# Some Misconceptions about Public Investment Efficiency and Growth

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**Abstract**

We reconsider the macroeconomic implications of public investment efficiency, defined as the ratio between the actual increment to public capital and the amount spent. We show that, in a simple and standard model, increases in public investment spending in inefficient countries do *not* have a lower impact on growth than in efficient countries, a result confirmed in a simple cross-country regression. This apparently counter-intuitive result, which contrasts with Pritchett (2000) and recent policy analyses, follows directly from the standard assumption that the marginal product of public capital declines with the capital/output ratio. The implication is that efficiency and scarcity of public capital are likely to be inversely related across countries. It follows that both efficiency and the rate of return need to be considered together in assessing the impact of increases in investment, and blanket recommendations against increased public investment spending in inefficient countries need to be reconsidered. *Changes* in efficiency, in contrast, have direct and potentially powerful impacts on growth: "investing in investing" through structural reforms that increase efficiency, for example, can have very high rates of return.

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# 1 Introduction

> "If the efficiency of the public investment process is relatively low—so that project selection and execution are poor and only a fraction of the amount invested is converted into productive capital stock—increased public investment leads to more limited long-term output gains" (IMF, 2014*b*)

> "Even where public *capital* has a *potentially* large contribution to production, public-investment *spending* may have a low impact" (Pritchett, 2000)

What is the growth impact of an increase in the rate of public investment spending? And in particular how does this depend on the efficiency of public investment spending? In an influential paper, Pritchett (2000) argues forcefully that it is incorrect to consider that one (real) dollar spent on public investment always yields one dollar of public capital. He argues instead that spending one dollar typically yields only a fraction in actual public capital and, plausibly, that the growth impact of additional investment spending will be lower countries that are inefficient in this sense.

In the context of exploring the implications of this sort of inefficiency, we came across an initially puzzling theoretical invariance result: the growth impact of public investment spending is not higher in countries with a (permanently) higher level of public investment efficiency.[1] This seemed counterintuitive as well as being inconsistent with the conclusions of Pritchett (2000), among other papers. However, we have come to the view that this result is generally correct and that it has important policy implications.

The essential intuition for the invariance result comes from the fact that the marginal contribution of an additional dollar of investment spending to output can be broken down into a product of two components: the amount of capital actually installed and the marginal productivity of that capital. Low public investment efficiency implies that less than a dollar of capital is installed. However, a country with (permanently) low efficiency has been installing less capital forever and as a result has a lower public capital stock. With the standard assumption of decreasing returns to any one factor of production, this implies a higher marginal productivity of public capital. These two effects go in opposite directions in terms of the effect of additional investment spending on output. Indeed, for the standard Cobb-Douglas case, the effects exactly offset: high- and low-efficiency countries have the same growth impact from additional public investment spending.

The logic of the invariance result is powerful and fairly general. The result hinges on the idea that there is a declining marginal product to the sum of all reproducible factors (notably physical and human capital). Thus, our findings

[1] Berg et al. (2010) and Buffie et al. (2012).

are obtained under the standard exogenous growth model (i.e. Solow, 1956; Mankiw, Romer and Weil, 1992). We focus on exogenous growth models because (i) they are the workhorse of growth theory and empirics that explains important features of post-war growth, such as conditional convergence in levels of income across countries, and (ii) Pritchett explicitly states that his assessment of the effects of public efficiency is based on exogenous growth models (see his footnote 15).[2]

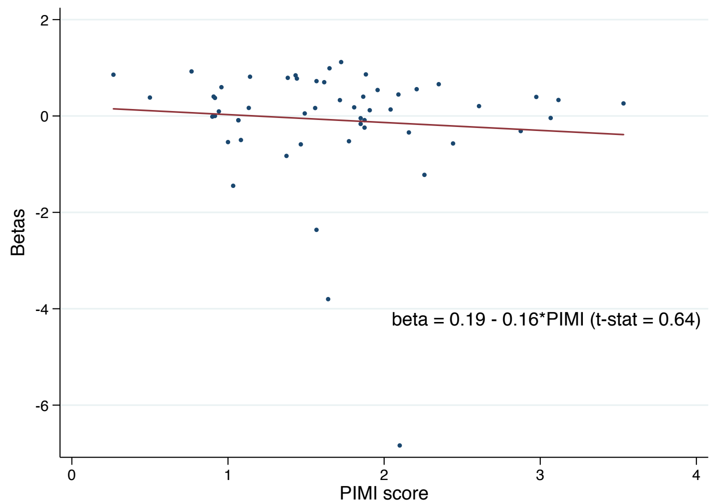
As should be clear by now, in this paper we subscribe to the concept of efficiency discussed in Pritchett (2000), Caselli (2005), and Gupta et al. (2014)—the ratio between the actual increment of public capital and the amount spent. This particular concept has been incorporated in macroeconomic models for developing economies such as those developed in Agénor (2014), Araujo et al. (2015), Berg et al. (2010, 2013); Berg, Yang and Zanna (2015), Buffie et al. (2012) and Melina, Yang and Zanna (2015), among others. There are, however, other approaches to modeling efficiency, taking into consideration, for instance, network effects (Agénor, 2010) or modeling explicitly rent-seeking bureaucracies (Chakraborty and Dabla-Norris, 2011).

This paper is essentially theoretical, but it may be useful to briefly examine the empirical relationship between measures of the output impact of public capital and measures of efficiency. We regress the log level of GDP per capita on the public capital stock country-by-country (controlling for private capital). The resulting coefficient is a measure of the growth effect of public investment from the supply side. Figure 1 (left hand side panel) shows a scatter of the estimated coefficients against the public investment management index (PIMI, see Dabla-Norris et al. (2012) for a discussion), a direct measure of investment efficiency calculated for for each country. As the figure shows, there is no significant correlation between the efficiency measure for the size of the growth impact.[3]

[2]In endogenous growth models the outcome depends on the structure of the model. The equilibrium growth rate is higher in countries with efficient public investment in a Barro-type model (Barro, 1990), but not necessarily in the models formulated by Lucas (1988); Manuelli and Jones (1990); Rebelo (1991). The results for growth on the transition path also appear to exhibit considerable variation. In preliminary research, we have found plausible cases in the Lucas model where growth on the transition path is continuously higher in the low-efficiency economy.

[3] Results vary, depending on the sample and the measure of "efficiency." IMF (2015) use an 'efficiency frontier' approach to map cumulative investment spending to a measure of the public capital stock that is itself a combination of a survey-based measure and an index of physical infrastructure. We find a positive but insignificant difference between the growth effect of efficient and inefficient countries according to this measure. (This result, available on request, differs from those in IMF (2015), because we allow for robust standard errors and we align the lag structure with Abiad, Furceri and Topalova (2015).) This insignificant effect nests a positive and significant growth impact of investment spending for a high-efficiency country, using the survey measure, and a *negative* and significant effect when using the physical infrastructure index. IMF (2014*b*) find a higher growth impact of public investment in high-efficiency countries in an advanced-country sample. The efficiency measure used is a survey-based index of the 'quality' of public infrastructure from the Global Competitiveness Report (GCR, Schwab, 2015). However, this index is meant to capture the effective *quantity* of infrastructure, not efficiency *per se*. Abiad, Furceri and Topalova (2015) update IMF (2014*b*) with a another survey measure of 'wastefulness of public spending' from the GCR and obtain a positive and significant effect of efficiency, in a similar sam-

Figure 1: Efficiency and the output impact of public capital stock




Notes: In the chart beta is the country-specific estimated coefficient obtained from a regression of the log of real per capita GDP on the log of the measured (i.e. unadjusted for efficiency) real public capital stock per capita, controlling for the log of real private capital stock per capita. The empirical model is estimated by the Common Correlated Effects Mean Group (CCEMG) estimator (Pesaran, 2006) on a balanced panel of 102 developing countries, with yearly data over the period 1970-2011. This estimator has been used in this context by Calderón, Moral-Benito and Servén (2015). GDP data are from the Penn World Tables (7.1), capital stock data are measured capital stocks (calculated as the discounted sum of investment spending), from Gupta et al. (2014). The chart report the betas and the corresponding values of the PIMI for 54 countries for which data on the PIMI are available (Dabla-Norris et al., 2012).

In this paper we derive and discuss the implications and limitations of the invariance result. We see it as much more than a technical point. Rather, much of the policy discussion about public investment scaling up has failed to note the trade-off between scarcity and efficiency, such that incorrect policy conclusions have been drawn. In section 2, we explore the economics of the issue with just two equations: a public capital accumulation equation and a Cobb-Douglas production function with only public capital as an input. In section 3, we explore some qualifications and extensions, including the addition of private capital and CES production functions. Section 4 concludes.

ple. Whether this survey question distinguishes the narrow conception of efficiency used in this paper from the broader question of the rate of return to investment spending is impossible to infer from the survey instrument.

## 2 The basic invariance result

Following Pritchett (2000), we define public investment efficiency as the ratio of the public capital actually installed to the amount of money spent on that capital. We capture this idea with the coefficient $\epsilon$ in the otherwise standard capital accumulation equation:

$$G_t = G_{t-1}(1-\delta) + \epsilon I_t^g, \tag{1}$$

where $G_t$ is the stock of public capital (or "infrastructure"), $\delta$ is the rate of depreciation, and $I_t^g$ is public investment spending. Efficiency $\epsilon$, typically implicitly to be 1, is assumed to take a value between 0 and 1 in Pritchett (2000) and related papers. Note that $\epsilon$ carries no time subscript and it is assumed to be time-invariant. This assumption is consistent with policy advice being often conditioned on the level of efficiency, with the idea that progresses in public investment efficiency take time (Pritchett, Woolcock and Andrews, 2013), and also with the fact that essentially all available measures of efficiency are time-invariant. In Section 3.5 we will relax this assumption to consider the growth effect of changes in efficiency.

There are various ways to think about efficiency here: (i) a fraction $\epsilon$ of spending may be redirected, e.g. misclassified as investment when it in fact covers transfers to civil servants ("corruption"); (ii) the costs of the project may be higher than they need to be ("waste"); (iii) the government may choose projects that yield a relatively low flow of capital services for the same investment spending ("poorly designed projects"); (iv) finally, governments may misallocate public investment spending across sectors or types of investment ("poor investment allocation").

As Appendix A discusses, the first three can be formalized as in equation (1). The algebra of the fourth is slightly different but has the same implications for the relationship between public investment and output growth. These four notions, which are not mutually exclusive, together yield a rich conception of inefficiency, despite the simplicity of the model: a country may choose the wrong mix of types of projects, and within types, it may choose especially wasteful or corrupt projects or ones where the service flow for a given dollar is relatively low.

It will be useful to define the measured public capital stock $G^m$, derived using the perpetual inventory method:

$$G_t^m \equiv \sum_{j=0}^{\infty}(1-\delta)^j I_{t-j}^g = \frac{G_t}{\epsilon}. \tag{2}$$

We now consider the simplest possible production function:

$$Y_t = A_t G_{t-1}^{\psi}, \tag{3}$$

where $Y_t$ is real output, $A_t$ is total factor productivity (TFP), and as before $G_t$

is the stock of public capital (or sometimes "infrastructure") at time t.[4]

It is straightforward to show that the growth effect of additional investment does not depend on the (constant) value of $\epsilon$. We start by noting that the production function (3) can be transformed by taking logs and differencing to to show that the growth rate of output is proportional to the percent change of the public capital stock:

$$\Delta y_t = a_t + \psi \Delta g_{t-1}, \tag{4}$$

where $a_t$ is the growth rate of TFP and small case $y$ and $g$ present natural logs of real output and the real public capital stock. Growth depends on the percent increase in the capital stock.

However, we also know from equation (2) that:

$$\Delta g_t = \Delta g_t^m. \tag{5}$$

That is, the percent change in the measured capital stock obtained under the assumption that $\epsilon = 1$ is the same as the percent change in the true capital stock. Thus, the value of $\epsilon$ does not matter for growth.

We can gain some intuition for this result by analyzing the marginal product of (public) capital (MPK). We can fix ideas by considering the steady-state analysis (though the results holds more generally, as we show later): in particular, $I^g$ is just adequate to sustain the capital stock and from equation (1):

$$G = \frac{\epsilon I^g}{\delta}. \tag{6}$$

We can then calculate the rate of return to an extra dollar of investment (permanently) as:

$$\begin{aligned}
\frac{dY}{dI^g} &= \overbrace{\frac{dY}{dG}}^{\text{MPK}} * \overbrace{\frac{dG}{dI^g}}^{\text{Capital per unit of investment spending}} && (7)\\
&= \psi \frac{Y}{G} * \frac{\epsilon}{\delta} && (8)\\
&= \psi \frac{Y}{\left(\frac{\epsilon I^g}{\delta}\right)} * \frac{\epsilon}{\delta} && (9)\\
&= \psi \frac{Y}{I^g}. && (10)
\end{aligned}$$

We can see that $\epsilon$ has two offsetting effects on the rate of return to investment. On the one hand, a higher $\epsilon$ lowers the marginal product of capital because it raises the stock of capital $G$ and hence lowers the capital output ratio. On the other, it raises the capital per unit of investment. With Cobb-Douglas, as above, the two effects exactly offset.

[4]We simplify by ignoring private capital and labor here, but this makes no difference to our main point, as we show in section 3.

Thus, and this is the main result of our paper, the effect of additional investment spending on the *growth rate* of output does not depend on the level of efficiency. From equation (10):

$$\frac{dY/Y}{dI^g/I^g} = \psi. \tag{11}$$

This *invariance* result is not a technical detail; rather, it speaks to different ways of thinking about public investment and development. One approach emphasizes the need to spend resources where they can be used well. Another emphasizes the need to invest where the need is greatest. The simple model above illustrates that both approaches have a point, that both need and efficiency matter. And it illustrates further that different levels of efficiency have two offsetting effects: one on the MPK and one on how much capital is built with a given expenditure.

## 2.1 Some Subtleties

It is easy to confuse the thought experiment of comparing countries with different (permanent) levels of efficiency with that of changing the level of efficiency at a point in time in a given country. *Changes* in efficiency in time have strong effects, as we will discuss below. But often we are interested in thinking about how high-efficiency and low-efficiency countries differ.

What we typically have in mind is that we want to compare two countries that are "otherwise the same" but differ with respect to efficiency. We cannot just change $\epsilon$, however; something else has to give to keep both sides of the production function equal. Different assumptions about what else changes result in different "thought experiments" about what we mean by comparing efficiency across countries. But we will see here that the invariance result holds in all cases.

To see this, we can substitute for output in our main result for the rate of return to investment (equation 10) using the steady-state production function:

$$Y = A\left(\frac{\epsilon I^g}{\delta}\right)^{\psi}. \tag{12}$$

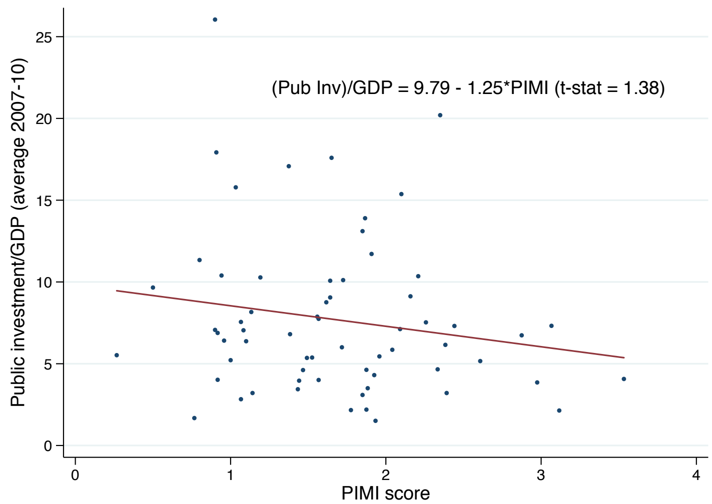
If two countries have different values of $\epsilon$, some combination of $Y$, $I^g$, and $A$ has to adjust.[5]

In section 2 above we implicitly followed what we think is the most natural approach of allowing the level of output to be lower in the lower-efficiency country while keeping investment/GDP the same.[6] This approach is implied by equation (10), which expresses the return to investment in terms of the investment/output share. This seems the most appropriate assumption for a

[5]We suppose that the parameters $\psi$ and $\delta$ are the same across the two cases.

[6]From equation (12), this implies that the ratio of the level of output in the two cases (call them $\epsilon_h$ and $\epsilon_l$) is equal to $\left(\frac{\epsilon_h}{\epsilon_l}\right)^{\psi/(1-\psi)}$, where the pair ($\epsilon_h$, $\epsilon_l$) are the levels of efficiency in the high- and low-efficiency countries, respectively ($\epsilon_h > \epsilon_l$).

Figure 2: Public investment spending and investment efficiency




Notes: The chart plots the average public investment-to-GDP ratio over the period 2007-10 (the same period used to calculate the PIMI) against the PIMI. The linear fit has a coefficient of $-1.25$, not statistically significant (p-value $= 0.17$).

variety of reasons. If, for example, public investment is financed by a value-added tax, then this tax rate would be the same in the two cases. It is also consistent with the empirical literature that focuses on investment *shares* as drivers of growth. Finally, it is also consistent with the empirical regularity that the ratio of investment spending to output is not correlated with efficiency (as measured by the PIMI) across countries (see Figure 2).

Another approach would be to allow the level of output to reflect the lower efficiency but keep the level of investment spending (not the output share) the same across the two cases.[7] To understand this case, we need to calculate the rate of return to an additional unit of investment not controlling for $I/Y$, as in equation (10), but $I$, obtaining:

$$\frac{dY}{dI^g} = \overbrace{\psi A \left(\frac{\epsilon I^g}{\delta}\right)^{\psi-1}}^{\text{MPK}} * \overbrace{\frac{\epsilon}{\delta}}^{\text{Capital per unit of investment}} . \tag{13}$$

We can see in this case that the two contrasting effects of different levels of $\epsilon$ are still present, but that they do not completely offset. Rather, the $MPK$ is inversely proportional to $\epsilon^{1-\psi}$, not $\epsilon$, and the net effect is that $\frac{dY}{dI^g}$ rises with $\epsilon$. What is going on here is that with investment the same across the two cases, the

[7]In this case, the ratio of the level of output in the two cases is equal to $\left(\frac{\epsilon_h}{\epsilon_l}\right)^{\psi}$.

investment share is higher when $\epsilon$ is low. And with the higher investment share comes a higher capital/output share than would be the case if, as in the first approach above, the investment share was the same across the two cases. This higher capital/output share implies a lower MPK than when the investment share is the same. However, equation (11) still holds, because the increase in output associated with the higher level of efficiency is exactly proportional to the higher level of output, so the *growth* effect of increasing investment is still invariant to efficiency.

A third possible "thought experiment" would be to consider what happens when we revise our assessment of a given country's level of efficiency, e.g. based on new evidence about the investment process. In this case it is natural to keep directly observed variables $Y$ and $I$ the same. With this approach, we are led to the view that $A$ (i.e. TFP) must be different in the two cases. We have revised down (for example) our assessment of the (permanent) level of efficiency. Given the observed investment spending data, we must lower our estimate of the public capital stock, and TFP must be higher to explain the observed levels of income and investment spending.

Stepping back, though, our main point holds whatever the thought experiment: the effect of additional investment spending on the *growth rate* of output does not depend on the level of efficiency.

## 2.2 Implications

Some policy-relevant corollaries follow from the main invariance result:

1. The assertion that the growth impact of public investment spending depends on $\epsilon$ is incorrect. Pritchett (2000), for instance, uses an equation like (1) to obtain an expression for the growth rate of public capital as:[8]

$$\Delta g_t = \epsilon \frac{I_t^g}{Y_t} \frac{Y_t}{G_t} - \delta.$$

Substituting into equation (4), he obtains:

$$\Delta y_t = a_t - \delta + \psi \epsilon \left( \frac{Y}{G} \right)_t \left( \frac{I^g}{Y} \right)_t .$$

He then argues that a regression of growth on the investment share creates an "identification problem", in that the estimated coefficient on the investment share is a combination of $\psi$ (the usual interpretation) and $\epsilon$. However, $G_t$ is unobserved. To create a regression based on observable data, $G_t$ needs to be replaced by $G_t^m = \epsilon G_t$, which means that $\epsilon$ drops out of the numerator and denominator. The "identification problem" is gone:

[8] We follow Pritchett (2000) in switching to continuous time here to simplify the algebra, and we abuse notation using $\Delta y_t = \frac{\dot{Y}_t}{Y_t}$ and $\Delta g_t = \frac{\dot{G}_t}{G_t}$.

$$\Delta y_t = a_t - \delta + \psi \left(\frac{Y}{G^m}\right)_t \left(\frac{I^g}{Y}\right)_t . \tag{14}$$

Footnote 16 of Pritchett (2000) states that "Using investment shares, one needs to divide by the capital/output ratio to recover the production function parameter, but the same lack of identification applies." However, because the capital stock in that equation is not measurable, that footnote actually applies to equation 14, where efficiency has dropped out.

Thus, controlling for the level of output and the history of investment spending, the output effect of additional investment spending depends on the production function parameter $\psi$ and the output/capital ratio (with capital measured as if efficiency is 1).

2. Empirical estimates of the rate of return to public investment in the literature should generally be understood as measuring the marginal product of *public investment spending*, not of an increment to public capital. In other words, even though they implicitly assume $\epsilon = 1$, their estimated rate of return includes the effects of inefficiency.

At first glance, this might be surprising, insofar an estimation of a neoclassical production function along the lines of equation (3), implicitly setting $\epsilon$ to 1 in measuring public investment and capital, produces an estimate of the production function parameter $\psi$ itself, uncontaminated by $\epsilon$.[9] As with the previous point, however, an analyst who ignores efficiency and calculates the gross marginal product of capital as $\psi \frac{Y}{G^m}$ will actually be calculating $\epsilon\psi\frac{Y}{G}$, which is the marginal product of public investment spending, already taking into account the effects of efficiency. Thus an analyst who looks at rate of return calculations such as the 15-30 percent range proposed in Dalgaard and Hansen (2015) and then factors in an additional inefficiency discount would be making a mistake.[10]

3. Efforts to infer efficiency from data on GDP growth and an assumed production function are misguided. Consider again Pritchett (2000), who notes that if $\epsilon$ were to equal 1, then:

$$a_t = \Delta y_t - \psi \Delta g_t^m.$$

He can measure $\Delta g_t^m$ and $\Delta y_t$ and assume a public capital share $\psi$ and deduce $a_t$. Arguing that $a_t$ is unlikely to be negative, he calculates a factor by which $\psi$

[9] This can be seen from equations (4) and (5). Of course we are putting various estimation issues aside such as endogeneity.

[10] Some of us have made such a mistake in some of our own calibrations, such as that in Buffie et al. (2012), despite having a brief appendix there on the topic of this paper. Of course, different sources for estimates of rate of return may or may not take into account efficiency. Project-specific analysis of rate of return may not, for example, depending on the nature of the estimate and the project, including whether the cost estimates incorporate the inefficiency. It seems plausible that they would if the inefficiency is related to "waste" or poor project selection, less clear if "corruption". Moreover, what we call the rate of return is the economy-wide increase in output associated with the project, holding other inputs constant. This may be hard to capture in a project-level analysis of rate of return.

must be scaled down to keep TFP growth positive. Assuming incorrectly that $\epsilon$ belongs in the growth equation (4), he asserts that that factor is a measure of $\epsilon$. In fact, though, equation (4) makes clear that low (time-invariant) efficiency cannot explain negative TFP growth. Alternative explanations for apparently negative TFP growth include very low values of $\psi$, time-varying $\epsilon$, or various mismeasurement issues. Alternatively, conflict, supply shocks, or policy reversals could after all imply negative TFP growth.[11]

4. TFP also does not matter for the output effect, once spending/GDP is controlled for. From equation (4), the level and change in TFP do not matter for the effect of public investment on growth, conditional on the level of output and the history of public investment spending.

5. "Efficiency-adjusted" capital stocks are generally uninformative for the analysis of growth. For example, Gupta et al. (2014) and IMF (2014*a*) construct public capital stock series using the perpetual inventory method, then *adjusted using the GCR 'quality of roads' index* (Schwab, 2015) as a proxy for efficiency. This quality index (as with other measures such as the PIMI) is time-invariant, so the adjusted infrastructure stock measures differ only by an equal percentage; the growth rates are identical.[12] That is, they are irrelevant for growth issues.

6. It is nonetheless true (as emphasized in Pritchett, 2000) that level decompositions of output into the contribution of public capital and TFP depend on $\epsilon$. Indeed, a low efficiency (and hence a capital stock) does imply an associated increase in the level of TFP, given the observed level of real income and history of investment spending. Consider the ratio of output levels of two countries $a$

[11]Pritchett (2000) recognizes the broader policy point that the MPK and $\epsilon$ effects are likely to offset when he notes (his footnote 19) that

> "The case of low efficacy is common in the developing countries reconciles a common paradox. For decades development "experts" have observed the lack of roads, power, schools, and health clinics and assumed that since the stock was so low, the marginal product of public-sector capital must be high and hence "more investment" was the appropriate answer. However, this has left a legacy in the poorest countries of large amounts of public-sector investment (often with official financing) but with little or now public-sector capital to show for it."

The "paradox" that is reconciled here is that public investment spending should generate very large growth effects, because the MPK is presumably high. But the analogous paradox would be that low-efficiency countries do benefit somewhat from public investment spending, despite wasting so much spending. And the reconciliation to both paradoxes is that it is necessary to take into account both that low efficiency makes the MPK high and that it means that spending generates only a little capital. Similarly, his discussion of the decision about whether to make a particular public investment recognizes the importance of both the MPK and efficiency, but he fails to note that likelihood of a general inverse relationship.

[12]That there is any time variation at all in the difference between 'efficiency-adjusted' and unadjusted capital stocks (with time-invariant efficiency measures) in IMF (2014*a*) is due to assumptions about how the initial capital stock is calculated and in particular whether the same efficiency measure is used for the initial capital stock as is used for subsequent accumulation. If the same efficiency value is used for the initial stock calculation and subsequent investment spending, there will be a constant percentage difference between the adjusted and unadjusted stocks.

and $b$ at time t. Output ($Y$) and investment spending ($I^g$ and the value of the associated measured capital stock $G^m$) are observed.

$$\frac{Y_t^b}{Y_t^a} = \frac{A_t^b \left(\epsilon^b G^{m^b}\right)^\psi}{A_t^a \left(\epsilon^a G^{m^a}\right)^\psi}. \tag{15}$$

Suppose for concreteness that $Y_t^b/Y_t^a = 2^{1/\psi}$ and both countries have the same history of investment spending captured by $G^m$ ($G^{m^a} = G^{m^b} = G^m$). From equation (15), an analysis (the "full efficiency case") that assumes $\epsilon = 1$ implies that $A_t^b/A_t^a = 2^{1/\psi}$: all the output difference between the two countries is due to TFP. If instead (the "different efficiency case") it is supposed that $\epsilon^b = 1$ and $\epsilon^a = 1/2$, then the two countries have the same TFP at time $t$ ($A_t^a = A_t^b$) and the output difference is due to the difference in the efficiency-adjusted capital stocks.

It is perhaps puzzling that the relationship between investment and output growth is invariant to the level of efficiency but the *level* of output and its decomposition into capital and TFP is not. But we can easily show that while, as we just saw, different assumptions about (time-invariant) efficiency imply different views about the source of the level differences at time $t$, they have no implications for the drivers of growth.

Suppose for example that some earlier time $t_0$ both countries had the same output level as had country $a$ at time $t_1$, so $Y_{t_0}^a = Y_{t_0}^b = Y_{t_1}^a$. In other words, only country $b$ grew between times $t_0$ and $t_1$. Assume also that $G^m$ was (observably) also identical for both countries at $t_0$. Take first the full efficiency case. It must have been the case that $A_{t_0}^b = A_{t_0}^a$, and all the growth in income in country $b$ between $t_0$ and $t$ must have been due to growth in TFP (i.e. $At_1{}^b/At_0{}^b$). Now, suppose instead that $\epsilon^a = 1/2$. This implies that $A_{t_0}^b/A_{t_0}^a = (1/2)^{1/\psi}$. And again all of country $b$'s output growth between $t_0$ and $t$ was due to TFP. Similar cases can be constructed for different assumptions about $G^m$ and so on, but in all of them the decomposition of growth between periods does not depend on $\epsilon$.

7. *Changes* in efficiency in time matter. For example holding historical efficiency constant, an increase in efficiency increases the output effect of public investment. There is no negative effect on the marginal product of public capital. For example increases might be associated with structural reforms or "investing in investing" (Collier, 2007). On the other hand decreases might result from investment surges that overwhelm administrative and implementation capacity. We will explore this issue below, see Section 3.5.

# 3 Qualifications and Extensions

In this section we examine several of the highly simplified assumptions above. While the exact offset we have observed in the invariance results does not always occur, the existence of largely offsetting effects is fairly general.

### 3.1 A CES Production Function

The previous analysis can be generalized to a CES production function that combines private capital ($K$) and public capital ($G$). As we will show, when $\eta > 1$ (the two inputs are substitutes), high $\epsilon$ implies a high growth effect of public investment spending, but that when the two inputs are complements, high $\epsilon$ implies a *low* growth effect.

For simplicity, assume that private capital is constant and satisfies $K = 1$. Then the production function at steady state reduces to

$$Y = A\left[\psi\,(G)^{\frac{\eta-1}{\eta}} + 1 - \psi\right]^{\frac{\eta}{\eta-1}}, \tag{16}$$

where $\eta$ is the elasticity of substitution. As is well known, when $\eta \to 1$ then the production function (16) becomes the Cobb-Douglas specification. Also when $\eta \to 0$, the production function (16) represents the Leontieff technology with inputs becoming perfect complements. On the other hand, when $\eta \to \infty$, inputs are perfect substitutes and the production function captures an "AK" technology, where the marginal product of public capital is constant.

We proceed as in the Cobb-Douglas case by calculating the return on public investment as the combination of two effects:

$$\frac{dY}{dI^g} = \frac{dY}{dG}\frac{dG}{dI^g}. \tag{17}$$

For the the production function (16), we have that the marginal product of public capital is

$$\frac{dY}{dG} = \psi A^{\frac{\eta-1}{\eta}}\left(\frac{Y}{G}\right)^{\frac{1}{\eta}}, \tag{18}$$

which is still decreasing in public capital. Moreover since it is still valid that

$$G = \frac{\epsilon I^g}{\delta}, \tag{19}$$

then

$$\frac{dY}{dG} = \psi A^{\frac{\eta-1}{\eta}}\left(\frac{\delta Y}{\epsilon I^g}\right)^{\frac{1}{\eta}}, \tag{20}$$

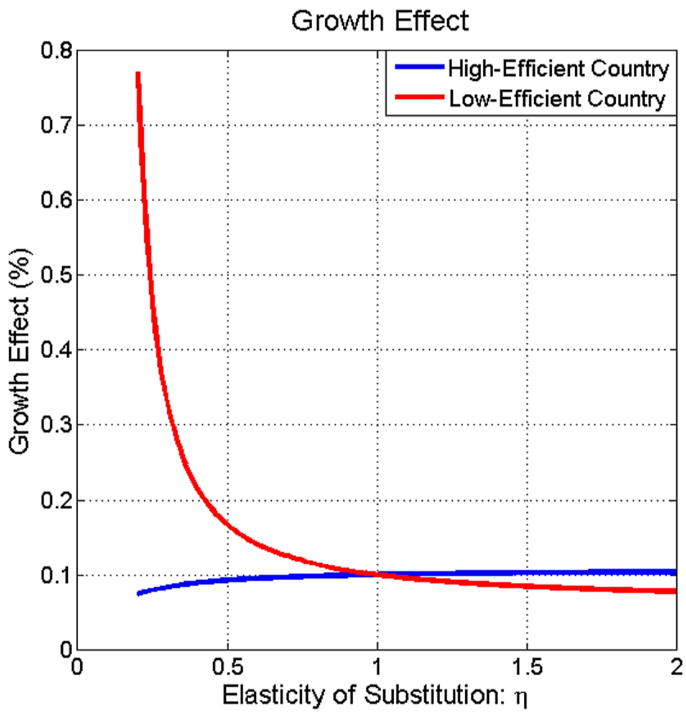
implying that the marginal product is still decreasing in efficiency.

As in the Cobb-Douglas case

$$\frac{dG}{dI^g} = \frac{\epsilon}{\delta}. \tag{21}$$

Combining this with equations (17) and (20) yields the following expression for the return on public investment.

Figure 3: The growth effect of additional investment spending in the CES case




Notes: authors' calculations (see text for details).

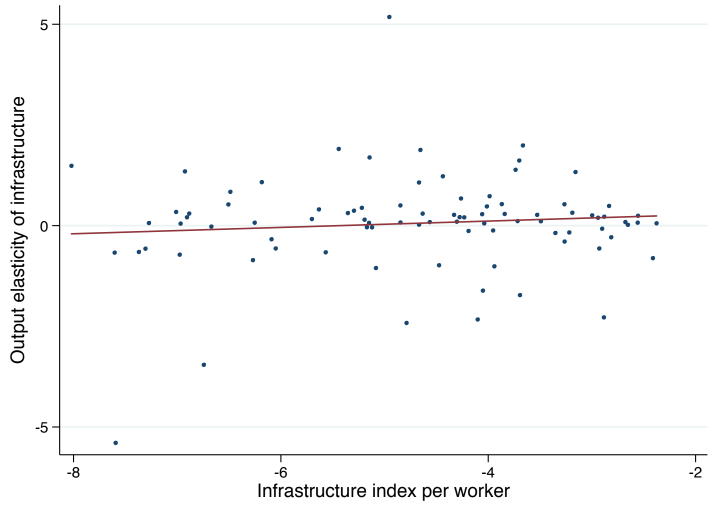
$$\frac{dY}{dI^g} = \psi \left(\frac{A}{\delta}\right)^{\frac{\eta-1}{\eta}} \left(\frac{1}{I^g/Y}\right)^{\frac{1}{\eta}} (\epsilon)^{\frac{\eta-1}{\eta}} . \tag{22}$$

Moreover as for the Cobb-Douglas case, we can rewrite this expression in terms of growth effects as follows

$$\frac{dY/Y}{dI^g/I^g} = \psi \left(\frac{AI^g}{\delta Y}\right)^{\frac{\eta-1}{\eta}} (\epsilon)^{\frac{\eta-1}{\eta}} . \tag{23}$$

This expression underscores the role of the elasticity of substitution $\eta$. As expected, if $\eta = 1$ then the output growth effect expression collapses to the expression for the Cobb-Douglas, which is independent on the level of efficiency. Instead, for $\eta > 1$, the growth effect is increasing in efficiency $\epsilon$. For the extreme case $\eta \to \infty$—the "AK" technology—the growth effect is directly proportional to efficiency. That is, $\frac{dY/Y}{dI^g/I^g} \approx \frac{\psi AI^g}{\delta Y}\epsilon$. On the other hand, for $\eta < 1$, the growth effect is *decreasing* in efficiency $\epsilon$!

Figure 4: The elasticity of output with respect to the true capital stock




Notes: Source is Calderón, Moral-Benito and Servén (2015). The capital stock is measured directly in terms of miles of roads etc. so it is a measure of $G$, not $G^m$.

To fix ideas and illustrate the role of the elasticity of substitution $\eta$, consider the following example. Assume there are two countries ($H$ and $L$) with the same TFP $A = 1$, productivity parameter $\psi = 0.1$, depreciation rates $\delta = 0.05$ and investment to GDP ratios $\frac{I^g}{Y} = 0.06$, but different levels of efficiency: $\epsilon^H = 0.9$ and $\epsilon^L = 0.5$. For each country, we can calculate and compare the growth effects of a one percent increase in investment, according to (23), for different elasticities $\eta$. Figure 3 illustrates this comparison. As can be seen, for the Cobb-Douglas case of $\eta = 1$, the invariance result implies that both countries, regardless of their level of efficiency, will have the same growth effect. For $\eta > 1$, the high-efficiency country will enjoy a higher growth effect than the low-efficiency country. In contrast for $\eta < 1$, the low-efficiency country will see a higher growth effect than the high-efficiency country.[13]

We choose to maintain $\eta = 1$ (Cobb-Douglas) as our baseline assumption, for two reasons. First, there is good evidence that in fact the elasticity of output with respect to the public capital stock is constant, that is it does not vary with the real capital stock (Calderón, Moral-Benito and Servén, 2015). In particular, the growth effect of an increment to the real public capital stock (measured as miles of roads etc. to avoid needing to think about efficiency) seems to be unrelated to the level of the public capital stock (Figure 4). This is a defining

[13]Interestingly, the calibrated CES production function in Eden and Kraay (2014) supports this case of higher complementarity between public and private capital. IMF (2014*b*, p. 78) suggests that complementary may be the more intuitive case: "…infrastructure is an indispensable input in an economy's production, one that is highly complementary to other, more conventional inputs such as labor and noninfrastructure capital."

feature of Cobb-Douglas (as in equation (3)). Second, when $\eta \approx 1$, the results are approximately Cobb-Douglas, so unless the deviation from Cobb-Douglas is large we are probably not making a major mistake.

### 3.2 Dynamics

The steady-state analysis has helped provide a simple intuition of the invariance result, but this result also holds in a dynamic setup. To show this, consider the continuous-time versions of equations (1) and (3):[14]

$$Y_t = AG_t^{\psi} \tag{24}$$

and

$$\dot{G}_t = \epsilon I_t^g - \delta G_t. \tag{25}$$

For simplicity assume that, at $t = 0$, public investment increases from the constant initial level $I_0^g$ to the new level $I_1^g$ and stays there forever, with $I_1^g > I_0^g$ . Using equations (24) and (25), it is possible to express the growth rate of this economy by

$$\frac{\dot{Y}_t}{Y_t} = \psi \left[ \frac{\epsilon I_1^g}{G_t} + 1 - \delta \right]; \tag{26}$$

while solving (25) for $G_t$ yields

$$G_t = (G_0 - G_1)\, e^{-\delta t} + G_1,$$

where $G_0 = \frac{\epsilon I_0^g}{\delta}$ and $G_1 = \frac{\epsilon I_1^g}{\delta}$ are the initial and the new steady-state public capital stocks. Substituting this last equation into (26) gives

$$\begin{aligned} \frac{\dot{Y}_t}{Y_t} &= \psi \left[ \frac{\delta \epsilon I_1^g}{(\epsilon I_0^g - \epsilon I_1^g) e^{-\delta t} + \epsilon I_1^g} - \delta \right] \\ &= \psi \left[ \frac{\delta \left( I_1^g / I_0^g \right)}{\left( I_1^g / I_0^g \right) \left( 1 - e^{-\delta t} \right) + e^{-\delta t}} - \delta \right], \end{aligned} \tag{27}$$

which implies that the growth rate at every point in time is independent of efficiency—the invariance result.[15]

[14] The following results also hold in discrete time. However, continuous time allows to derive simpler analytical expressions, while conveying the same message.

[15] In the long run ($t \to \infty$), the growth rate $\frac{\dot{Y}_t}{Y_t}$ tends to zero, since we assumed no technological progress–$\frac{\dot{A}_t}{A_t} = 0$. Moreover, the largest impact on growth occurs at $t = 0$, when investment jumps from $I_0^g$ to $I_1^g$ and growth corresponds to $\frac{\dot{Y}_t}{Y_t} = \psi \delta \left( \frac{I_1^g}{I_0^g} - 1 \right)$.

### 3.3 The Role of Private Capital

Adding private capital as well as intertemporal consumption decisions do not affect the invariance result. However, we will take the time to demonstrate this in an extension of the above dynamic model — the Cass-Koopmans-Ramsey model. We do so partly to illustrate a a further way in which we should be careful about how we think about "efficiency" and examine it empirically. In particular, we note that low efficiency may be associated in the cross-country data with more poorly-functioning private capital markets. In this case, the crowding in of private investment may be slower in "inefficient" countries (because of the poor private capital market performance). In a dynamic setting, this can affect the dynamics of output in response to a public investment shock.

Consider the following simple, one-sector, neoclassical growth model of a closed economy with perfectly competitive markets. The economy grows at a zero exogenous rate in the long run. The representative consumer owns private capital and maximizes intertemporally the utility that she derives from consumption $C_t$. Her preferences over consumption are logarithmic, so the intertemporal elasticity of substitution is one. The supply of labor $L_t$ is inelastic and equal to $1$. Firms hire capital $K_t$ and labor, which together with public capital $G_t$, are combined to produce output $Y_t$. The government invests $I_t^g$ units in public capital and finances it with lump-sum taxes $\mathcal{T}_t$.

The dynamics of this economy can be described by the following reduced set of equations: (i) the Cobb-Douglas production function that, besides public capital, includes private capital $K_t$

$$Y_t = A\left(G_{t-1}\right)^{\psi}\left(K_{t-1}\right)^{\alpha}\left(L_t\right)^{1-\alpha}, \tag{28}$$

where $A$ is a constant technology parameter (in equilibrium $L_t = 1$); (ii) the private capital accumulation equation

$$K_t = (1-\delta)K_{t-1} + I_t, \tag{29}$$

where $I_t$ corresponds to private investment; (iii) the Euler equation

$$\frac{1}{\beta}\left(\frac{C_{t+1}}{C_t}\right) = \alpha\frac{Y_{t+1}}{K_t} + 1 - \delta, \tag{30}$$

which implies that the marginal rate of substitution between consumption at times $t$ and $t+1$ is equal to the marginal rate of transformation, from production, between consumption at times $t$ and $t+1$; (iv) the public capital accumulation equation that accounts for investment inefficiencies

$$G_t = (1-\delta)G_{t-1} + \epsilon I_t^g; \tag{31}$$

and (v) the resource constraint of the economy

$$Y_t = C_t + I_t + I_t^g. \tag{32}$$

We focus first on the steady-state analysis and show analytically that our invariance result is still valid. Using equations (28) and (30) at the steady state we can derive

$$K = \Phi G^{\frac{\psi}{1-\alpha}}, \tag{33}$$

where $\Phi$ is a composite parameter that depends on $\beta$, $\alpha$, $A$, and $\delta$, satisfying $\Phi > 0$. This expression makes explicit the (long-run) crowding-in effect on private capital—increases in public capital raise the marginal product of private capital, stimulating more private capital accumulation. This effect depends on $\frac{\psi}{1-\alpha}$.

By substituting (33) into the steady-state version of (28) yields

$$Y = \Gamma G^{\frac{\psi}{1-\alpha}},$$

with $\Gamma$ being another composite parameter that depends on $\beta$, $\alpha$, $A$, and $\delta$. Using this and following the same steps described in section 2, it is possible to derive the following invariance result:

$$\frac{dY/Y}{dI^g/I^g} = \frac{\psi}{1-\alpha}.$$

Comparing this with equation (11) reveals that the impact of a percentage increase of public investment on growth now accounts for the crowding-in effect on private capital. However, this impact is still independent of the level of efficiency.

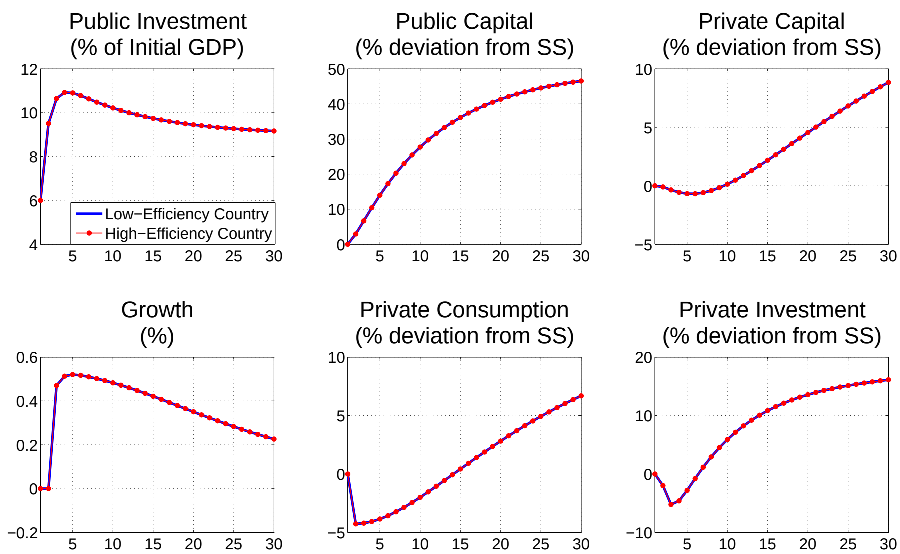
Outside of the steady state, we rely on numerical simulations of this model to pursue the dynamic analysis and explore the role of efficiency for the link between public investment increases and growth. For illustrative purposes, we focus on a scenario in which the path for public investment, in percent of initial GDP, is hump-shaped. It starts at 6 percent, then increases to almost 11 percent by the third year and finally tapers off gradually to a permanent new level of 9.3 percent (Figure 5). The calibration of the model is broadly in line with that of Buffie et al. (2012) for an average low-income country.[16] In what we will call the low-efficiency case, we set the efficiency parameter $\epsilon$ equal to $0.5$, in line with estimates by Arestoff and Hurlin (2010). We will compare this to a high-efficiency case where $\epsilon = 1$.[17]

In this neoclassical growth model, for the same increase in public investment, a high-efficiency country will enjoy the same growth benefits as those

[16]The time unit is a year and the discount factor $\beta = 0.94$. The initial infrastructure investment is set to be equal to 6 percent of GDP, which is close to the average for LICs in SSA reported by Briceño Garmendia, Smits and Foster (2008). The capital's share in value added corresponds to $\alpha = 0.5$ and the depreciation rates are set as $\delta = 0.05$. Lastly, the elasticity of output with respect to public capital $\psi$ is set to match a rate of return on public capital (net of depreciation) of 25 percent for the low-efficient country, which falls in the range of estimates provided by Briceño Garmendia and Foster (2010) for electricity, water and sanitation, irrigation, and roads in SSA.

[17]The numerical simulations track the global nonlinear saddle path. The solutions were generated by set of programs written in Matlab and Dynare 4.3.2. See http://www.cepremap.cnrs.fr/dynare.

Figure 5: Impulse Responses for High and Low-Efficiency Countries




Notes: authors' calculations (see text for details).

of the low-efficient country—the invariance result. This is confirmed in Figure 5, where the responses for the low and high-efficiency cases are indistinguishable (we present the results for comparison with some further results below). This scaling up of public investment translates into more public capital and growth—GDP increases by $0.5$ percent, on average, in the first $10$ years. Note, however, that private consumption and investment are crowded out, as taxes have to increase to finance the public investment scaling-up. As a result, resources are shifted away from the private sector to the public sector, as discussed in Buffie et al. (2012).[18] This has important consequences for the delay in the crowding-in effect on private capital as well as for the growth effects—if the public investment increase were to be financed with external resources, such as aid, the crowding out effects would be dampened and the effects on growth magnified.

We will now consider the role played by distortions that could slow the responsiveness of private capital formation to the relative price. We have in mind capital market imperfects and other policy or institutionally-induced frictions, which we will model, as a short-cut, by considering that they raise the cost of adjusting the private capital stock. Specifically, imagine now that the economy

[18]For these impulse responses, we keep TFP the same across the two cases and let initial income be lower in the low-efficiency case, so the initial public investment/GDP shares are the same. We then compare shocks that correspond to the same percentage increase in investment in the two cases.

faces some quadratic adjustment costs of the following type:

$$AC_t = \frac{\nu}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1},$$

where the parameter $\nu$ measures the severity of these costs. Note that the case of $\nu = 0$ embeds our previous model. Private capital adjustment costs will affect then the return on private investment as well as the resource constraint of the economy—some output will be allocated to cover these costs.[19] However, as long as both low-efficient and high-efficient countries face the same severity of these costs, as captured by an identical $\nu$, the growth effects of increases in public investment will still coincide.[20]

Suppose now that countries vary as to their adjustment costs $\nu$. Think about for example how the installation of a good road may have the same marginal product (holding private factors fixed) in two countries, one with relatively underdeveloped private capital and asset markets, the other with institutions that allow private agents to increase investment in particular sectors more easily. This difference in $\nu$ matters for cross-country differences in the dynamic (but not long-run) effects of public investment on growth, as confirmed in Figure 6, where, maintaining the same level of efficiency ($\epsilon = 0.5$), we compare the effects of having severe private capital adjustment costs ($\nu = 10$) versus the effects of no adjustment costs ($\nu = 0$).[21]

It should not be surprising, however, that cross-country variation in measures of efficiency are correlated with broader measures of institutions.[22] In this case, a finding that "efficiency" seems to matter might be capturing the weaker crowding-in due to dysfunctional private capital markets. Of course the policy implications are quite different.

## 3.4 Waste or Corruption

What is done with the inefficient part of investment spending may matter for some macroeconomic outcomes. So far, we have assumed that the portion of

[19] Formally, with these private capital adjustment costs, the budget constraint (32) becomes

$$Y_t = C_t + I_t + AC_t + I_t^g,$$

and the Euler equation (30) changes to

$$Q_t\left(\frac{1}{\beta}\right)\left(\frac{C_{t+1}}{C_t}\right) = \alpha\frac{Y_{t+1}}{K_t} + Q_{t+1}(1-\delta) - \frac{\partial AC_{t+1}}{\partial K_t},$$

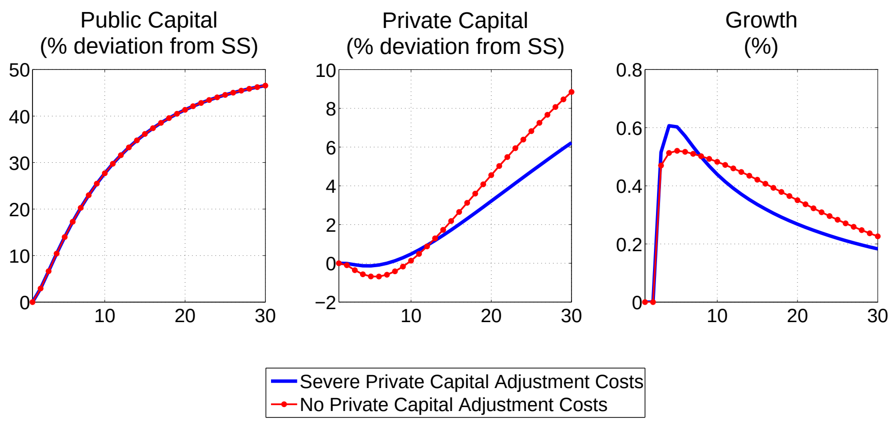
where $Q_t = 1 + \nu\left(\frac{I_t}{K_{t-1}} - \delta\right)$ is Tobin's $Q$.

[20] Simulation results are available from the authors upon request.

[21] In the figure, private capital remains higher in the low-adjustment-cost case after 30 years; eventually, though, the two lines converge.

[22] The PIMI and the ICRG measure of "institutions" (a composite indicator of the political, economic and financial risk of a country) have a correlation coefficient of 30 percent on a sample of 50 countries, with ICRG scores averaged over the same 2007-2010 period for which the PIMI is calculated.

Figure 6: Impulse Responses for High and Low Private Adjustment Costs




Notes: authors' calculations (see text for details).

spending that does not translate into public capital is non-productive spending ("waste"), i.e., real resources are used up with zero rate of return. But we can also consider a "pure corruption" case, where the portion of spending that does not translate into public capital is transferred as a lump sum back to households.[23]

It turns out that this distinction may matter for the real effects of increasing public investment spending, because of the interaction between the resource constraint of the economy and the behavior of private agents. Without private capital accumulation, the growth effects will be identical for countries with different efficiencies (Figure 7). This is not surprising, given that the only factor of production that can be accumulated is public capital. The supply side of these countries is similar to the one described by equations (1) and (3) above and, more importantly, is not affected directly or indirectly by the corrupt lump-sum transfer to households (which is $(1-\epsilon)I_t^g$).

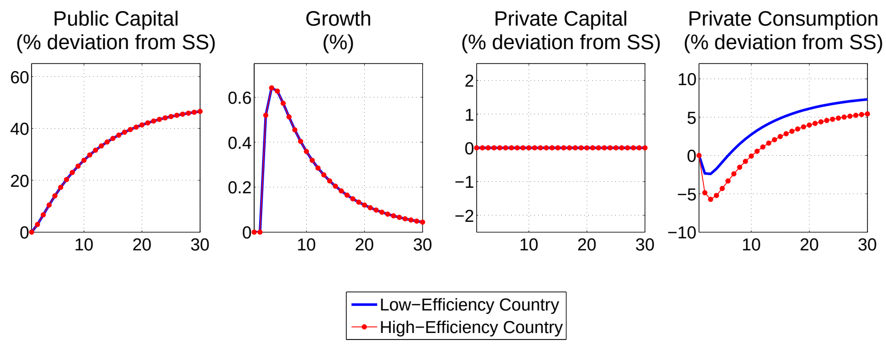
Consumption, however, increases more in the low-efficiency country than in the high-efficiency country. This is another perhaps surprising result. But consider two countries with the same level of income but different levels of efficiency. The low-efficient country must have higher TFP for them to have the same level of income. Now, when both increase investment spending by say 1 percentage point of GDP, they get the same increase in output. But the inefficient (but more highly productive) country gets this output increase with an smaller amount of actual productive (but costly) investment spending, trans-

[23] In this case, the resource constraint of the economy becomes

$$Y_t = C_t + I_t + \epsilon I_t^g,$$

since now the government transfers $(1-\epsilon)I_t^g$ to households.

Figure 7: Corruption: Impulse Responses Without Private Capital Accumulation




Notes: authors' calculations (see text for details).

ferring a larger amount back to the private sector in the form of "corruption". Thus it can increase consumption more.[24]

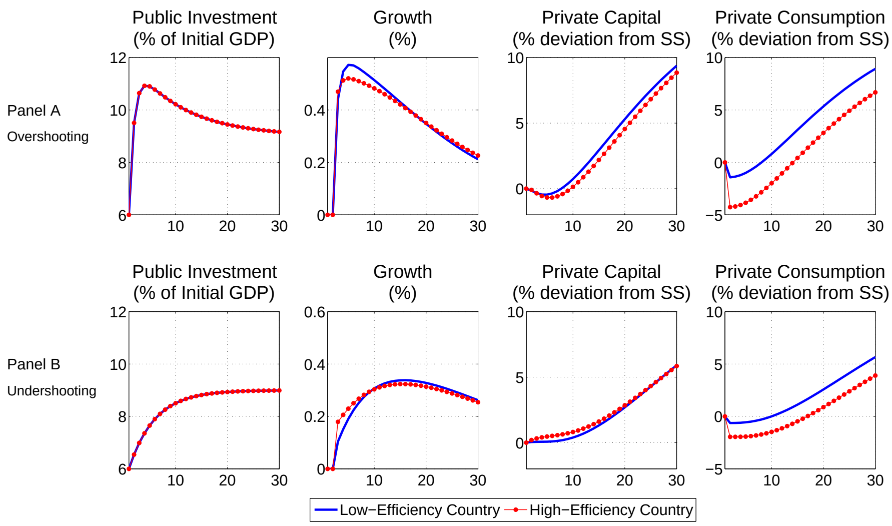
Even with private capital accumulation, the long-run effect of the investment scaling up on capital and output is invariant to efficiency, as before. In the long run, whatever the level of efficiency, households will consume the extra income, as the private investment rate remains determined by the marginal product of private capital (equation (33) still holds). Growth and private investment may differ, however, on the transition path.

To see this, return to the national budget constraint: $C + I = Y - \epsilon I^g$. When $I^g$ jumps immediately to its new steady state (call it $I^{g*}$), the constant term $\epsilon I^g$ acts like a decrease in permanent income. In this case, the impact is entirely absorbed by consumption, with no effect on the paths of private investment and growth. The invariance result holds. Suppose next that $I^g$ overshoots $I^{g*}$ at t=0 and then declines monotonically to $I^{g*}$. The income losses associated with the path of $\epsilon I^g$ decrease steadily over time. Importantly, the path is more steeply sloped in the high-efficiency economy. The incentive to smooth consumption by temporarily decreasing private investment is therefore greater in the high-efficiency economy. this produces the outcome shown in Panel A of Figure 8: less crowding out of private investment and higher growth in the medium (but not the long) run in the low-efficiency economy. Conversely, the output results favor the high-efficiency economy when public investment undershoots its long-run level (Panel B of Figure 8); in fact, high efficiency interacts with gradual scaling up to produce continuous crowding in of private investment along the adjustment trajectory.

To summarize, when low efficiency is due to corruption rather than waste, there is a stronger consumption case for increasing public investment, simply

[24]This is worked out in the steady state in Appendix B, which also shows that the result is not dependent on this particular characterization of the initial steady state.

Figure 8: Corruption: Impulse Responses With Private Capital Accumulation and Comparing Overshooting versus Gradual Increase in Public Investment




Notes: authors' calculations (see text for details).

because extra waste is no longer a cost of scaling up. Of course, corruption has presumably negative distributional and other consequences we do not consider. With respect to output, our invariance result largely holds. There are some potential differences in the output response during the transition to a new higher level of investment, but these differences can go either way, depending on the dynamics of the increase in public investment, and are quantitatively small in either case.

## 3.5 Investing in Investing

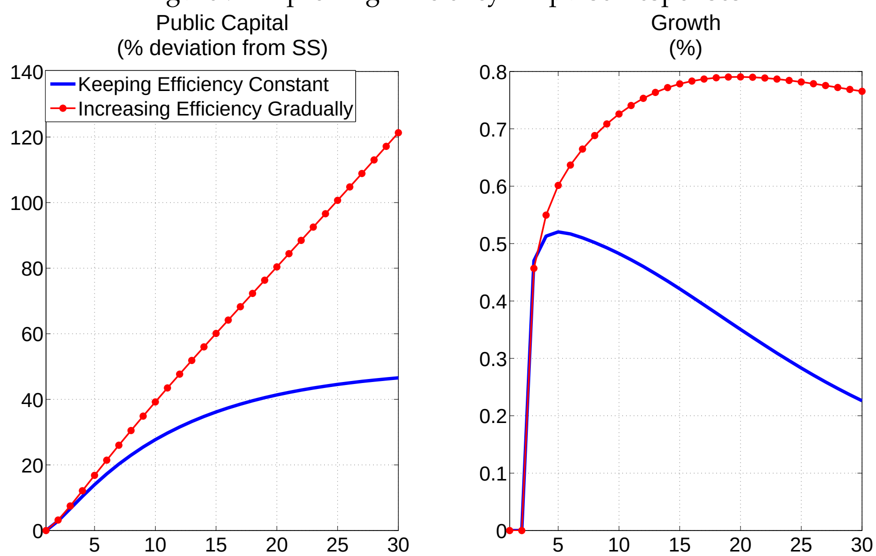
The conclusion that infrastructure investment increases growth the same amount in efficient and inefficient countries does not mean that efficiency is unimportant. Quite the contrary. Time-varying efficiency does matter for growth. In particular, what we can call "relative" efficiency matters.[25]

In equation (27), we calculated the dynamic response of output due to a move from some initial investment level $I_0^g$ to a new level $I_1^g$. If the investment paths $I_0^g$ and $I_1^g$ were associated with different efficiencies $\epsilon_0$ and $\epsilon_1$, respectively, then the growth rate of the economy would correspond to

$$\frac{\dot{Y}_t}{Y_t} = \psi \left[ \frac{\delta \left(\epsilon_1/\epsilon_0\right) \left(I_1^g/I_0^g\right)}{\left(\epsilon_1/\epsilon_0\right) \left(I_1^g/I_0^g\right) \left(1 - e^{-\delta t}\right) + e^{-\delta t}} - \delta \right]. \tag{34}$$

[25]See Berg et al. (2010, 2013) for a discussion.

Figure 9: Improving Efficiency: Impulse Responses




Notes: Same simulation as in Figure 5, except for the assumption of increasing efficiency.

Intuitively, the increase in efficiency to $\epsilon_1$ raises the impact of investment $I_1^g$ on growth because there is no offsetting effect on the scarcity of capital.[26]

This effect can be large, as shown in Figure 9, using the previous neoclassical growth model of Section 3.3. When efficiency $\epsilon$ gradually improves from $0.5$ to $1$ over a period of $30$ years, more public and private capital will be accumulated, raising growth. The opposite also holds, of course, when efficiency gradually decreases. This is relevant if one is of the view that, because of absorptive capacity constraints, public investment scaling-ups may have a negative impact on efficiency in developing countries.[27]

One way to think about these results is in terms of the concept of 'investing in investing' (Collier, 2007), in this context investing in investment efficiency. The rate of return to increased spending on raising efficiency may be higher—possibly much higher—than on raising the level of investment spending.

There are various ways to think specifically about investing in investing in our framework. At one extreme, we may consider that raising $\epsilon$ permanently requires a permanently higher level of spending on activities such as project analysis and selection—think for example that the government needs a certain

[26]To underscore, the "increase" here is in time in a given country. In the rest of the paper when we compare low- and high-efficiency countries, these are eternal differences across two cases, the comparison that is more relevant when we compare "low-efficiency" and "high-efficiency" countries using a cross-section indicator of efficiency.

[27]Berg et al. (2013) model absorptive capacity this way.

number of engineers to properly evaluate projects.[28] In this case, we can do some very simple calculations. For example, if it costs less than one percent of GDP per year (on the face of it a huge amount to pay for managing the public investment process) to raise the level of efficiency from $0.5$ to $0.6$, this would have a higher payoff than raising the rate of public investment from 5 to 6 percent of GDP.

Perhaps a more interesting way to think about investing in investing, though, is an investment in knowledge and institutions to manage public investments, which once built could be maintained at low cost. Suppose, to be concrete, that the rate of return to infrastructure is initially 20 percent and efficiency is 0.60, so the rate of return to investment spending is 9.8 percent. Suppose further that it requires an expenditure of 0.33 percent of GDP for three years to raise efficiency permanently to 0.65. The rate of return on this spending depends on the level of public investment spending—the bang for the buck is higher when the increased efficiency applies to a higher rate of investment spending. For a rate of public investment spending of 8 percent of GDP, the internal rate of return on this investing in investing would be 24 percent, much higher than the return to additional investment spending itself. Even if efficiency increases to just 0.62, the rate of return is a remarkable 15.5 percent.[29]

Two conclusions seem warranted: (i) if efforts to increase investment efficiency do in fact increase efficiency, then even large amounts of money (at least in terms of typical technical assistance budgets) would be well spent; and (ii) the return goes up with the size of investment scaling up.

At this point, unfortunately, these results are only suggestive. In particular, we need empirical evidence on whether, and how much, $\epsilon$ increases in response to spending on investment efficiency. Pritchett, Woolcock and Andrews (2013) argue that many developing countries have made little progress in state implementation capacity, and that "short-term programmatic efforts to build administrative capability in these countries are thus unlikely to be able to demonstrate actual success." On the other hand, increasing $\epsilon$ from 0.60 to 0.62 might look like "slow progress", but an investment that earns 15.5 percent qualifies as an "actual success."[30]

# 4 Conclusion

Our main result is that in a simple benchmark model, cross-country differences in the level of public investment efficiency do not matter for the growth impact of increases in public investment spending. This is no mathematical curiosity or technical detail. Countries are poor for many reasons, and this paper discusses two important ones: the scarcity of public capital, and the weak institu-

[28] An important corollary of this way of thinking is that efficiency is likely to fall if the investment rate increases, a notion of absorptive capacity limitations discussed briefly above and in Berg et al. (2013).

[29] Further calculations are available upon request.

[30] IMF (2015) emphasizes this point and provides a comprehensive analysis of the issues.

tions that make it difficult to convert public investment spending into usable public capital. As we show, public capital scarcity and inefficiency are likely to be inversely related, and this has important implications for policy. Most importantly, blanket recommendations that inefficient countries will likely see lower growth impact public investment spending (as in Pritchett (2000) and IMF (2014*a*)) need to be reconsidered.

The exact invariance result depends on the Cobb-Douglas specification for the production function. However, it is not a knife-edge result; rather, it is approximately true if the production function is Cobb-Douglas, as the empirical evidence suggests (Calderón, Moral-Benito and Servén, 2015). Moreover, insofar as public capital is highly complementary to private factors of production (as argued for example in IMF, 2014*a*), then the standard intuition and the results are overturned: investment spending in relatively low-efficiency countries would have a relatively *large* effect on real output growth, if they have the same investment-spending/output ratios. Moreover, careful treatments of private capital, adjustment costs, and different definitions of inefficiency do not change this broad conclusion.

We are not saying that low-efficiency countries should necessarily increase public investment. No would we say that high efficiency countries can expect higher output effects of increased investment. Ultimately, there is no short-cut: the merits of additional public investment spending in a particular case will depend on the marginal product of the resulting capital, efficiency, the cost of financing, the "fiscal space" and more generally the discretionary effects of taxation required to finance the investment, the prospects for and costs of required operations and maintenance, and the risks of debt distress, among other factors.[31]

Much of this discussion seems to presume that efficiency can be measured. In practice, though, this is difficult. The nonetheless useful PIMI cannot readily be mapped into $\epsilon$ quantitatively; we can safely assume only that it is monotonically related. Moreover, it is available only in cross-section. A promising way forward could be to compare physical indices of public capital stocks (miles of road etc.) against cumulative investment spending (Arestoff and Hurlin, 2010).

It is critical to distinguish between levels and rates of change of efficiency. Low levels of efficiency are worse than high for the level of output and for welfare, for any level of public investment. Much policy-related work focuses on the potential for increasing efficiency. And indeed, the rate of return to "investing in investing" to increase efficiency could be very high. But much discussion and most measures of inefficiency are static, and in these cases the lessons of this paper need to be kept closely in mind. Moreover, evidence from Pritchett, Woolcock and Andrews (2013) and Allen (2009) suggests that we may expect changes in public investment efficiency to be slow, so waiting for them to occur may not be a viable strategy in some cases.

[31] Buffie et al. (2012) emphasize the interaction of public investment/growth linkages with the fiscal reaction function, absorptive capacity, and other LIC-specific features; Adam and Bevan (2014) explore the role of distortionary taxation and operations-and-maintenance spending in conditioning the growth impact of public investment spending.

# Appendix A: Project Selection

In the main text we discuss four ways to think about public investment efficiency: (1) a fraction $\epsilon$ of spending is literally wasted ("corruption"); (2) the costs of the project are higher than they need to be, e.g. because of an inefficiently high use of inputs ("waste"); (3) government may choose projects that yield a greater or lesser flow of capital services for the same investment spending ("poorly designed projects"); and (4) governments may misallocate public investment spending across sectors or categories of investment ("poor investment allocation"). In this appendix we show that all four conceptions of efficiency have similar implications for the public investment/growth relationship.[32]

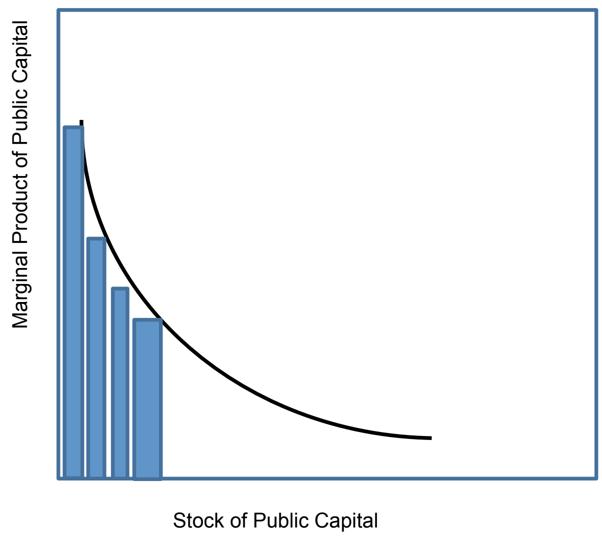
To understand public investment efficiency, it is tempting to imagine that all the available public investment projects at a given point in time can be ranked from highest to lowest rate of return. The marginal product of public investment is then the return of the best project available (Figure A-1). In a fully efficient investment process, when an additional dollar is spent, the next best project is chosen. With inefficient project selection, infra-marginal projects are chosen, resulting in a lower overall growth impact.

This notion is static, however, and thus potentially misleading: in general, the rate of return on one project will depend on the size of the capital stock that is already in place. The usual formulation of the public capital stock as the discounted sum of public investment implicitly assumes that all public capital goods are perfect substitutes. In this case, the downward slope of the schedule in Figure A-1 represents not the variety of available projects but simply the fact that capital becomes less productive as it becomes less scarce, as for example in a standard Cobb-Douglas production function. Each of our definitions of inefficiency, however, can be thought of in terms of poor project choice, in different ways.

The first two definitions are identical in terms of the basic equations of section (2), equations (1) and (3): only a fraction $\epsilon$ of the spending makes its way into public capital $G$, though we show in section 3.4 what is done with the $1 - \epsilon$ spending can matter for the general equilibrium outcome. What these definitions mean for project selection is relatively straightforward. If projects differ according to the degree of waste or corruption, then one can think of the schedule in Figure A-1 as measuring the amount of capital produced for given amount of spending. While the figure cannot readily capture the dynamics or even the steady state, the height of the curve would depend on the capital/output ratio, so the selection of more efficient projects would shift the

[32] There are other definitions of "efficiency" in the related literature. Hulten (1991) defines "efficiency" as the ratio between the amount of investment carried out some time in the past and the amount that would be needed now to provide equal productive capacity. When $\epsilon$ is equal to 1, this is related to the depreciation rate. In contrast, Hulten (1996) defines efficiency as the fraction of the capital *stock* that is available for productive use. This is a useful concept that is related to operations and maintenance expenditure and is also discussed in Adam and Bevan (2014). It is complementary to the concept analyzed in this paper. However, for current purposes it is worth noting that it is indistinguishable from TFP at the macroeconomic level.

Figure A-1: The MPK Schedule




Notes: Each vertical bar represents a hypothetical public investment "project", organized from highest to lowest. The marginal product is measured by height of the curve.

curve down more.

According to efficiency definition (3), different investment projects create capital that yields a greater or lesser flow of public capital services to the economy. So one dollar spent on a "bad project" is one that yields as much public capital as a good project, but the service flow from that project is lower by a factor of $\epsilon$.

Let the infrastructure stock be the sum of spending, discounted for depreciation, denoted $G^m$ and defined as in equation (2). The flow of infrastructure services from this stock depends on how well the particular projects were chosen and is equal to $\epsilon G^m$. Output then depends on this service flow:

$$Y = A\left(\epsilon G^m\right)^{\psi}.$$

Notice that this is exactly the same as what we get by discounting investment spending by $\epsilon$ and putting effective capital $G$ in the production function, as we do in the main text. Thus, all the results from the main text go through, reinterpreted. As before, an inefficient country (one that chooses more bad projects in this sense) does have a lower level of output, but it also has a higher marginal product of service flow (MPSF). If it always tends to choose inefficient projects, the growth impact of subsequent investments will be the same as in the country that has been choosing high-service-flow projects all along. In terms of Figure A-1, again a country could choose infra-marginal projects, but which projects it chooses would influence the overall height of the line through the scarcity of the service flow from public capital.

Finally, the fourth definition of inefficiency gets at the notion that projects differ in a more fundamental sense. In particular, different projects produce different types of public capital that are not perfect substitutes (nor are the service flows from these different stocks capital perfect substitutes). Suppose, for simplicity, there are two types of public capital, $G$ and $H$. These could be physical infrastructure and human capital, or they could represent roads in the south and roads in the north–the results generalize to any number of types of public capital. In this context, bad project choice is choosing the wrong type of project.

To be concrete:

$$Y = AG^{\psi}H^{\phi}. \tag{A-1}$$

And there are two associated capital accumulation equations:

$$G_t = (1-\delta_G)G_{t-1} + \epsilon_G I_G \tag{A-2}$$

and

$$H_t = (1-\delta_H)H_{t-1} + \epsilon_H I_H. \tag{A-3}$$

where $I_G$ is public investment spending on project type $G$. We can allow $\epsilon$ and $\delta$ to differ across types.

Now, to model project choice, let $\theta_G$ be the share of total investment spending going to projects of type $G$. So:

$$I_G = \theta_G I \text{ and } I_H = (1-\theta_G)I. \tag{A-4}$$

In steady state, we can rewrite equation (A-1) as:

$$Y = A\left(\frac{\epsilon_G \theta_G I}{\delta_G}\right)^{\psi}\left(\frac{\epsilon_H(1-\theta_G)I}{\delta_H}\right)^{\phi}. \tag{A-5}$$

There is an optimal allocation of spending across sectors (a $\theta_G^*$) that equalizes the marginal products. If the spending allocation is not optimal, it would be possible to rank projects of the two types by marginal product. Bad project choice means choosing the wrong type or, in steady state, choosing the wrong values for $\theta_G$.

We can now show that (1) choosing the wrong projects (i.e. the wrong value of $\theta_G$) lowers the *level* of output; and (2) the *level* of $\theta_G$ does *not* matter for the growth impact of additional public investment spending. Taking the derivative of equation (A-5) with respect to $\theta_G$ yields:

$$\frac{dY}{d\theta_G} = \frac{\psi Y}{\theta_G} - \frac{\phi Y}{(1-\theta_G)}.$$

Equalizing this to zero and solving for $\theta_G$ gives :

$$\theta_G^* = \frac{\psi}{\phi + \psi}. \tag{A-6}$$

This is a maximum, so choosing any other value of $\theta$ results in a lower level of output.[33] However, the growth impact of additional investment spending does not depend on $\theta_G$ (or on the values of $\epsilon$, for that matter):

$$\frac{dY/Y}{dI/I} = \psi + \phi.$$

In trying to match this definition of efficiency with Figure A-1, the different types could be aligned from highest to lowest marginal product (we have only two in the above equations but there is no reason this could not be generalized to many types). However, an efficient country would over time allocate investment spending to the highest-yielding types, reducing the scarcity of capital in those sectors so that in steady state the curve in the figure would be a horizontal line. An inefficient country would face a downward-sloping curve in steady state.

This fourth conception of "efficiency" as sectoral allocation of spending can be (and indeed in the above equations is) combined with any or all of the other three conceptions of efficiency as captured by $\epsilon$. This yields a fairly rich conception of inefficient project selection: a country may choose the wrong mix of types of projects, and within types, it may choose especially wasteful or corrupt projects or ones where the service flow for a given dollar is relatively low. All of this is consistent with the results in the main text.

[33] It may remain surprising that the values of spending efficiency $\epsilon_G$ and $\epsilon_H$ do not matter for the optimal allocation of spending across types of project, but this is just a reflection of the scarcity/efficiency trade-off emphasized in the main text.

## Appendix B: Consumption with Corruption

We confirm in a simple steady state analysis that countries with higher investment rates have higher growth independent of the level of efficiency (with corruption). Then we show that consumption rises more in the long run with higher investment in more inefficient (corrupt) countries.

Suppose for simplicity that two countries $a$ and $b$ at an initial steady state have equal output, but that country $b$ is fully efficient while country $a$ is not, and that there is no private capital. Thus:

$$Y_0^a = A^a \left(\epsilon G_0^a\right)^\psi = Y_0^b = A^b \left(G_0^b\right)^\psi \equiv 1 \tag{B-1}$$

Further, assume $I_0^b = I_0^a \equiv I_0$ . Thus $G_0^a = {}^{\epsilon I_0}/_{\delta}$, $\left(G_0^m\right)^a = {}^{G_0^m}/_{\epsilon}$, and $G_0^b = \left(G_0^m\right)^b = {}^{I_0}/_{\delta}$.

The equivalence of output in the two countries implies that $A^b = A^a \epsilon^\psi$.

From the budget constraint,

$$C_0^a = Y_0 - \epsilon I_0 \tag{B-2}$$

$$C_0^b = Y_0 - I_0 \tag{B-3}$$

Now, consider a new steady state, where the only difference is that I is at a new higher level $I_1$ in both countries.

It is apparent as usual that the change in output is invariant to efficiency:

$$\frac{Y_1^a}{Y_0^a} = \frac{Y_1^b}{Y_0^b} = \left(\frac{I_1}{I_0}\right)^\psi \equiv Y_1 \tag{B-4}$$

However, the increase in consumption is not invariant. For any $\epsilon$ less than 1, the increase in the inefficient (corrupt) country is bigger than in the efficient country.

$$\frac{C_1^b}{C_0^b} = \frac{\left({}^{I_1}/_{I_0}\right)^\psi - I_1}{1 - I_0} \tag{B-5}$$

$$\frac{C_1^a}{C_0^a} = \frac{\left({}^{I_1}/_{I_0}\right)^\psi - I_1}{1 - \epsilon I_0} \tag{B-6}$$

The intuition here is that the inefficient country gets more consumption out of an increase in investment spending because it takes less actual realized investment to generate the same output increase, and the rest can be spent on consumption (unlike in the "waste" case).

By assuming that the two countries have the same level of output in the initial steady state, we imply that the inefficient country has a higher level of TFP, and this higher level of TFP allows the country to get the same output (and more consumption) from the same increase in investment spending. Do things change if we assume instead that the two countries initially have the same TFP, and the only difference between the two is in efficiency?

We now have:

$$Y_0^a = A\left(\epsilon\frac{I^a}{\delta}\right)^{\psi} = \epsilon^{\psi} Y_0^b \equiv \epsilon^{\psi} \tag{B-7}$$

Again, there is invariance in output:

$$\frac{Y_1^a}{Y_0^a} = \frac{Y_1^b}{Y_0^b} = \left(\frac{I_1}{I_0}\right)^{\psi} \equiv Y_1 \tag{B-8}$$

and

$$\frac{C_1^b}{C_0^b} = \frac{\left({}^{I_1}/{}_{I_0}\right)^{\psi} - I_1}{1 - I_0} \tag{B-9}$$

$$\frac{C_1^a}{C_0^a} = \frac{\epsilon^{\psi}\left({}^{I_1}/{}_{I_0}\right)^{\psi} - \epsilon I_1}{\epsilon^{\psi} - \epsilon I_0} = \frac{\left({}^{I_1}/{}_{I_0}\right)^{\psi} - I_1\epsilon^{1-\psi}}{1 - I_0\epsilon^{1-\psi}} \tag{B-10}$$

When $\epsilon < 1$, consumption growth in the inefficient country $a$ is again higher.[34]

[34]Some algebra using equations (B-9) and (B-10) shows that $\frac{C_1^a}{C_0^a} > \frac{C_1^b}{C_0^b}$ as long as $0 \leq \epsilon < 1$, $\psi < 1$, and $I_1 > I_0$ (We thank Jing Wang for this observation).

## REFERENCES


Abiad, Abdul, Davide Furceri and Petia Topalova. 2015. The Macroeconomic Effects of Public Investment: Evidence from Advanced Economies. IMF Working Paper 15/95 International Monetary Fund Washington D.C.: .

Adam, Christopher S. and David L. Bevan. 2014. Public Investment, Public Finance, and Growth: The Impact of Distortionary Taxation, Recurrent Costs, and Incomplete. IMF Working Paper 14/73 International Monetary Fund Washington D.C.: .

Agénor, Pierre-Richard. 2010. "A Theory of Infrastructure-Led Development." *Journal of Economic Dynamics and Control* 34(5):932–950.

Agénor, Pierre-Richard. 2014. Optimal Fiscal Management of Commodity Price Shocks. Discussion Paper Series 197 Centre for Growth and Business Cycle Research Manchester: .

Allen, Richard. 2009. The Challenge of Reforming Budgetary Institutions in Developing Countries. IMF Working Paper 09/96 International Monetary Fund Washington D.C.: .

Araujo, Juliana Dutra, Grace Bin Li, Marcos Poplawski-Ribeiro and Luis-Felipe Zanna. 2015. "Current account norms in natural resource rich and capital scarce economies." *Journal of Development Economics* forthcoming.

Arestoff, Florence and Christophe Hurlin. 2010. "Are Public Investment efficient in Creating Capital Stocks in Developing Countries." *Economics Bulletin* 30(4):3177–3187.

Barro, Robert J. 1990. "Government Spending in a Simple Model of Endogeneous Growth." *Journal of Political Economy* 98(5):S103–S125.

Berg, Andrew, J Gottschalk, Rafael Portillo and Luis-Felipe Zanna. 2010. The Macroeconomics of Medium-Term Aid Scaling-Up Scenarios. IMF Working Paper 10/160 International Monetary Fund Washington D.C.: .

Berg, Andrew, Rafael Portillo, Shu-Chun S Yang and Luis-Felipe Zanna. 2013. "Public investment in resource-abundant developing countries." *IMF Economic Review* 61(1):92–129.

Berg, Andrew, Shu-Chun S Yang and Luis-Felipe Zanna. 2015. Modeling African Economies: A DSGE Approach. In *The Oxford Handbook of Africa and Economics: Context and Concepts*, ed. C Monga and Justin Y Lin. Vol. 1 Oxford: Oxford University Press.

Briceño Garmendia, Cecilia, Karlis Smits and Vivian Foster. 2008. Financing Public Infrastructure in Sub-Saharan Africa: Patterns and Emerging Issues. AICD Background Paper 15 The World Bank Washington D.C.: .

Briceño Garmendia, Cecilia and Vivian Foster, eds. 2010. *Africa's Infrastructure: A Time for Transformation*. Washington D.C.: Agence Francaise de Developpement and the World Bank.

Buffie, Edward F., Rafael Portillo, Luis-Felipe Zanna, Catherine A. Pattillo and Andrew Berg. 2012. Public Investment, Growth, and Debt Sustainability: Putting Together the Pieces. IMF Working Papers 12/144 International Monetary Fund.

Calderón, César, Enrique Moral-Benito and Luis Servén. 2015. "Is infrastructure capital productive? A dynamic heterogeneous approach." *Journal of Applied Econometrics* 30(2):177–198.

Caselli, Francesco. 2005. Accounting for cross-country income differences. In *Handbook of Economic Growth*, ed. Philippe Aghion and Steven N. Durlauf. Vol. 1 Elsevier chapter 4, pp. 679–741.

Chakraborty, Shankha and Era Dabla-Norris. 2011. "The Quality of Public Investment." *The B.E. Journal of Macroeconomics* 11(1):1–29.

Collier, Paul. 2007. *The Bottom Billion*. New York: Oxford University Press.

Dabla-Norris, Era, Jim Brumby, Annette Kyobe, Zac Mills and Chris Papageorgiou. 2012. "Investing in public investment: an index of public investment efficiency." *Journal of Economic Growth* 17(3):235–266.

Dalgaard, Carl-Johan and Henrik Hansen. 2015. The Return to Foreign Aid. UNU-WIDER Research Paper 53 World Institute for Development Economic Research (UNU-WIDER).

Eden, Maya and Aart Kraay. 2014. 'Crowding In' and the Returns to Government Investment in Low-Income Countries. Policy Research Working Paper 6781 The World Bank.

Gupta, Sanjeev, Alvar Kangur, Chris Papageorgiou and Abdoul Wane. 2014. "Efficiency-Adjusted Public Capital and Growth." *World Development* 57:164–178.

Hulten, C. 1996. Infrastructure Effectiveness as a Determinant of Economic Growth: How Well You Use it May Be More Important than How Much You Have. Working Paper 5847 National Bureau of Economic Research.

Hulten, Charles R. 1991. The Measurement of Capital. In *Fifty Years of Economic Measurement: The Jubilee of the Conference on Research in Income and Wealth*, ed. Ernst R. Berndt and Jack E. Triplett. Chicago: University of Chicago Press pp. 119–158.

IMF. 2014*a*. *Fiscal Monitor – Public Expenditure Reform*. Washington D.C.: International Monetary Fund.

IMF. 2014*b*. *World Ecoomic Outlook – Legacies, Clouds, Uncertainties*. Washington D.C.: International Monetary Fund.

IMF. 2015. "Making Public Investment More Efficient." Board Paper, June.

Lucas, Robert E. 1988. "On the mechanics of economic development." *Journal of Monetary Economics* 22(1):3–42.

Mankiw, N Gregory, David Romer and David N Weil. 1992. "A Contribution to the Empirics of Economic Growth." *The Quarterly Journal of Economics* 107(2):407–37.

Manuelli, Rodolfo and Larry E Jones. 1990. "A convex model of equilibrium growth." *Journal of Political Economy* 98(5):1008–1038.

Melina, Giovanni, Shu-Chun S Yang and Luis-Felipe Zanna. 2015. "Debt Sustainability, Public Investment and Natural Resources in Developing Countries: the DIGNAR Model." *Economic Modelling* forthcoming.

Pesaran, M. Hashem. 2006. "Estimation and inference in large heterogeneous panels with a multifactor error structure." *Econometrica* 74(4):967–1012.

Pritchett, Lant. 2000. "The tyranny of concepts: CUDIE (cumulated, depreciated, investment effort) is not capital." *Journal of Economic Growth* 5(4):361–

384.
Pritchett, Lant, Michael Woolcock and Matt Andrews. 2013. "Looking like a state: techniques of persistent failure in state capability for implementation." *The Journal of Development Studies* 49(1):1–18.
Rebelo, Sergio. 1991. "Long-Run Policy Analysis and Long-Run Growth." *Journal of Political Economy* 99(3):500–521.
Schwab, Klaus. 2015. *The Global Competitiveness Report 2014–2015*. Geneva: World Economic Forum.
Solow, Robert M. 1956. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics* 70(1):65–94.