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CONSTANT, DECREASING, OR INCREASING?
A NOTE ON THE RATE OF ECONOMIC GROWTH AND TECHNOLOGICAL PROGRESS, WITH A LITTLE BIT OF HISTORY

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Abstract

There is substantial disagreement regarding the rate of economic growth, particularly concerning economists' perspectives on the rate of total factor productivity (TFP) growth. Some envision theoretical mechanisms implying constant growth, others have recently suggested a potential secular slowdown in TFP growth, while still others foresee an acceleration of economic growth in the near future, driven by the diffusion of new technologies (Artificial Intelligence). Understanding the direction and speed of progress is critically important for planning our future. A part of the research gap is due to the limited temporal coverage of available data sources. In this article, I attempt to bridge this gap by using an alternative (and unconventional) dataset. The analysis is technically straightforward, based on a simple approach that considers learning mechanisms. Although also the dataset used here has its limitations, it allows for an empirical analysis spanning many centuries, if not millennia. The results, thus derived from a historical approach, are very clear and hold significant implications for the future. The key message is that we should expect a progressive slowdown in the rate of scientific progress, although substantial progress will continue for the next few centuries.

JEL codes: O47; O33; N00

Keywords: economic growth; technological progress; historical perspective

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Constant, decreasing, or increasing? A note on the rate of economic growth and technological progress, with a little bit of history[1]

1 - INTRODUCTION

The "state of the art" in economic growth theory appears to encompass a set of contrasting ideas. In addition to the classical exponential growth models (initially exogenous, later endogenous), two conflicting lines of analysis currently dominate the field. The first, more established line, posits the presence of diminishing returns in the production of "ideas" (semi-endogenous models), leading to a decreasing rate of TFP growth. The second line, often associated with the opposing hypothesis of a “law of accelerating returns,” predicts an increasing rate of TFP growth and the potential realization of a “technological singularity” in the near future.

To address this issue, I adopt an approach examining a millennial perspective to capture the full evolution of knowledge growth throughout human history. From a conceptual perspective, I rely on the “learning curve”, which practically suggests that a sigmoid curve best describes a learning process, and I will model the accumulation of human knowledge using a logistic curve (compared with other models). From the empirical perspective, this involves using a dataset that includes a timeline of inventions dating back to the prehistoric era. This approach allows us to account for varying rates of knowledge accumulation across different historical periods.

Calibrating and simulating the model with the given dataset yields several insights. The primary findings support the notion of a slowdown in the production of ideas and, consequently, a deceleration in economic growth.

This work is straightforward and still a work in progress. Consequently, its structure is equally straightforward to describe:

- Section 2 introduces the problem.
- Section 3 outlines a general framework for investigating and analyzing the problem.
- Section 4 presents and describes the dataset used.
- Section 5 discusses the results obtained.
- Section 6 provides the conclusions that close the paper.

[1] I thank Francesco Chiapparino, and the participants at the TIG2024 conference, Gdansk

2 – DIFFERENT IDEAS ON THE PACE OF ECONOMIC GROWTH

Economic growth was at the center of Adam Smith's analysis in the first chapters of his "Wealth of Nations" and since then, albeit intermittently, it has been one of the main topics on the research agenda of economists.

However, the current "state of the art" in economic growth theory resembles a mosaic of contrasting ideas rather than a clear, shared, and uniform field of analysis. While this diversity is partly a natural consequence of the historical evolution of science and societies, it warrants careful consideration. First, let me summarize the main schools of thought and their different implications for the future.

2.1 the standard: exponential growth

Since the introduction of the Solow model of economic growth, empirical observations of the long-term constancy in the growth rate of per capita GDP and TFP have supported an exponential growth model. Initially, this model was exogenous ("manna from heaven"), but it later evolved into an endogenous model, where economic growth is a result of agents' choices regarding the accumulation of knowledge. As well known, there are many contributions classified under the "endogenous growth theory" label, here I limit citations to Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

2.2 Going down: fishing-out and declining growth rates.

After the initial enthusiasm, some authors criticized the so-called "scale effect" (the notion that larger economies should grow faster) implicit in endogenous growth models. They noted that the constant growth of TFP was achieved by an increasing number of scientists involved in the production of ideas. Consequently, they suggested that the productivity of these scientists is decreasing over time, leading to the concept of a semi-endogenous growth model (Jones, 1995 and 2022; Bloom et al., 2020). [2] This perspective is sometimes referred to as the "fishing-out effect," implying that simpler inventions are made first, making it progressively more challenging to develop new ideas over time.

A point worth adding is that several (but not all) works that were concerned with the declining pace of A's advancement focused on specific sectors: electronics, medicine, agricultural research, etc.. In this case, a word of caution is necessary: it is possible to conceive that returns are constant at the aggregate level, but decreasing at the sectoral level: this can happen when "new ideas" consist in

[2] Even if Young (1998) showed that it is possible to have endogenous growth models without scale effects.

particular of new products, and economic growth is explained by the expansion of varieties, while this kind of dynamics is possibly weaker at a sectoral level and null at the product level.

As an extreme hypothesis within this line of thinking, we find Philippon's (2022) empirically driven idea that economic growth does not follow an exponential model but is intrinsically additive. This implies that the rate of growth is decreasing over time, representing a "radicalization" of the semi-endogenous model. The contrast between models suggesting a decreasing rate of growth and classical endogenous growth models is highlighted by the criticism of the semi-endogenous model found in the work of Ha and Howitt. (2007).

2.3 Going up: AI and (next) accelerating growth rates.

In contrast to previous ideas of decreasing growth rates, there is an alternative that imagines a rich, bright, and progressive world driven by the (expected) effects of new technologies. This perspective, supported by some futurologists, suggests that we are approaching a new era in which AI will become a “new superintelligence”. This superintelligence is expected to continually upgrade itself, advancing at an incomprehensible rate and leading us to a “technological singularity”, a hypothetical point where technological growth becomes uncontrollable and irreversible, resulting in unforeseeable changes to human civilization, aligning with Kurzweil's “law of accelerating returns” (2005).

This view of accelerating technological and economic advancement is supported by works analyzing the entire process of technological progress and economic development over millennia (Growiec, 2022; Grinin et al., 2020). These studies suggest that shorter and more dynamic economic phases have historically succeeded one another, and we are poised to enter a further phase of acceleration before the middle of this century. Additionally, recent investigations through economic growth models (Trammel and Korinek, 2023) indicate that transformative AI could drive a singularity characterized by ever-increasing growth rates. This view is further supported by Erdil and Besiroglu (2023), while a more skeptical perspective is offered by Aghion et al. (2017).

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2.4 A synoptic synthesis

We can synthesize the previous debate by employing the standard production function of the R&D sector, as outlined by Romer (1990). According to this framework, the growth of “ideas” can be expressed through the following equation: $g_A = \pi R^{\delta} A^{(\varphi-1)}$, where A is a measure the stock of "ideas", and R the amount of resources dedicated to the production of ideas (usually expressed in terms of labor, i.e. "scientists"); δ, φ, π are parameters.

I focus on the value of φ as the main determinant of the possibility of constancy, slowdown or acceleration of the rate of economic growth, as illustrated in Table 1.

Table 1

Economic growth by decreasing value of ϕ

value of ϕ	$\mathbf{g_A = \dot{A}/A = \cdots}$	**rate of economic growth**	**Basic Reference**
$\varphi>1$	$\pi R^{\delta} A^{(\varphi-1)}$ *N.B.* $(\varphi-1)>0$	increasing in A (explosive growth)	Artificial Intelligence case (Trammel, Korinec, 2023)
$\varphi=1$	πR^{δ}	constant (exponential growth)	Both exogenous and endogenous "traditional" growth theories
$0<\varphi<1$	$\dfrac{\pi R^{\delta}}{A^{(1-\varphi)}}$ *N.B.* $1>(1-\varphi)>0$	"slowly" decreasing	Semi-endogenous growth (Jones, 1995)
$\varphi=0$	$\dfrac{\pi R^{\delta}}{A}$	"rapidly" decreasing	Additive growth (Philippon, 2022)

If we move beyond these more consolidated models, it is worth mentioning the so-called unified growth model. Unified growth theory (Boucekkine et al., 2003; Galor, 2011) predicts an initial period of low growth, followed by a phase of high growth rates, and culminating in a final deceleration. A key feature of this model is the sudden transition between the first and second stages, driven by the presence of a threshold linked to demographic dynamics.[3]

3 – PUTTING (ALMOST) EVERYTHING INTO THE BOX

One fact is that the experience and pace of economic growth are not homogeneous phenomena; they have varied across different historical periods and geographical regions. This variability suggests

[3] We will see in section 4.2 that our results do not support the "break" hypothesis.

that a single model may struggle to accurately describe such a diverse and evolving situation. To address these challenges, I rely on the concept of the “learning curve.”

The learning curve, also known as the “experience curve,” describes the relationship between a learner's performance and the number of attempts or the amount of time required to complete a task.

It was initially referred to the behavior of an individual, but it was then applied to large organizations (Argote and Epple, 1990; Dar-El E.M., 2013; Grosse et al., 2015; Henderson, 1973; Peltokorpi and Jaber, 2020).
In general, the learning/experience curve can be adequately described by a logistic, sigmoid curve, although it is not the only possible representation. This learning curve is typically considered within the context of a specific, limited technological environment. However, can it be applied to the process of knowledge accumulation for an entire society?
According to Landes (1980, p. 111), “the heart of the whole process of industrialization and economic development is intellectual: it consists in the acquisition and application of a corpus of knowledge”; the online Oxford Dictionary defines science as “the intellectual and practical activity encompassing the systematic study of the structure and behavior of the physical and natural world through observation and experiment”. In sum, science advancement, with its consequent process of economic growth, consists of a learning process.
Following these lines of thought, we can consider the natural environment as the "given environment" that serves as the subject of humankind's learning activities. Inventions are applications of chemical, physical, and biological laws already existing in nature, which must be discovered and utilized. This concept aligns with the general notion of the learning curve, particularly if we assume that the quantity of natural elements and laws is finite. [4]

Moreover, it's evident that the learning process of humanity extends far beyond the past couple of centuries; it spans millennia. Consequently, we should contemplate a process of growth over very long historical periods, as previously proposed in some analyses (Growiec, 2022; Grinin, 2020).

Since the key variable in all growth models is "A," closely related to scientific progress, another valuable literature to consider is the SciSci (Science of Science) literature. This field aims to directly measure the productivity of science (scientists). It is an expanding area of analysis,

[4] On this last point, just to give some examples on a question of which I am completely unaware, I read that while the periodic table of elements continues to expand (slowly), physicists think it is probably limited. Even in the case parallel universes (“multiverse”) exist, physicists think that they are “all bound by the same laws of physics” (Swain, 2017, p. 12).

leveraging digital data on research inputs and outputs to explore the structure and evolution of science.

One of the most shared conclusions of this literature is that "papers and patents are becoming less disruptive over time," as indicated by the title of Park et al. (2022), while a useful synthesis is Clancy (2023). Analysis of millions of scientific articles shows that while the number of science and technology research papers has astonishingly increased in recent times, their ‘disruptiveness’ has declined. Today, scientific advancements are increasingly incremental rather than disruptive.

The ultimate determinants of this trend remain unclear, possibly linked to the "fishing-out" mechanism previously discussed, or to institutional aspects such as how research is rewarded (citation mechanisms) and patents granted.
This trend also appears to be associated with the rise of large, increasingly geographically dispersed research teams, as noted by Lin et al. (2022), Wu et al. (2019), and Wutchy et al. (2007), and, interestingly, large teams are often found to be less innovative than smaller teams.

In parallel with scientific articles, patent data are often utilized, potentially offering rich insights. Nevertheless, there are doubts about patents' ability to accurately measure scientific progress, leading to an investigation into a "patent crisis" characterized by low-quality patents: technologies vary greatly in their nature, and existing patent protection systems seem unable to adequately account for this diversity, often over-rewarding some technologies while under-rewarding others. Additionally, it's worth noting that many innovations are not patented, with only around 10% receiving patents according to Fontana et al. (2013).

Furthermore, both scientific literature and patent data have limited time availability compared to the long-term perspective of significant scientific progress. While patent data are accessible from around 1980 and scientific articles from around 1950, consider, for example, the significant scientific advancements made about four centuries ago, between the mid-1500s and the mid-1600s, by figures such as Copernicus, Galileo, Kepler, and Newton.

4 - DATA

4.1 The dataset and its description

To address some of the aforementioned challenges and to provide an alternative source of information, I utilize data that documents relevant inventions and discoveries over time. This type of data is essential when conducting analyses over a multi-secular timeframe, as is the case here.

For the current version of my work, I have used an uncoventional dataset, a timeline of human inventions and discoveries from a book intended for the general public (Bridgman in association with the Smithsonian Institution, 2006).[5] The list spans the entirety of human history, from the first stone tools (3 million years BCE) to the present day, encompassing 1007 items. While it primarily includes elements that can be classified as "scientific," some entries represent social innovations. However, these latter items constitute only a small fraction, and I have excluded them (39 items): the dataset was thus reduced to 968 innovations and discoveries.[6]

There are two limitations to consider. First, although the dataset includes inventions and innovations of varying impacts, assigning appropriate weights to them is challenging. For instance, how does one compare the significance of superphosphate fertilizer to that of the adding machine, both from the early 19th century? Similarly, is the theory of vision from 1000 CE more or less important than the wheelbarrow from 200 CE? Moreover, can the stained-glass window from 800 CE be fairly compared to the boomerang from 19,000 BCE? It is likely impossible to establish such a precise metric. Second, accounting for depreciation rates due to technological obsolescence for such a diverse range of items would be equally impractical. As a consequence, throughout the paper, I will simply use the stock of inventions/discoveries at any time, as a proxy for the cumulative knowledge: $A_t = \sum_{t=1}^{T} I_t$, where A is the stock of "ideas", computed, at any time, as the sum of all the inventions/discoveries from the first year and until that date (similar approach, with different data, in Park et al, cit.).

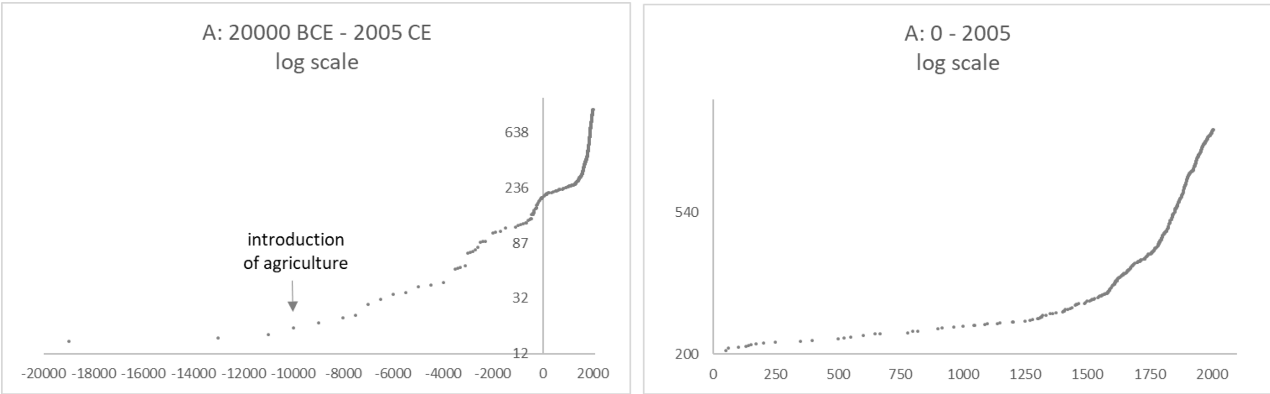
Given the unconventional nature of the database, it is appropriate to examine some aspects of it, at least in a preliminary manner, to ensure their coherence with contributions from the history of technology.

First, Figure 1 presents the value of the stock of A over time. Data are shown for two extensive periods: from 20,000 BCE and from year zero. Although it is theoretically possible to use the entire dataset, starting from 3 million years BCE, the resulting figure becomes nearly illegible, resembling an "L" rotated 90° to the left, lying on what is usually the vertical segment of the L.

[5] A new, richer, and more updated dataset will be used in the future, as soon as it will be available.
[6] The results are, however, completely unchanged whether the full or reduced dataset is used.

Figure 1

Stock of *A*, from 20000 BCE and from 0



From the first panel, we can observe the significant boost to inventiveness following the Agricultural Revolution around 10,000 BCE. There are some fluctuations, which may not be significant; a more comprehensive analysis with a richer dataset, as planned, could clarify this point.

From the second panel, we see a continuous increase in the stock of ideas since year 0. However, the growth rate (slope of the curve) remained steady and weak until the Middle Ages. Starting around 1300[7], there is a noticeable acceleration in growth continuing until 1800. Third, there is a steady and sustained growth (with a roughly constant slope) after the Industrial Revolution. It is important to note that this stage appears to be the culmination of a long historical process. Finally, there may be a slight weakening of growth in recent years.

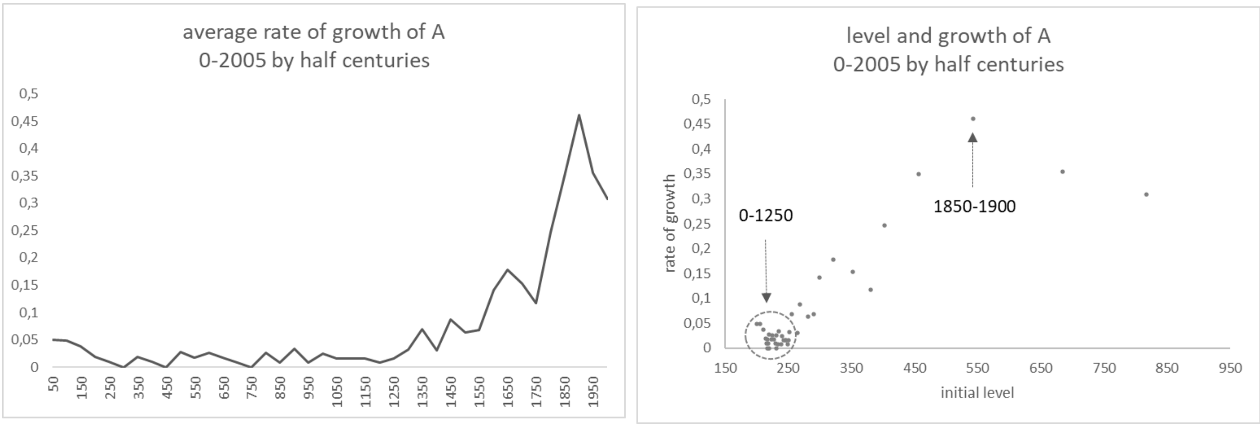
These results, while sharing some aspects with the unified growth theory, exhibit a notable difference: there is no sudden break. Unlike the cited theory, the transition from low to high growth rates appears gradual, with this progressive acceleration lasting about five centuries. This gradual transition might not surprise historians but is generally less recognized by economists, who often consider “modern economic growth” as a distinct era with specific characteristics.

To highlight the dynamic aspects of the evolution of the stock of knowledge, Figure 2 presents the growth rate of A. The first panel shows the historical evolution of this growth rate, while the second panel compares the growth rate to the level of A itself. In this second case, we are measuring something that is related to the parameter φ of the theoretical models.

[7] According to Crosby (1996) one relevant new feature was the growing process of “quantification” that started to be implemented in Western Europe after 1200.

Figure 2

growth of A, CE years



In the first panel, we observe that the rate of growth remained roughly constant and very low until about 1300. After this point, it progressively increased, peaking between 1850 and 1900. In the last two periods (1900-1950 and 1950-2005), the growth rate decreased, although it remained at high levels. This trend is a notable result that is only weakly evident in Figure 1. It's important to note that the real and continuous decline is concentrated after 1950. In the first half of the 20th century, the decrease is due to a temporary drop in the first 25 years, with the average growth rate in the second quarter of the 20th century being similar to that in the last quarter of the 19th century. This dynamic is obscured by the choice of 50-year periods. Generally speaking, the post-1950 decrease aligns with claims of a productivity slowdown in the second half of the 20th century, particularly after 1960 (Nordhaus, 1972; Erber et al., 2017; Baily and Gordon, 1988).
Interesting conclusions can be derived from the second panel. Two aspects are worth mentioning:

1. There is no relationship between the level and the rate of growth of A until 1250, as evidenced by the cloud of data points at the bottom left.
2. The relationship does not appear to be univocal. There is a long period of positive relationship where the rate of growth increases with A, followed by a period of negative relationship where the rate of growth declines at high levels of A.

The second panel's evidence may perhaps be interpreted as a change in φ: it is possible that φ was greater than 1 from 1300 to 1850 (indicating an increasing rate of growth in A) and less than 1 after 1850.

As a second step in describing the dataset, the entire dataset is divided into "significant" sub-periods. Historical thresholds are subjective, and my chosen divisions are as follows:

- from the most ancient times (Stone Age) to 800 BCE. This last year is the end of the bronze age, also significant because it is considered the beginning of classical Greece (archaic age).
- the second sub-period is from 800 BCE to 632 CE. This period of approximately 8 centuries corresponds to the Mediterranean civilization (the last year identifies the death of Muhammad and the beginning of the Arab expansion); moreover, it also roughly corresponds to the period of the Persian empires, whose start date is traditionally set in 550 BCE (Cyrus's overthrow of the Median kingdom, with the following conquest of Babylon in 539 BCE) and the end in 637 CE (when the Persians were defeated by the Arabs at Al Qadisiya).[8]
- the third spans from 632 to 1454, the year of the fall of the Eastern Roman Empire or, better, Byzantine Empire (formally the year of the conquest of Constantinople by the Ottoman Turks led by Muhammad II). Around the middle of the 15th century, two other dates are relevant: in 1433 the Chinese ocean expeditions on behalf of the Ming emperor ended; in 1492 America was discovered. Consequently, 1454 identifies the central date in this crucial period.
- a fourth sub-period goes until the beginning of the Industrial Revolution in 1760 (the year of the beginning of the kingdom of George III, as proposed by T. S. Ashton)
- Finally, I decided to use 1914 as the threshold year to witness the passing of the baton between Europe and the United States

Geographically, 5 areas are identified, plus a residual, relevant only in the first periods:

- *Asia* includes all Asian countries, from the Middle East to China, even if a finer division could be proposed.
- *Mediterranean* includes Rome, Greece, and Egypt, even if the latter country was initially connected to the civilizations of the Asian "Fertile Crescent", and only later close to the Greek and Roman experiences.
- *Europe* is any other country but Rome and Greece
- It is obvious what *USA* means
- Finally, the *Other* category includes all other items: those from the rest of the world, and not geographically classified items (very important initially)

[8] With the hiatus of the period of Alexander the Great

A presentation of the data, following the historical and geographical criteria just exposed, is in the next table 2:

Table 2

historical geographical distribution of A

period	Asia	Mediterranean	Europe	USA	Other
until 800 B.C.	31,0	23,8	8,7	0,0	36,5
800 BC-632 AD	26,6	63,3	3,7	0,0	6,4
632-1454	36,8	3,5	49,1	0,0	10,5
1454-1760	0,0	0,0	93,2	1,5	5,3
1760-1914	0,0	0,0	70,3	21,8	7,9
1914-2005	0,8	0,0	27,8	57,1	14,3
geographically identified values = 100					
until 800 B.C.	48,8	37,5	13,8	0,0	
800 BC-632 AD	28,4	67,6	3,9	0,0	
632-1454	41,2	3,9	54,9	0,0	
1454-1760	0,0	0,0	98,4	1,6	
1760-1914	0,0	0,0	76,3	23,7	
1914-2005	0,9	0,0	32,5	66,7	

From the values of Table 2, it emerges that the scientific progress was largely shared by Asia and the Mediterranean area, from the beginning until about 600 CE, and, with a more general perspective, by Asia and Europe until about 1500. Only successively there has been a strong concentration, with a reversal of positions between Europe and USA after WWI.

4.2. Simple notes on the productivity of innovative activity

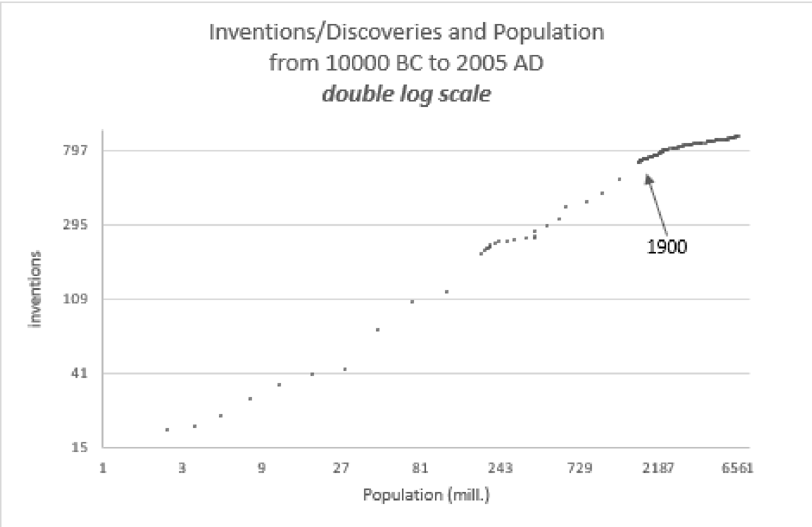
A further step is to highlight the relationship with the population by introducing a measure of productivity. While we have reliable data concerning the world population for recent decades, estimates for past centuries are sparse; I use a synthesis presented in OWID (Roser et al, 2023), which considers three already existing datasets: the History Database of the Global Environment, or HYDE 3.2 (10000 BCE - 1799 CE), Gapminder (based on Maddison), version 6 (1800 – 1949), and UN World Population Prospect (1950 on).

In Figure 3 the stock of *A* and the population are matched, with data presented with a double proportional scale.

The linkage between the size of the population and the "profitability" of inventions has been stressed in some past papers. Figure 3, where the slope of the curve is a measure of the elasticity of inventions to the population, does not seem to evidence the existence of a "threshold" after which the innovation exploded (as proposed in the "unified growth theory"); rather, the evidence seems to suggest that the linkage between the two variables evolved gradually, with an apparent constant elasticity until a level of population of about 1500 million, roughly corresponding to 1900, a slope bending after 1900 (and more after 1950): with all the necessary precautions, these data also seem to confirm the existence of some difficulties in producing, in recent times, advancement of knowledge with the same intensity as in the past, and a need to increase society's efforts to maintain a sustained pace of innovation

Figure 3

Inventions and population over millennia



In Figure 4, I then present a rough estimate of "productivity" in terms of inventions, measured as the production of inventions relative to population. The production of inventions is simply the difference in the stock of A over a time interval (A_t - A_{t-n}), while the population is the average population of the same period. I present this productivity measure for two different time intervals: 100 years and 10 years. The second interval focuses on a relevant feature highlighted by the first for the most recent period. Given the limitations of historical population data, the 100-year interval analysis begins from year 0, providing about a millennium of observations. The 10-year interval analysis is limited to the 20th century. The results are very clear:

1. Productivity was positive but low and roughly stable until about 1200 (first panel).
2. Productivity increased after 1200 but declined in the last century (first panel).
3. The final decrease, as shown in the second panel, is concentrated almost entirely in the years following World War II.

Figure 4

Productivity for time intervals

5. Calibration: the past and the future of knowledge

In principle and following the idea of the input of the "learning curve" we could model the whole process of accumulation of growth through a logistic curve.

To test if this kind of hypothesis holds, I explored several periods of different lengths: 30000 BCE - 2005 CE; 0-2005; 1000-2005; 1300-2500; 1600-2005; 1800-2005.
In any period, I calibrated the data, comparing four possible scenarios: one with a variable rate of growth of A, according to a *logistic model*; the second with a constant rate of growth of A (*exponential model*); the third with a positive but decreasing rate of growth of A over the whole analyzed period (*semi-endogenous model*); the fourth with a positive and progressively increasing rate of growth (*explosive model*),
In sum: 6 periods and 4 models.

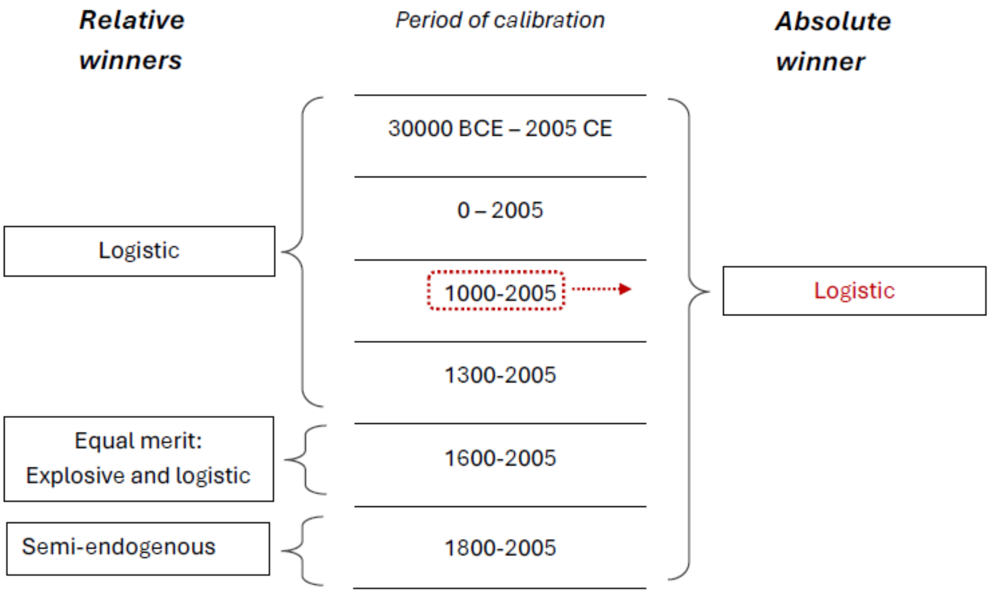
I used a simple iterative manual process in which the relevant parameters were adjusted to find the best fit. This best fit was selected by minimizing the root-mean-square deviation (RMSD) of the differences between the historical and simulated series. I aimed to identify:

1. The best fit among models for a given period (relative winner).
2. The best fit across different periods and models (absolute winner).

Some of the results were expected, but overall, they are intriguing and provide valuable insights. These findings are summarized in Table 3.

Table 3

Results of calibration

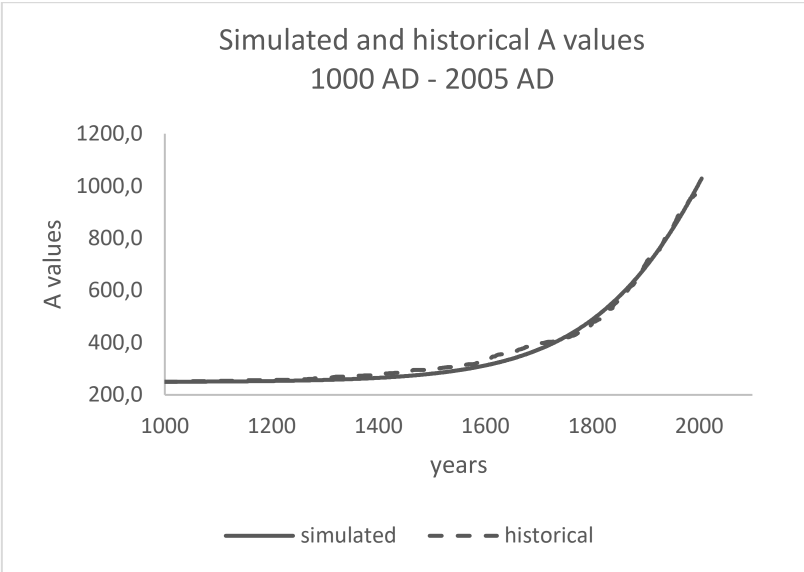


1. The constant rate model (exponential model) consistently fails to outperform alternative models in all simulated periods. This finding is somewhat surprising (at least to me).
2. The logistic curve consistently performs better than other models, except for two instances: it shares the best performance with the explosive model in the period from 1600 to 2005, and, notably, it is outperformed by the semi-endogenous model in the final period 1800-2005.
3. The simulation using the logistic model from 1000 to 2005 emerges as the absolute best fit across all models and periods. This finding is significant given the extensive period covered.

In summary, both the logistic and semi-endogenous hypotheses appear to be strong choices. However, while the semi-endogenous model seems particularly useful for describing recent periods, the logistic model provides a more accurate depiction of the long-term evolution of knowledge accumulation. This last result serves as the basis for the following Figure 5

Figure 5

Calibrated stock of A - best fit with the logistic

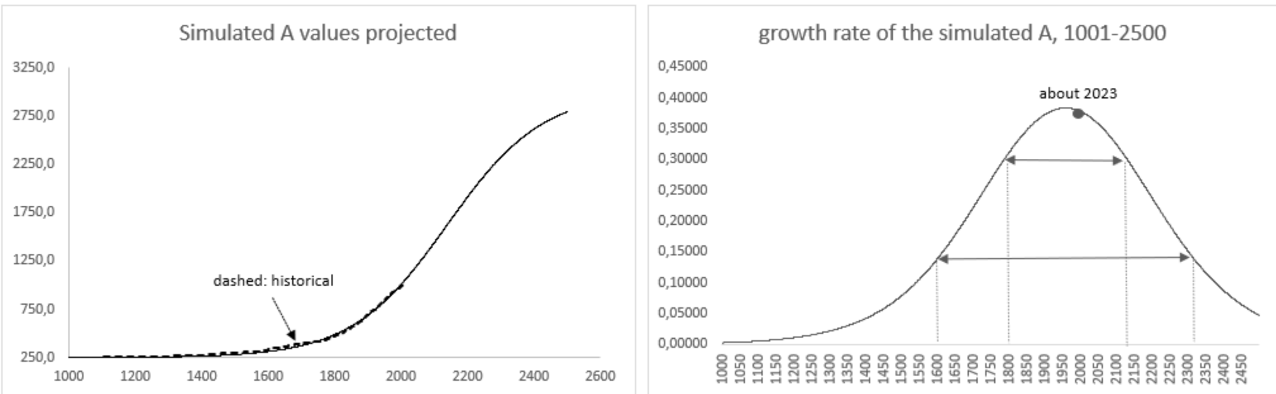


It is easy to see that the fit is very good[9]. I don't know if the fact that 1000-2005 comes out as the best fit has any particular meaning. However, I can suggest that, according to some historians, medieval Europe saw a radical change in the rate of new inventions and innovations.

Now, we can look ahead to the future with a simulation of future values obtained by extending the fitted logistic curve 500 years into the future. The results are intriguing.:

Figure 6

Logistic: past and future level and growth of A (1000-2500)



Continuing with our analysis, we can examine, in Figure 6, the future projections based on the simulations. The first panel provides an overview based on the level of A: it suggests that A will

[9] This fit, realtive to a logistic equation where A_{max} is tha maximum achievable value of A and g is the growth rate, comes from values of $A_{max} \approx 3000$, and g≈0,007

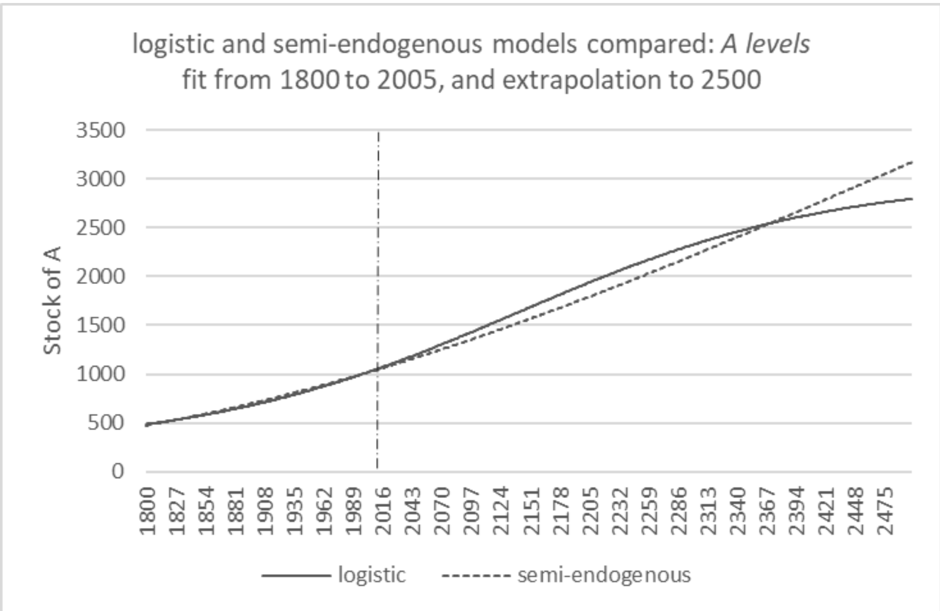
continue to grow, albeit at a progressively slower pace. Determining the inflection point precisely is challenging. Therefore, focusing on the simulated growth rate A provides clearer indications: the turning point occurs in the second half of the 1960s, with the growth rate still close to its maximum in 2023.

According to this simulation, by 2100, the rate of growth of A is projected to approach the speed seen around 1800, and by 2300 or so, it would approximate the relatively low growth rates observed in 1600. However, it's important to note that by these dates, the stock of A would be significantly higher than the levels of today.

To shed more light on these projections, I decided to compare the extrapolation of the logistic model over the period 1000-2005 with that of the semi-endogenous model fitted over the period 1800-2005. Despite being calibrated over different periods, both models exhibit very good fits, with similarly low distances from the original values. We can compare these models over their overlapping period (1800-2005) and also extend their projections into the future. This comparison is presented in Figure 7, where the vertical dashed and dotted lines denote 2005, the endpoint of the historical series.

Figure 7

Logistic and semi-endogenous hypothesis (from their best fits): comparison of A levels for the calibrated and the simulated periods



The comparison yields clear results:

1. Both models show very similar levels during the calibrated period 1800-2005, as expected since they were chosen for their best fit, despite being calibrated for different historical periods.
2. While they exhibit different shapes and progressions, there is a limited deviation between the two models for an extended period of time.
3. Significantly, they begin to diverge notably only after 2370, and even then, their values remain relatively close through 2400.

In summary, while we cannot claim complete overlap between the two models for the analyzed period, they generally portray a similar narrative and predict comparable scenarios for the next approximately 400 years.

6 - Conclusions

This simple study, which can be viewed as an exercise, suggests that we can still anticipate future growth at the scientific frontier. However, the pace of progress may progressively slow down, which differs from the historical experience and perception following the Industrial Revolution. In historical data, the stock of A in 2000 was just over three times that of 400 years earlier. The extrapolation exercise in the final section, also suggests that, despite progressively decreasing growth rates, the same stock could be, in 2400, only slightly less than three times that of 2000. Therefore, we can conclude that the "scientific distance" separating 1600 from 2000 will be approximately the same as that between 2000 and 2400. This finding is quite reassuring.
Why is this reassuring? An intuition of the meaning and scope of this "technological distance" can be gained from the comparison between two pandemics: the plague of the 1600s and COVID-19 in 2019. Despite the different infectious nature of the two, it is clear that the duration (almost the entire century for the plague), the mortality rate (for example, about ¼ of the population in Northern Italy died in the early 1600s), awareness of the causes, and the availability of remedies were radically different (at the beginning of the 17th century, the existence of bacteria and viruses was still unknown). Therefore, even with an alleged slowdown in the pace of advancement of science, the scientific level of the future will be radically different from the current one, probably contributing to a much better standard of living for the world's population.
Nevertheless, a word of caution is instead the consideration that if population growth stimulates inventiveness, the probable reduction in the world population starting from the end of this century

could lead to a sensible reduction in the growth rate of the advancement of knowledge.

What remains to be considered? The possibility of an imminent "technological singularity" and the recovery or even acceleration of the growth rate of the scientific frontier. This is a prediction and, while not considered probable by many authors and not supported by current data (which are, however, backward-looking), it remains a possibility. Given that the time horizon within which this should occur is, according to proponents, a few decades, many will be able to witness it directly.

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