Are Antitrust Fines Friendly to Competition? An Endogenous Coalition Formation Approach to Collusive Cartels

David Bartolini
Università Politecnica delle Marche, Department of Economics
OPERA

Alberto Zazzaro
Università Politecnica delle Marche, Department of Economics
MoFiR
CFEPSR

MoFiR working paper n° 19

February 2009

Are antitrust fines friendly to competition? An endogenous coalition formation approach to collusive cartels *

David Bartolini[†] Alberto Zazzaro[‡]

February 12, 2009

Abstract

A well-established result of the theory of antitrust policy is that it might be optimal to tolerate some degree of collusion among firms if the authority in charge is constrained by limited resources and imperfect information. However, few doubts are cast on the common opinion by which stricter enforcement of antitrust laws definitely makes market structure more competitive and prices lower. In this paper we challenge this presumption of effectiveness and show that the introduction of a positive (expected) antitrust fine may drive firms from partial cartels to a monopolistic cartel. Moreover, introducing uncertainty on market demand, we show that the socially optimal competition policy can call for a finite or even zero antitrust penalty even if there are no enforcement costs. We first show our results in a Cournot industry with five symmetric firms and a specific rule of cartel formation. Then we extend the analysis to the case of N symmetric firms and a generic rule of coalition formation. Finally, we consider the case of asymmetric firms and show that our results still hold for an industry populated by one Stackelberg leader and two followers.

JEL Classification: *C70, L40, L41*
Key words: *Coalition formation; Collusive cartels, Antitrust policy.*

*We wish to thank Francis Bloch, Ben Lockwood, Marco Marini, Luca Riccetti and Alessandro Sterlacchini for useful comments. Our thanks go also to seminar participants at the Universities of Ancona, Cagliari, Verona, Warwick and participants at the conference "The Economics of Imperfect Markets" (Rome, May 2008). Usual disclaimer applies.

[†]Università Politecnica delle Marche and OPERA, Dipartimento di Economia, Piazzale Martelli 8, 60121 Ancona, Italy, e-mail: d.bartolini@univpm.it.

[‡]Università Politecnica delle Marche, MoFiR and CFESPR, e-mail: a.zazzaro@univpm.it

1 Introduction

A well-established result of the theory of antitrust policy is that greater expected penalties on colluding firms (i.e., the probability of proving collusion in a trial times the monetary fine) *can* hurt social welfare. When the authority in charge is constrained by limited resources or imperfect information, it might be optimal to tolerate some collusion among firms, while saving on auditing costs and reducing the probability of erroneously prosecuting and punishing firms that have not colluded (Besanko and Spulber, 1989; Souam, 2001; Frezal, 2006; Martin, 2006). However, few doubts are cast on the common opinion that stricter enforcement of antitrust laws definitely helps to make the market structure more competitive and prices lower.

In this paper we challenge this view. Confidence in the beneficial effects of antitrust penalties on competition seems to be grounded on the very restrictive assumption that firms either form a monopolistic cartel (the grand coalition) or compete independently as singletons. In this case, an increase in the expected penalty reduces the benefit of explicit agreements to fix prices, reduce output and other collusive practices.

However, industries are often characterised by the presence of one (or more) partial cartels and a number of independent agents, as shown by the recent cases of the Dutch beer cartel and the international cartel in the elevator industry, sanctioned by the European Antitrust authority.[1]

To take this possibility into consideration, we focus on the process of cartel formation, modelling it as an endogenous coalition game, where firms can react to competitors' deviation and to the introduction of an antitrust fine by forming a different cartel structure.[2] In this setting, we show that if the equilibrium market structure in the absence of any antitrust policy is characterised by partial cartels, the introduction of an antitrust expected fine may drive firms towards greater, at most monopolistic, cartels and only eventually towards "full" competition.

The anti-competitive effect of the antitrust penalty rests on two basic ingredients of many cartel formation models: grand coalition superadditivity and positive externalities. From Stigler (1950) onward, industrial economists have recognised that collusion is characterised by a prisoner-dilemma-type instability. Firms could obtain higher profits by colluding rather than competing, but individual members of each cartel have an incentive to breach collusive agreements by increasing their output or reducing prices. Implicit and explicit cartels can be sustained by the repeated interaction among member firms (Friedman, 1971), by the uncertainty on market demand (Green and Porter, 1984) or by firms' awareness of the impact of their actions on the market structure itself (D'Aspremont et al., 1983; Bloch, 1996; Ray and Vohra, 1997). Whatever the device that ensures cartel stability, however, firms outside cartels would earn higher profits than firms inside cartels due to the presence of positive externalities. At the same time, grand coalition superadditivity implies that aggregate profits are greater in the monopolistic cartel (grand coalition) than in any other market structure with partial cartels. Therefore, in any market structure different from the grand coalition there exists a

[1] For a survey on cartel organisations see Levenstein and Suslow (2006).

[2] Useful surveys of the endogenous coalition formation approach are Yi and Shin (1995), Bloch (2003), Marini (2007) and Ray (2007).

cartel whose members earn less than they would earn in the grand coalition. Putting things together, positive externalities and profit superadditivity are sufficient to show that there might exist values of the expected antitrust fine that break up partial cartels but do not deter the formation of a monopolistic cartel.

In what follows, we first provide an example of the anti-competitive effects of antitrust fines in a Cournot oligopoly with five symmetric firms, where cartels are supposed to form according to the equilibrium binding agreement rule introduced by Ray and Vohra (1997). In this environment, we show that the introduction of a positive expected antitrust fine on colluding firms modifies the market structure, making it initially less competitive. As the authority raises the antitrust fine, the equilibrium shifts progressively from a coalition structure formed by two cartels and a non-colluding firm, to a two-cartel structure, and, then, to a monopolistic market structure. Only when the expected antitrust fine is sufficiently high do the five firms find it worth not forming any cartel and the market structure becomes more competitive. In this setting it can be shown that, if the antitrust authority does not observe the level of market demand, even a costless welfare-maximising competition policy might consist in setting an *intermediate*, at most a zero, antitrust penalty. It is worth noting that in this case what drives the welfare-reduction effect is not the cost of antitrust enforcement but the incentives to form greater coalitions introduced by the antitrust fine.

We then generalise our results by showing that in a generic oligopolistic market with symmetric firms, the introduction of an antitrust expected fine can have anti-competitive effects whatever the number of firms and irrespective of the coalition formation rule. Finally, we extend our analysis to the case of asymmetric firms by studying a three-firm Cournot industry with one Stackelberg leader and two followers.

The possibility that the antitrust penalty indirectly produces an anti-competitive effect has been analysed in different settings by McCutcheon (1997), Cyrenne (1999) and Harrington (2004)[3]. McCutcheon considers a model of explicit collusion where the stability of the cartel is hindered by the possibility to renegotiate the original agreement – which specifies the punishment scheme in the case of deviation. In this framework, if renegotiation were costless, no collusive agreement would be sustained. The introduction of a penalty, therefore, can strengthen cartel stability by making renegotiation costly, and the punishment of defections credible. In a similar vein, Cyrenne and Harrington show that, when the probability of cartel detection depends on price discontinuity, the introduction of an antitrust fine reduces the benefit from cheating, thereby bolstering higher collusive prices.

All these papers assume the formation of a monopolistic cartel and argue that the antitrust fine can indirectly facilitate the formation of a collusive agreement by increasing the cost of defection. Our approach is different as we focus on the initial process of coalition formation, considering the direct effect that antitrust penalties have on the profitability of signing collusive agreements. As we relax the assumption of a monopolistic cartel, allowing for the formation of partial cartels, we show that the perverse effect of the antitrust policy arises also when agreements are binding (i.e.,

[3]Similar perverse effects are remarked upon also by the literature on leniency policy (Spagnolo, 2000; Ellis and Wilson, 2001). The literature on the anti-competitive effects of antitrust is reviewed by Bartolini and Zazzaro (2009).

renegotiation is impossible) and enforcement is costless.

The rest of the paper is organised as follows. In section 2, we set up the basic features of the coalition formation game; in Section 3, we consider a 5-firm Cournot competition model; in Section 4, we generalise our result to symmetric industries; in Section 5 we consider the case of asymmetric firms; in Section 6 we discuss the results and policy implications of our model.

2 The environment

2.1 The coalition formation game

Consider an industry with a given number of risk-neutral firms, N, and no possibility of entry. Although the number of firms is fixed, the market structure is endogenous. Each firm either explicitly colludes with other firms forming a cartel or acts independently as a singleton. Firms can join only one cartel at a time, and participation in a cartel does not result in any cost or information synergy.

We model the formation of cartels as a transferable utility two-stage coalition game. In the first stage, firms negotiate the formation of coalitions (cartels) and sign binding agreements on the basis of an exogenous rule to divide the coalition worth among members. Once formed, in the second stage coalitions non-cooperatively compete in the market, thereby determining their worth (profit). Assuming that the Nash equilibrium in the second stage is unique, in the first stage, when facing the choice to form a cartel, each firm is able to perfectly anticipate the payoff it is going to achieve in any possible coalition structure and, on this basis, to make the payoff-maximising choice.

This game can be summarised by a triple (N, Ω, v), where Ω is the set of partition structures, whose elements, $\mathcal{P}$, represent partitions of the set of N firms into coalitions. Each $\mathcal{P} \in \Omega$ consists of $m \in [1, N]$ coalitions S with cardinality s (i.e. s represents the number of firms in cartel S). The partition function v assigns payoffs (worth) to coalitions according to the coalition structure they belong to: $v(S_j, \mathcal{P})$ is the worth of coalition $S_j \in \mathcal{P}$. The individual (per-firm) payoff is represented by a *valuation* π_i, mapping the set of coalition structures into vectors of individual payoffs, i.e. $\pi_i(S_j, \mathcal{P})$ is the payoff of firm i in coalition S_j belonging to the coalition structure $\mathcal{P}$. As a consequence, $v(S_j, \mathcal{P}) = \sum_{i=1}^{s_j} \pi_i(S_j, \pi)$. The aggregate worth of a coalition structure is denoted by $v(\mathcal{P}) = \sum_{j=1}^{m} v(S_j, \mathcal{P})$. The worth of the grand coalition is represented by $v(N)$. Now, we introduce some useful definitions.

Definition 1 (Finer Coalition Structures) *A coalition structure $\mathcal{P}'$ is said to be* finer *than $\mathcal{P}$ if $m' > m$.*

This is our measure of competition: the higher the number of coalitions in the market, the higher is the degree of competition.

Definition 2 (Coalitional symmetry — CS) *A partition function satisfies coalitional symmetry if $v(S_j, \mathcal{P}) = \frac{v(\mathcal{P})}{m}$ for all coalition $S_j \in \mathcal{P}$ and any coalition structure $\mathcal{P} \in \Omega$*

This condition is shared by all standard models of cartel formation, in which the only result of signing collusive agreements is to reduce competition in the market. This simply means that the agreement does not lead to any synergy among cartel members, which may arise, for instance, through research, advertising or distribution. Interestingly, coalitional symmetry can hold only if individual players are symmetric and, moreover, it implies that cartel formation produces positive externalities (PE) on firms outside the cartel.[4]

Definition 3 (Grand Coalition Superadditivity — GCS) *A partition function exhibits grand coalition superadditivity if* $v(N) \geq v(\mathcal{P})$ *for any* $\mathcal{P} \in \Omega$.

GCS is a less restrictive assumption than coalition superadditivity, as it leaves the possibility of the payoff of players decreasing when forming partial coalitions. In particular, GCS only requires that the aggregate payoff in any coalition structure is smaller than the worth of the grand coalition — see Ray (2007).

2.2 Antitrust policy

A cartel is defined as an explicit agreement among $s \geq 2$ firms that binds them to cooperate in order to maximise the cartel payoff. We assume that an antitrust law prohibits such agreements and an authority is in charge to enforce a monetary fine $f > 0$ on firms that collude. Firms face a probability q of being audited and found guilty by a court of law. Basically, we assume that the unit of investigation is the single firm, and that the *joint* probability of being audited by the antitrust agency and being sentenced to pay the fine f by a court of law does not depend on the size of the cartel.[5].

Given firms are wealth-constrained, the monetary fine f is upward bound. In order to focus on the anti-competitive effect of the fine, we assume that the Antitrust authority can costlessly implement any detecting-prosecuting technology, i.e. can costlessly choose the value of q. This implies that, without loss of generality, we can consider as the Authority's choice variable the expected fine $F = q \cdot f$, where $F \in [0, \bar{f}]$, and ignore the Beckerian trade-off between the level of the monetary sanctions and the strictness of the enforcement (Becker, 1968; Polinsky and Shavell, 2000). Finally, we assume that the level of the expected fine is publicly announced at the beginning of the game, before firms decide whether or not to form a cartel, and that the Authority can fully commit to this announcement.

[4]Formally, a coalition game exhibits positive externalities (PE) if $v(S_j, \mathcal{P}) > v(S_j, \mathcal{P}')$, where $\mathcal{P}'$ is a finer coalition than $\mathcal{P}$, and $S_j \in \mathcal{P}, \mathcal{P}'$.

[5]It could be argued that the larger the number of firms in a cartel the higher the probability that at least one of them is audited. However, the anti-trust cannot automatically prove the presence of the other firms in the cartel. Moreover, a large cartel have access to greater financial resources in order to defend its position (and that of their members) in front of a court of law. As a consequence, even though the probability of firms being discovered by the anti-trust would increase with the size of the cartel, the probability of being found guilty should decrease with the cartel size. In the absence of a detailed model which directly addresses both the issue of auditing and providing evidence of collusive behaviour in front of a court of law (which is not not the focus of our paper), we chose to consider the joint probability of the two events identical for all firms, regardless of the cartel size. Nevertheless, in Section 6.1, we provide an informal discussion of the implications of dropping the assumption of an exogenously fixed q.

3 Equilibrium binding agreements and antitrust penalties in a 5-firm Cournot oligopoly

In this section, we analyse the effects of an antitrust policy on competition and social welfare in the case of a Cournot oligopoly with five symmetric firms. In particular, we assume that the market is characterised by homogeneous goods and a linear (inverse) demand function $p = \alpha - \beta Q$, with $\alpha, \beta > 0$, and where $Q = \sum_i q_i$ is the aggregate output. Marginal costs are constant and, for simplicity, normalised to zero. This set-up ensures the existence of a unique Nash equilibrium in the competition game, for any coalition structure $\mathcal{P}$, and satisfies both GCS and CS definitions, and as a consequence also the assumption of positive externality.

In order to investigate the formation of cartels and characterise the coalitional equilibrium of the game, we need to introduce a rule of coalition formation (from which a notion of coalition stability follows). In this section, we rely on the concept of *equilibrium binding agreement* (EBA), introduced by Ray and Vohra (1997). According to this concept of equilibrium, a strategy profile is an EBA for the coalition structure $\mathcal{P}$ if it is the best response for each player given $\mathcal{P}$, and there is no other *finer* coalition structure $\mathcal{P}'$ which is sustained as an EBA and can be induced by a profitable deviation of some agents. Therefore, the concept of EBA only allows cartels to break up into smaller coalitions, while it excludes the possibility of deviating from a given coalition by creating cartels with players outside that coalition.

When considering a deviation from an arbitrary coalition structure $\mathcal{P}$, each player looks ahead and takes into account additional deviations that may be induced by its own initial deviation (farsighted players). The set of EBAs is recursively defined starting from the singleton coalition structure, which is an EBA by construction, as it is the finest possible coalition structure.

The payoff of each coalition is imputed to coalition members according to an exogenous sharing rule, that we assume to be the equal sharing rule. As Ray and Vohra (1997) show, in coalition formation games with symmetric players, transferable utility and positive externalities, there is no loss of generality in restricting the game to strategies with equal division of coalition worth, in the sense that "the set of equilibrium coalition structure is unchanged by restricting attention to equal division" (Ray and Vohra, 1997, Proposition 6.3, p. 67).

Following Ray and Vohra (1997), in a Cournot oligopoly with linear demand and equal sharing rule the per-member payoff in the coalition $S_j \in \mathcal{P}$ decreases with the number of competing coalitions, m, and the number of firms in the coalition, s_j. In particular, when production costs are zero we have the following *valuation*:[6]

$$\pi_i(S_j, \mathcal{P}) = \frac{1}{s_j} \frac{\alpha^2}{\beta(m+1)^2} \tag{1}$$

In Table 1 we report the per-firm payoffs in any possible cartel structure. For the sake of notation, since firms are symmetric we can neglect their identity and consider just one possible permutation of firms. For instance, in coalition structure $\mathcal{P}_2$, the

[6]Since the hypothesis of symmetry $\pi(S_j, \mathcal{P}) = \pi_i(S_j, \mathcal{P})$ for all $i \in S_j$.

non-colluding firm obtains a profit $\frac{1}{9}$ regardless of its identity.

Table 1: Firms' payoff (each entry must be multiplied by $\frac{\alpha^2}{\beta}$)

coalition structure		π_1	π_2	π_3	π_4	π_5
$\mathcal{P}_1$	{1,2,3,4,5}	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{1}{20}$
$\mathcal{P}_2$	{1,2,3,4} {5}	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{9}$
$\mathcal{P}_3$	{1,2,3} {4,5}	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{18}$	$\frac{1}{18}$
$\mathcal{P}_4$	{1,2,3} {4} {5}	$\frac{1}{48}$	$\frac{1}{48}$	$\frac{1}{48}$	$\frac{1}{16}$	$\frac{1}{16}$
$\boldsymbol{\mathcal{P}_5}$	**{1,2} {3,4} {5}**	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{16}$
$\mathcal{P}_6$	{1,2} {3} {4} {5}	$\frac{1}{50}$	$\frac{1}{50}$	$\frac{1}{25}$	$\frac{1}{25}$	$\frac{1}{25}$
$\mathcal{P}^*$	{1} {2} {3} {4} {5}	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$

In this scenario the only non-trivial[7] stable coalition structure is $\mathcal{P}_5$, where two equilibrium binding agreements are reached, both leading to the formation of a cartel between two firms.

Proposition 1 (Ray and Vohra 1997) *In a Cournot oligopoly game with five symmetric firms, linear demand and constant (zero) marginal costs, the equilibrium coalition structure is formed by two 2-firm cartels and one independent firm.*

Proof. The proof can run intuitively. Since the EBA is a recursive concept we start from the singleton coalition which is an EBA by construction. Then, we check whether coalition structures that can "directly" lead to the singleton are stable. Let us start from coalition $\mathcal{P}_6$. This coalition structure is not stable because firms in the cartel $\{1, 2\}$ prefer to split and compete in the singleton structure, where they can gain higher profits.

Moving up to $\mathcal{P}_5$, it is easy to check that this is a stable coalition structure: although firms in one of the two cartels have an incentive to deviate in order to induce $\mathcal{P}_6$, we have already noted that the latter is not an EBA. Therefore, a deviation from $\mathcal{P}_5$ would necessarily lead to the singleton structure. Anticipating this conclusion, no firm in $\mathcal{P}_5$ has any incentive to deviate, as their payoff in the cartel is higher than the payoff they would achieve by competing as singletons in $\mathcal{P}^*$.

Coalition structure $\mathcal{P}_4$ is not a stable structure because firms in the 3-member cartel would earn less than in $\mathcal{P}^*$ and hence do not find it profitable to sign a binding agreement (note that starting from $\mathcal{P}_4$ firms could induce another coalition structure, $\mathcal{P}_6$, which is, however, not a stable EBA).

[7] That is, excluding the singleton coalition structure which is a stable EBA by definition.

Finally, coalition structures $\mathcal{P}_3$, $\mathcal{P}_2$ and $\mathcal{P}_1$ are not stable because they are blocked by the equilibrium coalition structure $\mathcal{P}_5$. Starting from $\mathcal{P}_3$, one of the firms in the 3-member cartel 1,2,3 can directly induce the coalition $\mathcal{P}_5$ where it earns greater profits: $\frac{\alpha^2}{16\beta}$ versus $\frac{\alpha^2}{36\beta}$. Similarly, from $\mathcal{P}_2$ firms in the 4-member cartel have incentive to split and form two 2-member cartels as in $\mathcal{P}_5$. Starting from $\mathcal{P}_1$ no firms can directly induce $\mathcal{P}_5$. However, one of the firms finds it profitable to unilaterally deviate towards $\mathcal{P}_4$, anticipating the reaction of other players and the formation of the coalition structure $\mathcal{P}_5$, where the payoff of the deviator is higher than in the grand coalition: $\frac{\alpha^2}{16\beta}$ versus $\frac{\alpha^2}{20\beta}$. ■

As this example clearly shows, when the choice of firms is not restricted to either colluding in a monopolistic cartel or not colluding, the equilibrium market structure can be characterised by the presence of partial cartels. This result does not depend on the concept of coalition formation employed in the analysis, which only allows the equilibrium coalition structure to be established. For instance, if we consider a sequential coalition formation game, the equilibrium coalition structure would be $\mathcal{P}_2$ rather than $\mathcal{P}_5$, but it would still include a partial cartel (Bloch, 1996).

The output produced by each cartel, as well as by the singleton player in $\mathcal{P}_5$ is $q = \frac{1}{4}\frac{\alpha}{\beta}$, such that the total output is $Q = \frac{3}{4}\frac{\alpha}{\beta}$, resulting in an equilibrium price of $p = \frac{1}{4}\alpha$. The social welfare, given by the sum of consumer and producer surplus, is:

$$S_5 = \left(\frac{3}{16} + \frac{9}{32}\right)\frac{\alpha^2}{\beta} = \frac{15}{32}\frac{\alpha^2}{\beta} \tag{2}$$

where the subscript 5 refers to the equilibrium coalition structure $\mathcal{P}_5$.

3.1 Antitrust policy

We now modify the scenario by adding the antitrust authority which imposes an expected penalty $F \in [0, \bar{f}]$. Since firms are risk-neutral, their expected payoffs are simply those reported in table 1 minus the fine F. As a consequence the equilibrium of the game depends on the exogenous level of the penalty.

Let us consider the coalition structure $\mathcal{P}_5$. In the presence of the antitrust penalty, the two cartels, between firms 1 and 2 and firms 3 and 4, break up if the individual expected payoff from colluding is lower than the payoff of not colluding, that is, if

$$F \quad > \quad F_5 = \left(\frac{1}{32} - \frac{1}{36}\right)\frac{\alpha^2}{\beta} \simeq 0.003\frac{\alpha^2}{\beta} \tag{3}$$

When the expected fine announced by the Authority is greater than F_5, firms would prefer to split into singletons rather than stick to the agreement in their cartel, and hence $\mathcal{P}_5$ can no longer be considered an EBA. Note that a fine equal to F_5 would dissolve any cartel in which the per-member payoff is lower than $\frac{1}{32}$, i.e. all the cartels in $\mathcal{P}_2$, $\mathcal{P}_4$, and $\mathcal{P}_6$. However, it might induce the formation of cartels with higher per-firm payoff, which, in the absence of any antitrust penalty, were not EBAs.

In our example, fines higher than F_5 could induce firms to partition themselves either into coalition structure $\mathcal{P}_3$ or $\mathcal{P}_1$. In particular, $\mathcal{P}_3$ becomes an EBA for

$$F_5 \quad \leq \quad F < F_3 = \left(\frac{1}{27} - \frac{1}{36}\right)\frac{\alpha^2}{\beta} \simeq 0.009\frac{\alpha^2}{\beta} \tag{4}$$

When engaging in the coalition formation process, firm 5 anticipates that the expected fine announced by the Authority would break both the cartels between firms 1 and 2 and between firms 3 and 4, making it not worth inducing $\mathcal{P}_5$. Yet, firms find it profitable to form two cartels of size three and two, respectively, inducing $\mathcal{P}_3$: on the one hand, firms in the 2-member cartel receive a higher individual expected payoff than in the grand coalition, hence $\mathcal{P}_3$ blocks $\mathcal{P}_1$; on the other, firms in the 3-member cartel have no incentive to split because they receive a profit, net of the expected fine, higher than in the singleton structure.

In fact, firms belonging to the 3-member cartel in $\mathcal{P}_3$ receive an individual payoff higher than in $\mathcal{P}_5$, such that the fine F_5 does not prevent them from signing a collusive agreement. Note, however, that the per-firm payoff in the 3-member cartel in $\mathcal{P}_3$ is lower than in the grand coalition, $\mathcal{P}_1$. This implies that there exists an antitrust fine high enough to dissolve this cartel, making $\mathcal{P}_3$ unstable, but at the same time low enough for the grand coalition to be more profitable than the singleton coalition $\mathcal{P}^*$,

$$F_3 \quad \leq \quad F < F_1 = \left(\frac{1}{20} - \frac{1}{36}\right)\frac{\alpha^2}{\beta} \simeq 0.022\frac{\alpha^2}{\beta} \tag{5}$$

In this case, the only EBA is the grand coalition because the policy announced by the antitrust Authority would dissolve the 3-member cartel in $\mathcal{P}_3$, but not the monopolistic cartel which, due to profit superadditivity, is still more rewarding than atomistic competition.

The following proposition summarises our results.

Proposition 2 *In a 5-firm Cournot oligopoly with linear demand and zero production costs, the degree of market competition does not monotonically increase with the expected antitrust fine announced by the Authority. For F in the interval* $\left(0.003\frac{\alpha^2}{\beta}, 0.022\frac{\alpha^2}{\beta}\right]$*, the market progressively becomes less competitive, moving from a triopoly ($\mathcal{P}_5$) to a duopoly ($\mathcal{P}_3$) and, eventually, to a monopoly ($\mathcal{P}_1$).*

A surprising warning against tough antitrust penalties emerges from Proposition (2). As the value of the expected fine increases up to F_1, the degree of competition in the market decreases because it induces initial deviators to reconsider participation in a collusive cartel and drives firms to partition into a lower number of competing cartels. Only when F is greater than F_1 is the antitrust policy effective, as the only EBA is the singleton coalition structure and the market becomes more competitive than in the absence of antitrust.

3.2 Welfare analysis: the optimal fine under demand uncertainty

Even assuming that the enforcement of the antitrust law is costless and non-distortionary, and that the fines paid by firms found guilty of signing collusive agreements are purely transferred to consumers, social welfare changes with the market structure and, hence, with F. In Table 2 we report the social welfare values in the space F.

Table 2: Social Welfare for different levels of expected fine

	$F < 0.003\frac{\alpha^2}{\beta}$	$0.022\frac{\alpha^2}{\beta} \leq F < 0.009\frac{\alpha^2}{\beta}$	$0.009\frac{\alpha^2}{\beta} \leq F < 0.022\frac{\alpha^2}{\beta}$	$F \geq 0.022\frac{\alpha^2}{\beta}$
Eq.m structure	Partial coalition: $\mathcal{P}_5$	Partial coalition: $\mathcal{P}_3$	Grand coalition: $\mathcal{P}_1$	Singleton: $\mathcal{P}^*$
Social welfare	$S_5 = 0.47\frac{\alpha^2}{\beta}$	$S_3 = 0.44\frac{\alpha^2}{\beta}$	$S_1 = 0.37\frac{\alpha^2}{\beta}$	$S^* = 0.48\frac{\alpha^2}{\beta}$

The obvious corollary of Proposition (2) is that the relationship between social welfare and the toughness of a costless competition policy is not monotone. This is in sharp contrast with the typical conclusion of the Beckerian-Stiglerian approach to crime (trust) and punishment, where the increase in antitrust penalties can be welfare-decreasing only in the presence of costly actions or imperfect knowledge on the part of the enforcing Authority regarding the occurrence and gravity of the offence.

In a world of certainty, and if $F_1 \leq \bar{f}$, the non-monotonicity of the social welfare with respect to the fine would not be a problem, as the Authority could simply set the expected penalty at a level higher than F_1. Things change with uncertainty or when $F_1 > \bar{f}$, in which case the optimal policy might even be *doing nothing*.

To illustrate, let us assume that the Authority does not know the exact value of the demand parameter α, which distributes as a continuous random variable with cumulative distribution function $\Phi(\alpha)$. Firms observe the realisation of α, while the Antitrust authority only knows its distribution. In this case, the Authority cannot compute the actual threshold values F_5, F_3 and F_1. This means that for any F there is a positive probability that the level of market demand is such that the stable market structure is $\mathcal{P}_5$, $\mathcal{P}_3$, $\mathcal{P}_1$ or $\mathcal{P}^*$. In particular, we have:

$$Prob[\mathcal{P}_5] = Prob\left[\alpha > \sqrt{\frac{\beta F}{0.003}}\right] = 1 - \Phi\left(\sqrt{\frac{\beta F}{0.003}}\right) \tag{6}$$

$$Prob[\mathcal{P}_3] = Prob\left[\sqrt{\frac{\beta F}{0.003}} < \alpha \leq \sqrt{\frac{\beta F}{0.009}}\right] = \Phi\left(\sqrt{\frac{\beta F}{0.003}}\right) - \Phi\left(\sqrt{\frac{\beta F}{0.009}}\right) \tag{7}$$

$$Prob[\mathcal{P}_1] = Prob\left[\sqrt{\frac{\beta F}{0.009}} < \alpha \leq \sqrt{\frac{\beta F}{0.022}}\right] = \Phi\left(\sqrt{\frac{\beta F}{0.009}}\right) - \Phi\left(\sqrt{\frac{\beta F}{0.022}}\right) \tag{8}$$

$$Prob[\mathcal{P}^*] = Prob\left[\alpha \leq \sqrt{\frac{\beta F}{0.022}}\right] = \Phi\left(\sqrt{\frac{\beta F}{0.022}}\right) \tag{9}$$

As expected, a large antitrust penalty increases the probability that no firm colludes (equation 9); however, it also decreases the probability that firms form partial cartels (equation 6). As a consequence, the effect of F on the probability of firms being in coalition $\mathcal{P}_3$ or $\mathcal{P}_1$, both coarser than $\mathcal{P}_5$, is ambiguous.

The optimal antitrust policy is the level of F that maximises the expected social welfare, which is computed as the sum of any possible equilibrium coalition structure weighted by the probability that such an event occurs:

$$
\begin{aligned}
ES = S_5\left[1-\Phi\left(\sqrt{\frac{\beta F}{0.003}}\right)\right] + S_3\left[\Phi\left(\sqrt{\frac{\beta F}{0.003}}\right)-\Phi\left(\sqrt{\frac{\beta F}{0.009}}\right)\right]+ \\
+S_1\left[\Phi\left(\sqrt{\frac{\beta F}{0.009}}\right)-\Phi\left(\sqrt{\frac{\beta F}{0.022}}\right)\right]+S^*\left[\Phi\left(\sqrt{\frac{\beta F}{0.022}}\right)\right]
\end{aligned}
\tag{10}
$$

If Φ is differentiable on the domain of α, taking the derivative of equation (10) with respect to F and substituting out S_5, S_3, S_1 and S^*, with the values of table 2, we get the following F.O.C. for an interior solution,

$$
\frac{0.11}{\sqrt{0.022}}\phi\left(\sqrt{\frac{\beta F}{0.022}}\right)-\frac{0.07}{\sqrt{0.009}}\phi\left(\sqrt{\frac{\beta F}{0.009}}\right)-\frac{0.03}{\sqrt{0.003}}\phi\left(\sqrt{\frac{\beta F}{0.003}}\right)=0 \tag{11}
$$

where $\phi(\cdot)$ represents the probability density function of α.

The optimal F is either the solution of equation (11), or a corner solution $F = \{0, \overline{f}\}$. Therefore, the expected welfare is not necessarily increasing with F and it could even monotonically decrease with it. To give an example, assume that α distributes uniformly between 0 and $\overline{\alpha}$, with $\overline{\alpha} > \sqrt{\frac{\beta \overline{f}}{0.022}}$. It is straightforward to verify that the sign of condition (11) is always negative (i.e., $\frac{dES}{dF} = \frac{1}{\overline{\alpha}}[0.742 - 0.738 - 0.548]$). Hence, the antitrust fine that maximises the social welfare is $F = 0$.

4 Cartel formation and antitrust policy in an N-firm oligopolistic market

The model presented in the previous section rests on specific assumptions on the number of firms (five), the rule of cartel formation (equilibrium binding agreements) and the structure of the market (linear Cournot competition). Therefore, a critical question is whether the anti-competitive and welfare-decreasing effects of the antitrust policy are robust to changes in these features of the model. In this section, we generalise the result reported in Proposition 2 and show that the same underlying economic mechanism works in an oligopolistic market with N symmetric firms, regardless of the rule of cartel formation, the structure of the market, and the strategy variable (prices or output).

The intuition is quite simple: if in the absence of antitrust fines partial cartels form, it means that at least one firm has found it profitable not to subscribe to the monopolistic cartel agreement. Since profits in the industry are grand-coalition superadditive and the partition function exhibits coalition symmetry, in any possible coalition structure the per-member payoff in at least one of the partial cartels in which firms are partitioned has to be lower than in the grand coalition. Consequently, the level of the expected antitrust fine that makes intermediate coalitions unstable is lower than the level which would dissolve the grand coalition. Furthermore, as the break-up of a partial coalition makes competition harsher, the initial deviators might reconsider the grand coalition and propose the monopolistic agreement to the residual players.

Our demonstration strategy is to prove that a value of F exists for which all possible coalition structures are dissolved, except the grand coalition. In other words, what we provide is a sufficiency result, where *coalitional symmetry* and *grand coalition superadditivity* are shown to be sufficient conditions for the antitrust expected fine to adversely affect competition and social welfare.

Let F_1 denote the level of the expected antitrust fine above which firms in the grand coalition prefer to deviate to the singleton coalition structure

$$F_1 = \frac{v(N)}{N} - \pi^* \tag{12}$$

where the first term is the per-member payoff in the grand coalition, while the second term is the per-firm payoff in the singleton coalition structure $\mathcal{P}^*$.

If any intermediate coalition structure can be broken up by an expected antitrust fine strictly lower than F_1, then by continuity a positive interval of antitrust penalty values certainly exists by which the grand coalition becomes a stable equilibrium. In order to dissolve a coalition structure it is sufficient that for one partial cartel (one coalition of size $s \geq 2$) the individual payoff after the expected fine is lower than the payoff in the singleton structure. Let us define $\hat{S}$ as the coalition with the lowest per-member payoff in each coalition structure $\mathcal{P}$.

Lemma 1 *In a coalitional symmetric game $\Gamma(N, \Omega, v)$, $\hat{s} > 1$ for any $\mathcal{P} \in \Omega \setminus \mathcal{P}^*$.*

Proof. Assume by contrast that a coalition structure $\mathcal{P}$ different from $\mathcal{P}^*$ with $\hat{s} = 1$ exists. In this case, $v(\hat{S}, \mathcal{P}) < \frac{v(S_j, \mathcal{P})}{s_j}$ for any $s_j \geq 2$. However, this condition contradicts the assumption of CS for which $v(\hat{S}, \mathcal{P}) = v(S_j, \mathcal{P})$. ■

Lemma 1 implies that for any $\mathcal{P} \in \Omega \setminus \mathcal{P}^*$ the coalition with the lowest per-member payoff is not a singleton. Therefore, the threshold level of the antitrust penalty, $F_{\mathcal{P}}$, above which the coalition structure $\mathcal{P}$ would dissolve is:

$$F_{\mathcal{P}} = \frac{v(\hat{S}, \mathcal{P})}{\hat{s}} - \pi^* \tag{13}$$

We can now state our main result:

Proposition 3 *In a coalition game $\Gamma(N, \Omega, v)$, where v satisfies coalitional symmetry and grand coalition superadditivity, $F_1 \geq F_{\mathcal{P}}$ for any $\mathcal{P} \in \Omega \setminus \mathcal{P}^*$.*

Proof. This result is easy to prove by contradiction. Assume that a $\mathcal{P} \in \Omega \setminus \mathcal{P}^*$ such that $F_{\mathcal{P}} > F_1$ it exists. Then, given the definitions of $F_{\mathcal{P}}$ and F_1, we have

$$\frac{v(\hat{S}, \mathcal{P})}{\hat{s}} > \frac{v(N)}{N} \tag{14}$$

By GCS we also have that the average payoff is greatest in the grand conalition,

$$\frac{v(N)}{N} \geq \frac{v(\mathcal{P})}{N} \tag{15}$$

Therefore, combining conditions (14) and (15), we get

$$\frac{v(\mathcal{P})}{N} < \frac{v(\hat{S}, \mathcal{P})}{\hat{s}}$$

Given that $\hat{S}$ is defined as the coalition which receives the lowest per capita payoff in $\mathcal{P}$, the above condition cannot be satisfied as it contradicts the simple rule of arithmetic mean. ■

According to proposition (3), the antitrust fine that would dissolve the less profitable cartel in any possible coalition structure will be lower than the antitrust fine needed to dissolve the grand coalition, regardless of the rule followed to form coalitions. This implies that if the equilibrium market structure in the absence of an antitrust law provides for the presence of partial cartels, the introduction of an expected fine $max\{F_{\mathcal{P}}\} \leq F < F_1$ makes market competition definitely weaker. Furthermore, under demand uncertainty, even a costless competition policy can end up being welfare-reducing, such that the optimal choice for the Authority might be *doing nothing*.

It is worth noting that the ingredients we use to prove the anti-competitive effects of the antitrust policy are only coalitional symmetry and grand coalition superadditivity in cartel formation. However, these are only sufficient conditions for the antitrust policy to be anti-competitive. The exact impact of an expected fine F on market competition and expected welfare depends on the number of firms in the industry and the rule of cartel formation, besides the distribution function of the unknown parameter. For instance, we cannot exclude that a given expected fine which breaks the intermediate coalition structure (which would be stable in the absence of antitrust) improves competition by driving market structure towards a previously unstable *finer* intermediate coalition structure. However, our proposition implies that even in this case there is a range of F for which the grand coalition is an equilibrium. Moreover, the range of F values for which competition reduces may be very narrow, and the expected welfare effect of a tougher competition policy always positive.

5 Cartel formation and antitrust policy with asymmetric firms

So far we have assumed that firms and coalitions are symmetric. Although this is quite a restrictive assumption, the extension of Proposition 3 to the case of asymmetric firms is very complex. To the best of our knowledge, the current literature on endogenous coalition formation does not provide general results for games with N asymmetric players. The issue of player asymmetry has been addressed by restricting either the number of players or the set of feasible coalition structures.[8]

In what follows we provide a very simple example of coalition formation in an asymmetric setting, and then by imposing a restriction on the set of coalition structures we extend our general result to asymmetric firms.

5.1 EBAs in a 3-firm Stackelberg industry

Consider an industry populated by a quantity Stackelberg leader A and two followers b and c. The process of coalition formation proceeds as in Section 3: in the first stage firms sign binding agreements simultaneously and form coalitions; in the second stage they compete non-cooperatively, setting quantities in a market characterised by linear demand.

The notion of coalition stability we use is still the EBA. However, we dismiss the equal sharing rule, an unduly restrictive assumption when firms are asymmetric, in favour of a more realistic rule under which the worth of a cartel is divided among its members on the basis of their own contribution to the cartel worth.

A well known weighted sharing rule based on members' contribution is the Shapley value. The Shapley value, however, is built on the marginal contribution of a player to all possible coalitions at which he/she can participate, including coalitions that are not supported by an EBA. The application of this sharing rule seems hardly practicable in writing binding agreements consistent with further deviations. For this reason, we introduce an *equilibrium-weighted* sharing rule, where the contribution of an agent to a coalition depends only on alternative coalition structures that are supported as EBAs (Bartolini, 2008). According to this rule the contribution of agent i to coalition S depends on the difference between the worth of coalition S with player i, and the worth of coalition S without player i, where the payoff without the agent must be computed in the induced stable coalition structure.

Application of this sharing rule to a given coalition structure $\mathcal{P}$ requires the definition of EBAs for coalition structures finer than $\mathcal{P}$. Starting from the singleton coalition $\mathcal{P}^*$, which is an EBA by definition, the sharing rule is applied to impute per-firm payoff in the next coarser coalition structures $\mathcal{P}_k$. Then EBAs are computed such that, when computing the contribution of a player to cartels in a still coarser structure $\mathcal{P}_{k-1}$, we take into account only finer coalition structures that are EBAs. It is worth noting that, in the case of symmetric players (like in Sections 3 and 4), the $equilibrium - weighted$ sharing rule coincides with the equal sharing rule.

Market competition is such that the leader chooses the amount of output first. Then the followers observe the leader's output choice and simultaneously choose their optimal level of output. We assume that, when the leader forms a cartel with one follower, the coalition keeps the first mover's advantage and acts as a leader. Since there is one leader and two followers, six possible coalition structures arise, as shown in Table 3.

[8]For an application to collusive cartels see Donsimoni (1985); for merging in oligopolistic markets see Fauli-Oller (2000) and Bartolini (2008); for international trade agreements see Carraro (1997).

Table 3: Coalition structures

Coalition structure		Type of competition
$\mathcal{P}_1$	{A,b,c,}	Monopoly
$\mathcal{P}_2$	{b,c} {A}	Stackelberg duopoly
$\mathcal{P}_3$	{A,b} {c}	Stackelberg duopoly
$\mathcal{P}_4$	{A,c} {b}	Stackelberg duopoly
$\mathcal{P}^*$	{A} {b} {c}	One-leader two-follower (singletons)

In order to derive the subgame perfect equilibrium of the game, we move backward by computing coalition payoffs in the second stage where they non-cooperatively compete and per-member payoffs are computed following the equilibrium-weighted sharing rule previously described (in the Appendix we report a detailed computation of the individual payoffs displayed in table 4).

Table 4: Firms' payoff: each value must be multiplied by $\frac{\alpha^2}{\beta}$

		π_A	π_b	π_c
$\mathcal{P}_1$	{A,b,c,}	0.107	0.0715	0.0715
$\mathcal{P}_2$	{b,c} {A}	0.125	0.031	0.031
$\mathcal{P}_3$	{A,b} {c}	0.0875	0.0375	0.0625
$\mathcal{P}_4$	{A,c} {b}	0.0875	0.0625	0.0375
$\mathcal{P}^*$	{A} {b} {c}	0.083	0.028	0.028

From table 4, it can be easily proved that:

Proposition 4 *The intermediate coalition structures $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ can all be supported by equilibrium binding agreements, while the grand coalition $\mathcal{P}_1$ is not an EBA.*

Proof. Since coalition structure $\mathcal{P}^*$ is an EBA by definition, the proof consists in showing that members of the cartels in $\mathcal{P}_2$, $\mathcal{P}_3$, and $\mathcal{P}_4$ have no incentive to deviate to a singleton coalition, and that at least one player in the grand coalition has an incentive to deviate. The payoff each firm receives in the two-firm coalition (in any of the intermediate coalitions) is higher than its payoff in the singleton structure. Hence, once such a coalition structure is reached, there is no incentive to deviate. The grand coalition is blocked by coalition structure $\mathcal{P}_2$, because the leader has an incentive to deviate and the followers in that case have an incentive to form a two-member cartel. ■

The intuition of Proposition 4 is straightforward. In the absence of any cost synergy among firms, the leader prefers the market structure $\mathcal{P}_2$ because this allows it to add the positive externality, derived from the formation of a cartel between the two followers, to its first mover's advantage. On the other hand, the followers find it profitable to make competition as weaker as possible in order to reduce their competitive liability, but they can only form a cartel among themselves. However, according to the concept of EBA, also a cartel between the leader and one follower is stable, because if, for some reasons, they end up in this situation nobody has an incentive to deviate.

In all the EBA coalition structures competition is between a Stackelberg cartel and a follower. Therefore, in all the intermediate equilibrium coalition structures, the aggregate equilibrium output is $Q = \frac{3\alpha}{4\beta}$, and the equilibrium price is $p = \frac{1}{4}\alpha$. The social welfare is

$$S_{\mathcal{P}} = \frac{3\alpha^2}{16\beta} + \frac{9\alpha^2}{32\beta} = 0.47\frac{\alpha^2}{\beta} \tag{16}$$

Now, let us look at the effect an antitrust fine has on the outcome of this game. For the sake of clarity, we focus on coalition structure $\mathcal{P}_2$, which is the only one that blocks the grand coalition.[9] In the presence of the antitrust fine, the cartel between firms b and c is stable if the expected payoff from colluding is higher than the payoff of not colluding, that is, if

$$F \leq F_2 = (0.031 - 0.028)\frac{\alpha^2}{\beta} \simeq 0.003\frac{\alpha^2}{\beta} \tag{17}$$

When the expected fine announced by the Authority is higher than F_2, the followers would not form the cartel and two possible scenarios emerge: either the expected penalty is not large enough to make the grand coalition unprofitable, such that the leader, anticipating the splitting of the followers, proposes to form a monopolistic cartel; or the expected penalty is so large that the grand coalition is unprofitable, leading all firms to compete individually as singletons.

Since the difference between the payoff in the grand coalition and the singleton payoffs is smaller for the leader, the scenario which prevails depends only on the leader's incentive to deviate,

$$F_1 = (0.107 - 0.083)\frac{\alpha^2}{\beta} \simeq 0.024\frac{\alpha^2}{\beta} \tag{18}$$

Therefore, we can state the following proposition:

Proposition 5 *When the expected fine is $F < 0.003\frac{\alpha^2}{\beta}$, the equilibrium coalition structure is $\mathcal{P}_2$ where a partial cartel between the followers prevails; for $0.003\frac{\alpha^2}{\beta} \leq F \leq$*

[9] It is easy to show that the introduction of an expected fine has the same effect on the other two equilibria. However, since our objective is to show that there exist some cases in which the antitrust fine reduces competition, it is sufficient to focus on one of the three equilibria.

$0.024\frac{\alpha^2}{\beta}$ *the equilibrium coalition structure is the grand coalition* $\mathcal{P}_1$*; for* $F > 0.024\frac{\alpha^2}{\beta}$ *no cartel forms.*

Once again, according to Proposition 5, there exists a range of antitrust penalties such that the market becomes less competitive, moving from a duopoly to a monopoly with all firms joining the same cartel. Therefore, as in the 5-firm Cournot oligopoly model, social welfare changes with F (this is shown in table 5).

Table 5: Social welfare for different levels of expected fines

	$F < 0.003\frac{\alpha^2}{\beta}$	$0.003\frac{\alpha^2}{\beta} \leq F \leq 0.017\frac{\alpha^2}{\beta}$	$F > 0.017\frac{\alpha^2}{\beta}$
Eq.m structure	Partial coalition: $\mathcal{P}_2$	Grand coalition: $\mathcal{P}_1$	Singleton: $\mathcal{P}^*$
Social welfare	$S_2 = 0.47\frac{\alpha^2}{\beta}$	$S_1 = 0.37\frac{\alpha^2}{\beta}$	$S^* = 0.48\frac{\alpha^2}{\beta}$

Introducing uncertainty on the demand parameter α, the expected social welfare is

$$ES = S_2\left[1 - \Phi(\sqrt{\frac{\beta F}{0.003}})\right] + S_1\left[\Phi(\sqrt{\frac{\beta F}{0.003}}) - \Phi(\sqrt{\frac{\beta F}{0.024}})\right] + S^*\left[\Phi(\sqrt{\frac{\beta F}{0.024}})\right] \quad (19)$$

where $\left[1 - \Phi(\sqrt{\frac{\beta F}{0.003}})\right]$, $\left[\Phi(\sqrt{\frac{\beta F}{0.003}}) - \Phi(\sqrt{\frac{\beta F}{0.024}})\right]$ and $\Phi(\sqrt{\frac{\beta F}{0.024}})$ are the probabilities that the actual fine is below F_2, between F_2 and F_1 and above F_1, respectively.

Following the same steps as in Section 3, it is easy to prove the following proposition:

Proposition 6 *The socially optimal expected fine announced by the antitrust authority is:*

$$F = \begin{cases} \infty & \textit{if} \quad \frac{\phi\left(\sqrt{\frac{\beta F}{0.003}}\right)}{\phi\left(\sqrt{\frac{\beta F}{0.017}}\right)} < 0.46 \textit{ for any } F \\ F^* \in [0, \infty] & \textit{if} \quad \frac{\phi\left(\sqrt{\frac{\beta F}{0.003}}\right)}{\phi\left(\sqrt{\frac{\beta F}{0.017}}\right)} = 0.46 \textit{ for } F = F^* \\ 0 & \textit{if} \quad \frac{\phi\left(\sqrt{\frac{\beta F}{0.003}}\right)}{\phi\left(\sqrt{\frac{\beta F}{0.017}}\right)} > 0.46 \textit{ for any } F \end{cases} \quad (20)$$

As in the case of symmetric firms, the socially optimal antitrust policy depends on how parameter α distributes, and *doing nothing* may be the best choice.

5.2 A general result for N asymmetric firms

In the previous section we showed that the anticompetitive effect of the antitrust penalty can arise also in games with asymmetric players. However, when firms differ in some characteristics, the assumption of coalitional symmetry (CS) no longer holds and it is very difficult to provide a general result similar to Proposition (3). The reason is that, lacking coalitional symmetry, in some coalition structures $\mathcal{P} \in \Omega \setminus \mathcal{P}^*$ the lowest payoff can be attached to a singleton, allowing all cartels to guarantee their members a larger payoff than in the grand coalition.

Nevertheless, if we restrict the set of "feasible" coalition structures to structures with no singletons, we are able to prove a sufficient condition for the anti-competitiveness of antitrust penalties for generic asymmetric games. Let $\bar{\Omega} \subset \Omega$ be the set of coalition structures where $s \geq 2$ for all $S \in \mathcal{P}$ and $\mathcal{P} \in \bar{\Omega}$, that is where there is no singleton, and let $\bar{\Omega}'$ be the complementary set, where there is at least one singleton.

Proposition 7 *In a game of coalition formation $\Gamma(N, \bar{\Omega}, v)$, where v satisfies GCS, $F_1 \geq F_{\mathcal{P}}$ for $\mathcal{P} \in \bar{\Omega}$.*

Proof. The proof runs analogously to the proof of Proposition 3. The only difference is that in all $\mathcal{P} \in \bar{\Omega}$, $\hat{s} \geq 2$ by construction. ■

This result is obtained without considering the assumption of symmetry or any assumption on the nature of the externalities and it applies in all games where coalition structures $\mathcal{P} \in \bar{\Omega}'$ are not admissible. How severe this restriction is, however, depends on the structure of the game used to describe the industry.

6 Discussion

6.1 Extensions and caveats

Throughout the analysis we abstracted from many policy issues that are relevant to the implementation of antitrust laws in the real world: the optimal mix of monetary fines and enforcement; auditing technology; the definition of what constitutes a proof of collusion in front of a court of law; etc. Despite their crucial importance when designing antitrust policies, such issues do not affect the mechanism which drives the anti-competitive effect of the fine in our model. On the contrary, the way we modelled the monetary fine deserves a more detailed discussion.

We considered a "lump-sum" monetary fine. In fact, many antitrust laws only establish an upper bound to the penalty, usually proportional to firms' sales volume.[10] This leaves courts a large degree of flexibility in determining the actual penalty imposed on firms. For instance, if courts were more severe with large cartel, the expected fine would increase with the size of the cartel, creating incentives to form smaller cartels.

On the other hand, a fine which is proportional to sales volume does not modify our analysis, unless the participation to a collusive cartel produces cost synergy. In this case, the greater worth of large cartels would partly derive from cost savings, such that the weight of the fine on members' profit is reduced and the anti-competitive effect of the fine is reinforced.

[10] For example, Article 23 of EU Regulation 1/2003 states that at the outset the monetary fine cannot be greater than 10% of the firm's total annual turnover.

A similar result — increase in competition — could be reached if the antitrust law prescribes penalties increasing with the size of the cartel. Such a penalty scheme results either by providing for a monetary fine linked to the size of the cartel, or by auditing smaller cartels less frequently. In this case, while the basic mechanism driving our results – reacting to the imposition of a fine by changing the coalition structure – is still in place, it would be possible to devise a specific shape of the penalty scheme where firms have *only* incentives to dissolve into smaller cartels.

6.2 Policy implications

The obvious policy implication arising from our analysis is a note of caution for antitrust authorities in introducing large fines or aggressive implementation policies, and in prosecuting small and large cartels indifferently.

As we have already mentioned, antitrust laws provide for a ceiling to the maximal enforceable fine. The rationale behind this rule is to avoid throwing cartel's members into an irreversible financial crisis risking to drive them bankrupt and reduce competition in the industry. Many commentators, however, forcefully argue that the risks of bankruptcy for firms belonging to convicted cartels are limited and should not be a concern when devising antitrust laws, which should only look at the deterrence effect produced by the fine (Buccirossi and Spagnolo, 2007a,b). We offer another reason to justify an upper limit on socially costless monetary fines, which stems from their pro-collusive effects. Increasing the maximum level of the fine risks leading firms towards greater cartels, especially in not very concentrated industries and in boom periods when the market demand is quite high.

A second note of caution relates to cartel detection policy. The shared idea that the antitrust authority should be mainly concerned with concentrated industries and big cartels descends from the empirical (but not well documented[11]) argument that they produce high social losses and are relatively less costly to detect and convict. Our model provides a complementary theoretical justification for allocating the resources of the Antitrust authority in this direction. First, the more the investigated industry (market) is concentrated the more likely the cartel is a quasi-monopolistic one and, hence, the less pronounced are the incentives to form greater cartels. Secondly, by devoting more resources (and effort) to the prosecution of big collusive agreements, the Authority implicitly creates a structure of penalties which decreases with cartel size. This, as we have already noted, provides incentives to form smaller cartels and increase competition.

7 Conclusions

In this paper we challenge the common view that a stricter antitrust law makes the market structure more competitive and prices lower to the benefit of consumers. In particular, we show that when in the market one or more partial cartels are in action,

[11]See Levenstein and Suslow (2006).

the introduction of an expected antitrust fine may lead firms to form coarser coalitions. This anti-competitive effect of the antitrust policy is mainly due to the coalition superadditivity of profits. This element introduces a discontinuity in the effect of the antitrust fine, that can increase competition only if it is so high as to discourage the formation of the monopolistic cartel. However, this is enough to prove that under demand uncertainty the optimal antitrust policy may be *to do nothing* even when enforcement is socially costless.

Appendix: Individual payoffs with the *equilibrium-weighted* sharing rule

In this appendix we go through the details of computing the per-firm payoffs of the Stackelberg oligopoly model presented in Section 5.1.

Under the *equilibrium-weighted* sharing rule, individual payoffs and EBAs are computed simultaneously and recursively, starting from the singleton coalition structure up to more concentrated coalition structures. In our case, there are three levels: singleton, two-player coalitions and the grand coalition. For the singleton structure firms' payoff coincides with the coalition worth. In coalition structures $\mathcal{P}_2$, $\mathcal{P}_3$ and $\mathcal{P}_4$ there is a two-player coalition and a one-player coalition. The share of worth of the two-player coalition depends on their contribution to this coalition with respect to the singleton structure (which is stable by definition).

Let us consider coalition $\{A, b\}$ in $\mathcal{P}_3$. The contributions of firm A and firm b are computed in the following way: take a absolute contribution of each player,

$$\begin{aligned} w_A &= v(A,b) - v(b) = \frac{1}{8} - \frac{1}{36} = \frac{7}{72} \\ w_b &= v(A,b) - v(A) = \frac{1}{8} - \frac{1}{12} = \frac{1}{24} \end{aligned}$$

then, compute the relative contribution,

$$\begin{aligned} \omega_A &= \frac{w_A}{w_A + w_b} = \frac{7}{72} \cdot \frac{36}{5} = \frac{7}{10} \\ \omega_b &= \frac{w_b}{w_A + w_b} = \frac{1}{24} \cdot \frac{36}{5} = \frac{3}{10} \end{aligned}$$

and, finally, with these relative weights we get the share of coalition worth that goes to firm A and firm b:

$$\begin{aligned} \pi_A &= v(A,b) \cdot \omega_A = \frac{1}{8} \cdot \frac{7}{10} = \frac{7}{80} = 0.0875 \\ \pi_b &= v(A,b) \cdot \omega_b = \frac{1}{8} \cdot \frac{9}{30} = \frac{9}{240} = 0.0375 \end{aligned}$$

The payoff of firm A would be the same in coalition $\{A, c\}$ in $\mathcal{P}_4$, with the payoffs of firm b and c inverted.

As regards coalition $\{b, c\}$ in $\mathcal{P}_2$, since the two firms are identical we would expect them to receive the same share of coalition worth. Indeed, following the *equilibrium-weighted* sharing rule we have:

$$\begin{aligned} w_b &= v(b,c) - v(c) = \frac{1}{16} - \frac{1}{36} = \frac{5}{144} \\ w_c &= v(b,c) - v(b) = \frac{1}{16} - \frac{1}{36} = \frac{5}{144} \end{aligned}$$

Hence, the relative contribution of each member in the cartel is

$$\begin{aligned}\omega_b &= \frac{w_c}{w_b + w_c} = \frac{5}{144} \cdot \frac{72}{5} = \frac{1}{2} \\ \omega_c &= \frac{w_b}{w_b + w_c} = \frac{5}{144} \cdot \frac{72}{5} = \frac{1}{2}\end{aligned}$$

and the individual payoffs are

$$\begin{aligned}\pi_b &= v(b,c) \cdot \omega_b = \frac{1}{16} \cdot \frac{1}{2} = \frac{1}{32} = 0.031 \\ \pi_c &= v(b,c) \cdot \omega_c = \frac{1}{16} \cdot \frac{1}{2} = \frac{1}{32} = 0.031\end{aligned}$$

In the case of the grand coalition we need to compute the contribution of each player with respect to a stable coalition structure. The individual contributions to the grand coalition are:

$$\begin{aligned}w_A &= v(A,b,c) - v(b,c) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16} \\ w_b &= v(A,b,c) - v(A,c) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8} \\ w_c &= v(A,b,c) - v(A,b) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}\end{aligned}$$

from which we compute the weights

$$\begin{aligned}\omega_A &= \frac{w_A}{w_A + w_b + w_c} = \frac{3}{16} \cdot \frac{16}{7} = \frac{3}{7} \\ \omega_b &= \frac{w_b}{w_A + w_b + w_c} = \frac{1}{8} \cdot \frac{16}{7} = \frac{2}{7} \\ \omega_c &= \frac{w_c}{w_A + w_b + w_c} = \frac{1}{8} \cdot \frac{16}{7} = \frac{2}{7}\end{aligned}$$

and, consequently, the payoff of each firm in the grand coalition is

$$\begin{aligned}\pi_A &= v(A,b,c) \cdot \omega_A = \frac{1}{4} \cdot \frac{3}{7} = \frac{3}{28} = 0.107 \\ \pi_b &= v(A,b,c) \cdot \omega_b = \frac{1}{4} \cdot \frac{2}{7} = \frac{2}{28} = 0.071 \\ \pi_c &= v(A,b,c) \cdot \omega_c = \frac{1}{4} \cdot \frac{2}{7} = \frac{2}{28} = 0.071\end{aligned}$$

References

Bartolini, D. (2008). Investment strategies and market structure. Available at `http://utenti.dea.univpm.it/bartolini` (a previous version appeared as a University of Essex Discussion Paper No. 628/2007).

Bartolini, D. and A. Zazzaro (2009). Non-cooperative models of coalition formation in games with spillovers. In G. Calcagnini and E. Saltari (Eds.), *The Anticompetitive Effects of Antitrust Policies*, The Fondazione ENI Enrico Mattei series on Economics and the Environment, Chapter 2, pp. 35–79. Springer.

Becker, G. S. (1968). Crime and punishment: An economic approach. *Journal of Political Economy 76*, 169–217.

Besanko, D. and D. F. Spulber (1989). Antitrust enforcement under asymmetric information. *Economic Journal 99*(396), 408–425.

Bloch, F. (1996). Sequential formation of coalitions in games with externalities and fixed payoff division. *Games and Economic Behavior 14*, 90–123.

Bloch, F. (2003). Non-cooperative models of coalition formation in games with spillovers. In C. Carraro (Ed.), *The Endogenous Formation of Economic Coalitions*, The Fondazione ENI Enrico Mattei series on Economics and the Environment, Chapter 2, pp. 35–79. Edward Elgar.

Buccirossi, P. and G. Spagnolo (2007a). Antitrust sanction policy in the presence of leniency programs. Concurrences, No. 4.

Buccirossi, P. and G. Spagnolo (2007b). Optimal fines in the era of whistleblowers. should price fixers still go to prison? In V. Goshal and J. Stennek (Eds.), *The Political Economy of Antitrust*. Elsevier: North Holland.

Carraro, C. (1997). *International Environmental Negotiations: Strategy Policy Issues*. Edward Elgar.

Cyrenne, P. (1999). On antritrust enforcement and the deterrence of collusive behaviour. *Review of Industrial Organization 14*(3), 257–272.

D'Aspremont, C., A. Jacquemin, J. J. Gabszewicz, and J. A. Weymark (1983). On the stability of collusive price leadership. *Canadian Journal of Economics 16*(1), 17–25.

Donsimoni, M. P. (1985). Stable heterogeneous cartels. *International Journal of Industrial Organization 3*(4), 451–467.

Ellis, C. J. and W. W. Wilson (2001). What doesn't kill us makes us stronger: An analysis of corporate leniency policy. University of Oregon.

Fauli-Oller, R. (2000). Takeover waves. *Journal of Economics & Management Strategy 9*(2), 189–210.

Frezal, S. (2006). On optimal cartel deterrence policies. *International Journal of Industrial Organization 24*(6), 1231–1240.

Friedman, J. W. (1971). A non-cooperative equilibrium for supergames. *Review of Economic Studies 38*(113), 1–12.

Green, E. J. and R. H. Porter (1984). Noncooperative collusion under imperfect price information. *Econometrica 52*(1), 87–100.

Harrington, Joseph E., J. (2004). Cartel pricing dynamics in the presence of an antitrust authority. *The RAND Journal of Economics 35*(4), 651–673.

Levenstein, M. C. and V. Y. Suslow (2006). What determines cartel success? *Journal of Economic Literature 44*(1), 43–95.

Marini, M. (2007). An overview of coalitions and networks formation models for economic applications. CREI Working Paper no. 7/2007, Università Roma Tre, Italy.

Martin, S. (2006). Competition policy, collusion, and tacit collusion. *International Journal of Industrial Organization 24*, 1299–1332.

McCutcheon, B. (1997). Do meetings in smoke-filled rooms facilitate collusion? *Journal of Political Economy 105*(2), 330–350.

Polinsky, M. A. and S. Shavell (2000). The economic theory of public enforcement of law. *Journal of Economic Literature 38*(1), 45–76.

Ray, D. (2007). *A Game-Theoretic Perspective on Coalition Formation*. The Lipsey Lectures. Oxford University Press.

Ray, D. and R. Vohra (1997). Equilibrium binding agreements. *Journal of Economic Theory 73*, 30–78.

Souam, S. (2001). Optimal antitrust policy under different regimes of fines. *International Journal of Industrial Organization 19*(1-2), 1–26.

Spagnolo, G. (2000). Self-defeating antitrust laws: How leniency programs solve Bertrand's paradox and enforce collution in auctions. FEEM.

Stigler, G. J. (1950). Monopoly and oligopoly by merger. *American Economic Review Proceedings* (40), 23–34.

Yi, S.-S. and H. Shin (1995). Endogenous formation of coalitions in oligopoly. Dartmouth College Department of Economics WP No. 95-2.