \geq 0 \implies i_\pi \leq \frac{\tilde{i}}{\overline{\rho}} \tag{A.4}$$

Since $\rho^+_{\mathcal{I}} > \overline{\rho}$, it must be that:

$$i_\pi < \iota < \frac{\tilde{i}}{\overline{\rho}}$$

Hence, $\rho^+_{\mathcal{I}}$ increases in π. Let us now consider the derivative with respect to ω. The derivative of the R-H-S with respect to ω is negative if:

$$-1 - \frac{1}{\tilde{i}(1-\pi)}\Big[\frac{R}{d} - R + \pi(i_\pi - \tilde{i})\Big] < 0 \tag{A.5}$$

Since $d < 1$ and $i_\pi > \tilde{i}$, the above term is negative. Therefore, the equilibrium mass of banks investing in the long-term asset decreases in ω.

(ii) The value $\rho_{\mathcal{I}}^{-}$ is implicitly defined by the equality condition between (10) and (11). This condition can be stated as:

$$\omega\Big[\pi R + (1-\pi)p^\star\Big] + (1-\omega)R = \omega + (1-\omega)\Big[\pi(1+i_\pi) + (1-\pi)\frac{R}{p}\Big] \tag{A.6}$$

By taking the derivative with respect to π of both sides, we get:

$$\Bigg[\omega(1-\pi) + (1-\omega)\frac{\pi R}{\Big(p^\star\Big)^2}\Bigg]\frac{\partial p^\star}{\partial \pi} = (1-\omega)\Big(i_\pi - \frac{R}{p^\star}\Big) - \omega\Big(R - p^\star\Big) \tag{A.7}$$

As $R/p^\star > 1$, for reasonable value of i_π, the R-H-S of the above equation is likely to be negative. Therefore, $\partial p^\star/\partial\pi < 0$. Since the market clearing price is negatively related to the mass of banks investing in the long-term asset, the equilibrium mass of banks investing in the long-term asset increases in π.

By taking the derivative with respect to ω of both sides of equation (A.6), we get:

$$(1-\pi)\Bigg[\omega + \frac{(1-\omega)R}{\Big(p^\star\Big)^2}\Bigg]\frac{\partial p^\star}{\partial \omega} = \Big(R - p^\star + 1 - \frac{R}{p^\star}\Big)\Big(1-\pi\Big) - \pi i_\pi \tag{A.8}$$

As the mass of banks investing in the long-term asset is smaller than $\overline{\rho}$, the market clearing price is in the interval (d, R). For $p^\star \to R$, the R-H-S of the above equation is negative. Therefore, $\partial p^\star/\partial\omega < 0$. In addition, the R-H-S is increasing in price. Hence, $\partial p^\star/\partial\omega < 0$ always holds.

Since the market clearing price is negatively related to the mass of banks investing in the long-term asset, the equilibrium mass of banks investing in the long-term asset increases in ω.

B Proof of lemma 5

To begin with the proof, notice that from equations (11), and (27), at ρ_1 we have the following condition:

$$\mathcal{R}_{\mathcal{I}}^{-} > \mathcal{R}_{\mathcal{D}}^{\mathcal{P}^-} \tag{B.1}$$

To demonstrate the statement, we have to show that discontinuity points, ρ_1 and ρ_2, cannot constitute equilibria.

Suppose that, at ρ_1, we have $\Delta_D < 0$. That is:

$$\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-} - \mathcal{R}_{\mathcal{D}}^{\mathcal{P}^-} < 0 \tag{B.2}$$

Starting from ρ_1, a bank investing in the long-term asset is better off in switching investment decisions. In fact, by doing so, the mass of banks investing in the long-term asset will fall outside the set $\mathcal{P}$ and all banks investing in reserves will earn $\mathcal{R}_{\mathcal{I}}^{-}$. If the bank does not switch, the expected return is equal to $\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-}$. The conditions (B.1) and (B.2) imply that $\mathcal{R}_{\mathcal{I}}^{-} > \mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-}$, meaning that to switch investment decision is profitable.

Suppose now that, at ρ_1, we have $\Delta_D > 0$. That is:

$$\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^-} - \mathcal{R}_{\mathcal{D}}^{\mathcal{P}^-} > 0 \tag{B.3}$$

In this case, the continuity of Δ_D over the interval $\mathcal{P}$ implies that any bank will be better off by switching the investment in reserves with the long-term asset. Therefore, at ρ_1 there are always profitable deviations.

The fact that at ρ_2 there are profitable deviations immediately follows from the comparison between equations (28) and (29). That is, since

$$\mathcal{A}_{\mathcal{D}}^{\mathcal{P}^+} - \mathcal{R}_{\mathcal{D}}^{\mathcal{P}^+} < 0$$

holds for any $\rho^e \in \mathcal{P}^+$, and since Δ_D is decreasing and continuous in $\mathcal{P}$, any bank will be better off by switching the investment in the long-term asset with reserves.

C Proof of Proposition 2

Consider cases where $i_\pi \geq \iota$. That is, $\rho^{\star}_{\mathcal{I}} = \rho^{-}_{\mathcal{I}} < \overline{\rho}$.

1. Suppose that the measure of the set $\mathcal{P}$ is so high that $\rho^{-}_{\mathcal{I}} \in \mathcal{P}$. In this case, as outside the set $\mathcal{P}$ the central bank behaves as if the inflation targeting mandate and $\rho^{-}_{\mathcal{I}}$ is the only value for ρ that constitute an equilibrium in the $\mathcal{I}-$mandate, there are no equilibria outside $\mathcal{P}$. Thus, Δ_D must have at least one point in $\mathcal{P}$ where $\Delta_D = 0$. As we stated in the text, the only point where this is possible is $\rho^{\diamond}$.

   Since the measure of the set $\mathcal{P}$ depends on β, we can easily characterize the condition $\rho^{-}_{\mathcal{I}} \in \mathcal{P}$ in terms of β. As $\rho^{-}_{\mathcal{I}} < \overline{\rho}$, $\rho^{-}_{\mathcal{I}} \in \mathcal{P}$ holds whenever the lower bound of $\mathcal{P}$ is no larger than $\rho^{-}_{\mathcal{I}}$. That is:

$$\rho_1(\beta) \leq \rho^{-}_{\mathcal{I}} \implies \beta \geq \overline{\beta} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d \rho^{-}_{\mathcal{I}}} \tag{C.1}$$

   Thus, when β satisfies the above condition, the subgame perfect Nash equilibrium is as described by the the first bullet of the proposition and $\overline{\beta}$ coincides with $\overline{\beta} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d \rho^{-}_{\mathcal{I}}}$.

2. Suppose now that β is such that $\rho^{\diamond} \in \mathcal{P}$ and $\rho^{-}_{\mathcal{I}} \notin \mathcal{P}$. In this case, there are two values for ρ where $\Delta_{\mathcal{D}} = 0$. This means that two equilibria are possible. One where the mass of banks investing in the long-term asset arising in equilibrium is $\rho^{-}_{\mathcal{I}}$, leading to the same equilibrium arising under the inflation targeting, and the other where the mass of banks investing in the long-term asset arising in equilibrium is $\rho^{\diamond}$, leading to the same equilibrium we discuss in the previous bullet.

   We can characterize the conditions $\rho^{-}_{\mathcal{I}} \notin \mathcal{P}$ and $\rho^{\diamond} \in \mathcal{P}$ in terms of β. As $\rho^{-}_{\mathcal{I}} < \rho^{\diamond} < \overline{\rho}$, the two conditions hold whenever the lower bound of $\mathcal{P}$ is larger than $\rho^{-}_{\mathcal{I}}$ but lower than $\rho^{\diamond}$. That is, whenever:

$$\rho_1(\beta) > \rho^{-}_{\mathcal{I}} \implies \beta > \underline{\beta} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d \rho^{-}_{\mathcal{I}}} \tag{C.2}$$

$$\rho_1(\beta) \leq \rho^{\diamond} \implies \beta > \underline{\beta} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho^{\diamond}} \tag{C.3}$$

Hence:

$$\beta \in [\frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho^{\diamond}}, \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho_{\mathcal{I}}^{-}}] \tag{C.4}$$

3. Finally, suppose that the measure of the set $\mathcal{P}$ is so small that $\rho^{\diamond}, \rho_{\mathcal{I}}^{-} \notin \mathcal{P}$. Hence, the only point where $\Delta_{\mathcal{D}} = 0$ is equal to $\rho_{\mathcal{I}}^{-}$, meaning that the subgame perfect Nash equilibria coincides with the one arising under the inflation targeting mandate.

   We can characterize the condition $\rho^{\diamond}, \rho_{\mathcal{I}}^{-} \notin \mathcal{P}$ in terms of β. As $\rho_{\mathcal{I}}^{-} < \rho^{\diamond} < \overline{\rho}$, the condition holds whenever the lower bound of $\mathcal{P}$ is larger than $\rho^{\diamond}$. That is, whenever:

$$\rho_1(\beta) > \rho^{\diamond} \implies \beta < \overline{\beta} = \frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho^{\diamond}} \tag{C.5}$$

Note that the case for $i_\pi < \iota$ requires similar steps. However, as in this case $\rho_{\mathcal{I}}^{\star} = \rho_{\mathcal{I}}^{+} > \overline{\rho}$ the relevant threshold to define the upper bound for β is ρ_2. Following similar computations as the one stated above, we can easily end up with a value for $\overline{\beta}$ coinciding with $\frac{\gamma(i_\pi - \tilde{i})}{\alpha\omega d\rho^{\diamond}}$.

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