Università Politecnica delle Marche
Dipartimento di Scienze Economiche e Sociali

Risk and Strategic Complementarities: Banks Behavior, Supervision and Macroprudential Policies

T. Carraro E. Gaffeo M. Gallegati

QUADERNI DI RICERCA n. 452
ISSN: 2279-9575

February 2021

Comitato scientifico:

Giulia Bettin
Marco Gallegati
Stefano Staffolani
Alessandro Sterlacchini

Collana curata da Massimo Tamberi

Abstract

In this paper we present a model where frictions in the supervision process may set the stage for strategic complementarities among banks. We derive the conditions for strategic complementarities in the behavior of banks in a banking system in which the supervisory authority has a budget constraint on the resources to allocate for monitoring, and supervision is costly for banks. In such a framework, the goal of macroprudential policies consists in simultaneously restraining the incentive of banks in extending risky loans, without forcing the system towards a corner solution where all or none of the banks provide credit. We point out that the countercyclical buffer is a proper tool to reduce the number of banks issuing a higher amount of credit during booms, while a loan-support-program can increase the number of banks issuing higher credit during downturns.

JEL Class.: C49, E44, G32
Keywords: Banking Crisis; Strategic complementarity; Macroprudential Supervision

Indirizzo: Marco Gallegati (corresponding author), Department of Economics and Social Sciences, Universit Politecnica delle Marche (Italy). E-mail: marco.gallegati@univpm.it

Risk and Strategic Complementarities: Banks Behavior, Supervision and Macroprudential Policies

T. Carraro *E. Gaffeo* *M. Gallegati*

1 Introduction and Related Literature

After the crisis, there was a wide consensus regarding the financial system need of a new macro-prudential framework to resist procyclicality. Some proposed a more active role of monetary policy to address financial imbalances (White 2009, Taylor 2010). Others suggested a regulation reform which introduced new macro-prudential tools (e.g. leverage control, an additional capital buffer or a mandatory insurance) to bridle the credit fluctuations and to increase the resilience of the financial system (Lowe and Borio 2002, Repullo and Saurina 2011, Hanson et al. 2011, BoE 2011). Both the solutions had a common denominator: identify an efficient way to control and limit the exaggerated fluctuation of the credit in the economy.

The political agenda after the crisis included both the solutions. Central banks returned to their roots by re-assuming a broad mandate that encompassed the financial stability, and, many of them, started the so-called unconventional monetary policy characterised by the worldwide use of *the Quantitative Easing* (QE[1]) (Fawley and Neely (2013)). On the other hand, the regulatory authorities of all advanced economies have already introduced macro-prudential instruments in their toolbox e.g. the countercyclical buffer. The main goal of these measures is not only to narrow the gap between the banks' perceived risk and the real risk (Borio and Zhu 2012, Shin 2010, Borio et al. 2001, Brunnermeir and Krishnamurthy 2014), but also to reduce the strategic complementarity forces that are present in the bank market both in an economic boom and in a crisis period (Shin 2011, Danielsson and Shin 2003, Aikman et al. 2015).

The macro-prudential approach to regulation has to consider the systemic implications of the collective behaviour of financial firms. Specifically, it is about identifying those endogenous processes that turn heterogeneity

[1]Although the beginning the scope of *QE* was more general, at a later stage, the ECB and the Bank of Japan (BoJ) used the QE whit the scope of boost the credit in the economy. The BoJ started *Loan Support Program* (LSP) in 2012 (Fawley and Neely 2013) while the ECB started *Targeted longer-term refinancing operations* (TLTRO) in 2016 (Bucalossi et al. 2016, Alvarez et al. 2017).

banks decisions into homogeneity and make the financial system more fragile. As a corollary, the macro-prudential paradigm stresses the possibility that actions that may seem desirable from a bank's point of view may result in unwelcome system outcomes. There are several examples of such fallacies of composition. For instance, for a single bank it is natural to tighten lending standards in a downturn, but if all banks do the same the resulting impact on economic activity can cause a further deterioration in the credit quality of its portfolio. The mirror image during the upturn could generate an unsustainable credit boom, sowing the seeds of following financial turmoil (Crockett 2000). In last few years, researchers have made the maximum effort to respond to the multitude of questions that the introduction of macro-prudential approach has raised, however several questions are still open and the literature arguing these topics is growing fast (Brunnermeier et al. 2009, Claessens 2014, Schoenmaker 2014, Freixas et al. 2015).

The idea proposed in this study is close related to the findings of Aikman et al. (2015) which show how several frictions, such as risk shifting, reputational concerns and moral hazard behaviour of economic agents, can generate strategic complementarity behaviour among banks. The general idea is that if any bank system fragility triggers a strategic complementarity behaviour among banks, then each one takes into account the decisions of all others. In the case that the strategic complementarity is strong enough, banks decisions could be led by lemmings, losing the connection with the economic fundamentals.

In our model we assume that CBs supervision may suffer from rigidities that, in turn, can generate strategic complementarity among banks. In particular, in a banking system in which the supervisory authority has a budget constraint on the resources to allocate for monitoring, and supervision is costly for banks, we derive the conditions for strategic complementarities in the behavior of banks. The model presented in this study follows the standard framework explained and intensively used in the literature to describe strategic complementarities[2]. We can resume the original contributions of this work in three main points. First, we point out that the simultaneous presence of constraints on supervision that makes it not flexible and the fact that monitoring is costly for banks, springs strategic complementarities among banks. Secondly, the countercyclical buffer enables the authority to

[2]This type of model it is a handy way to take into account the interdependence decisions through the players. The micro-foundations of this class of models is obtained through the balance sheet of financial intermediaries, and it allows explaining the movement observe on credit, leverage and asset prices (Geanakoplos 2010, Kiyotaki and Moore 1997, Adrian and Shin 2013, He and Krishnamurthy 2013). The model follows the standard framework explained and intensively used in the literature to describe strategic complementarities, for instance attacking a currency (Morris and Shin 1998), refusing to roll over debt (Morris and Shin 2004), participating in a run on a bank or SIV and declining to renew a certificate of deposit in the interbank market (Rochet and Vives 2004) and the credit freezes during the latest crisis (Bebchuk and Goldstein 2011).

affect the banks incentive of issuing more credit. Thirdly, accordingly with Stein (2012), the CB can use the monetary policy to affect the probability of a credit crunch or boom. All in all, the model here proposed can be used to provide new evidence on how the new macroprudential tools introduced after the crisis are useful to manage the strategic complementarity forces in the bank system. Indeed, one of the purposes of these measures is to prevent that the equilibrium of the bank system escalates towards a corner solution where all or none of the banks provide credit.

The paper is organized as follow. Section 2 briefly summarizes the literature studying CBs banks supervision, its effects, and why its rigidities can generate strategic complementarity among banks. Section 3 presents the model, and section 4 is dedicated to discussing the CB problem, the general framework of regulation, and the presentation of macro-prudential instruments. Section 5 concludes.

2 How supervision can generate strategic complementarity among banks

In this section, we present briefly the literature that studies banks' supervision, its effects, and why its rigidities can generate strategic complementarity among banks. Two assumptions are needed in the model to observe strategic complementarity. The authority in charge of the banks' supervision has to have a budget constraint, and the supervision has to be costly for supervised banks. If these hypotheses are acceptable, then it is trivially understand why there is strategic complementarity among banks. Indeed, the supervision authority cannot effort to supervise all banks that it would, and each bank knows that higher is the number of banks that should be supervised, lower is the probability to be supervised.

Academic works studying bank supervision are quite limited, reflecting the opacity of supervisory activities which results from supervisors reliance on confidential information. Regarding the economic impact of supervision, there are few recent contributions as Rezende and Wu (2014), Eisenbach et al. (2016), Hirtle et al. (2016), and Ivanov et al. (2017) in which the focus is on how the supervision affects the performance and the behavior of banks. The results suggest that an improvement of Continuum Monitoring supervision decreases the risk, as measured by less risky lending, lower earnings volatility, and more conservative accounting practices.[3] supervision

[3]Banks supervision can be divided into two areas (Eisenbach et al. 2017). The Continued Monitoring activities which are intended to enable each firm-focused supervisory team to develop and maintain an understanding of the organization, its risk profile, and associated policies and practices (BGFRS 2015a), as well as to identify gaps or issues that might lead the team to do a more in-depth analysis. In contrast to Continuous Monitoring, which consists of on-going, repeated activities of the firm-focused supervisory teams, the Enhanced Continuous Monitoring and Formal Examinations involve discrete

decreases the risk as measured by less risky lending, lower earnings volatility, and more conservative accounting practices. Kandrac and Schlusche (2017) point out how that, absent routine and ongoing supervisory oversight and examination, banks seek to accumulate riskier lending portfolios. The expansion in risk-taking goes hand in hand with a delay in the resolution of failed institutions, so the consequence of low supervisory capacity results in a higher incidence of failures, as well as more costly failures. Therefore, the Continuum Monitoring may lead to a positive spillover effect which increases banks efficiency, but, by limiting banks risk taking, it may also reduce the expected interest rate by shareholders. As Ivanov et al. (2017) figure out, if banks perceive costs of supervision exceeding its benefits, they may attempt to avoid oversight. This potentially reduces the effectiveness of supervision. Concerning the effect of Enhanced Monitoring, as shown by Eisenbach et al. (2016)it can be costly for banks, and usually ends with fines and costly decisions. Moreover, it could generate a stigma effect which reduces its credibility and in turn increases internal and external costs of capital.

Notwithstanding the authoritys budget and how it is used represent confidential information, we may get (extract) some information from Staff Reports of The Federal Reserve of New York.

> *The goals of the planning process are to identify the key supervisory objectives for the coming year – for example, what areas or topics will the team analyze in depth or what topics will be the key focus of continuous monitoring and enhanced continuous monitoring and to ensure that there are sufficient resources to achieve those objectives. Resources are often constrained in the sense that there is often more work that could be done than there is time or staff to do it. There is also a desire to leave space in the supervision plan to deal with for unforeseen developments so that these do not crowd out other important work.* (Eisenbach et al. 2017)

supervisory projects that are generally conducted on a one-time basis. The Enhanced Continuous Monitoring is intended to provide insight into a particular topic, business strategy, risk levels and risk management practices and controls. It can also be used to learn more about an area or fill a knowledge gap. When the information-gathering and analysis conducted reveal deficiencies in a firms risk management, governance, or other controls, or if a firm is found to be in a financial condition that threatens its safety and soundness, then supervisors can take various actions to compel the firm to address the deficiencies. From the mildest of these supervisory actions: matters requiring attention (MRAs), matters requiring immediate attention (MRIAs), Memoranda of Understanding (MOUs) and the formal supervisory actions known as 4(m) agreements (4Ms). These are all the confidential actions while such as written agreements, cease and desist orders, and fines are publicly disclosed by the Federal Reserve Board. These formal supervisory actions have legal force. If should the firm fail to meet the terms of the action, it can face fines and other actions, such as a requirement to restrict its growth or to divest certain assets.

It is evident how the supervision authority suffers from a budget constrain which does not allow it to set the supervision freely. Furthermore, Figure 1 displays the evolution of Federal Reserve employees in supervision and regulation both in absolute and relative terms. The workforce was steady from 1995 to 2006 despite before the crisis the financial system was not only growing but becoming more complex. This indicates that during an upturn phase the regulatory authority is not incentivized to increase supervision and the budget allocated to it. There are several possible explanations for this behavior. For instance, when economic and financial indexes are good the authority may suffer from a positive bias of the specific and systemic risk, and face political pressures to have a relaxed supervision to enhance the credit.

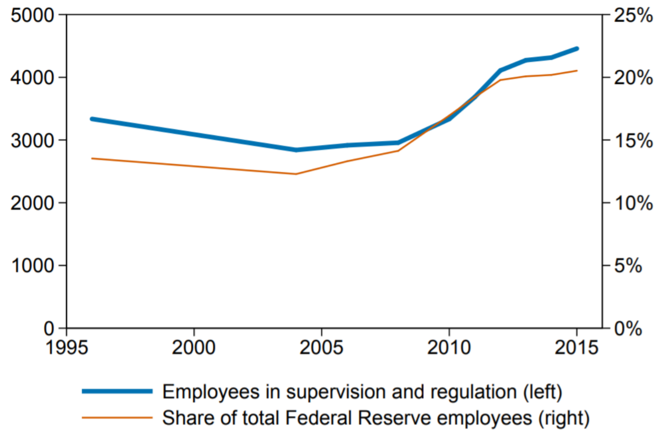


Figure 1: Federal Reserve supervisory staff. (Source: Eisenbach et al. 2016)

Finally, as explained by Dudley (2014) and Dudley (2016), banking supervision works in concert with regulation as a more flexible element of banking policy. The accounting standards and the processes for identifying, measuring, monitoring and controlling risks can influence the result of supervision. In particular, a tighter regulation can reflect an authority which makes more punctilious supervision, and vice versa for a less stringent one. However, it was the crisis that pushed the FED, the European Central Bank (ECB) and, the vast majority of the Central Banks (CBs), to reform the accounting standards and the process of risk evaluation, by making them more stringent, while no intervention has been adopted in the early 2000s during the boom phase (BIS 2015, ECB 2014, ECB 2017).

Given the above considerations, it is reasonable to assume that there are some rigidities on supervision authorities. More specifically, they suffer a budget constraint that does not let them make a timely adjustment of the supervision resources.

3 A simplified bank credit model

If the authority cannot adjust the amount of supervision, as it suffers from a budget constraint, and the supervision is costly for banks, then there is the proper framework for a banks' strategic complementarity behavior. Indeed, if the number of banks that has to be supervised increase then the bank's probability to be supervised decrease. This, in turn, reduces the expected cost of supervision and therefore it boosts the banks' incentive to take risks. In this section, we present a general framework for analysis, including the game played by banks managers which causes the strategic complementarity within the model.

3.1 Preliminaries

Consider the following binary action game among a continuum of risk-neutral bank managers with a unit mass. The task of the manager is to choose the leverage of the bank to maximize the return on equity. The action set of the manager i is $\{0, 1\}$, where $y_i = 1$ is interpreted as set a high leverage; while, $y_i = 0$ set a lower one.

Let $\pi^1 = \pi(y_i = 1, y; \theta)$ and $\pi^0 = \pi(y_i = 0, y; \theta)$ denote, respectively, the managers' payoffs of action 1 and 0. y is the fraction of managers that play $y_i = 1$ and θ is the state of the world variable. The differential payoff between the two actions is $\pi^1 - \pi^0 = B > 0$ if $y \geq h(\theta)$ and is $\pi^1 - \pi^0 = -G < 0$ if $y < h(\theta)$, where $h(\theta)$ is the *resistance function*, namely the critical fraction (of managers) above which it is convenient to play $y_i = 1$. Therefore, from a manager's point of view, it is convenient to play $y_i = 1$ if enough managers choose to play 1, and so the resistance function is overcome ($y > h(\theta)$).

Let p be the probability that enough managers play $y_i = 1$, and thus the manager is indifferent between the two strategy. This means that $pB + (1 - p)(-G) = 0$. We can define this critical probability with $\gamma \equiv \frac{G}{G+B}$, $\gamma \in (0, 1)$. A manager will play $y_i = 1$ if his assessed probability of $y > h(\theta)$ exceeds γ. We shall assume that $h(\cdot)$ is strictly increasing, it is smooth on $(\underset{\sim}{\theta},\tilde{\theta})$, crossed 0 at $\theta = \underset{\sim}{\theta}$ (with $\lim_{\theta \to \underset{\sim}{\theta}} h(\theta) = 0$) and crossed 1 at $\theta = \tilde{\theta}$.

The managers' game has the standard feature of strategic complementarity games, namely the differential payoff $\pi^1 - \pi^0$ is increasing in the mass of managers y.

Under the hypothesis of complete information among managers, we have that if $\theta < \underset{\sim}{\theta}$ the dominant strategy is to play $y_i = 1$, if $\theta > \tilde{\theta}$ the dominant strategy is to play $y_i = 0$, and finally if $\underset{\sim}{\theta} \leq \theta < \tilde{\theta}$ there are multiple equilibria, including one in which all managers play the action 1 and one in which all managers play action 0. So $\underset{\sim}{\theta}$ is the threshold below which there is the maximum level of credit in the economy. All managers opt for the high level of leverage and thus the system's leverage is at its maximum. On the other hand, $\tilde{\theta}$ is the threshold above which there is the minimum amount of credit in the whole bank system, and accordingly the system's leverage is at its minimum. Several crises have thought us that the proper level of credit in the economy has to be far from extremes as both too much and too little are costly for the real economy. Therefore, we can already define $\underset{\sim}{\theta}$ as the threshold below which we observe a dramatic credit boom in the economy and $\tilde{\theta}$ as the threshold above which we observe a tremendous credit crunch.

From now on, just between $\underset{\sim}{\theta}$ and $\tilde{\theta}$ we consider an incomplete information[4] version of the game where managers, given them information set, have a Gaussian prior on the state of the world $\theta \sim N(\mu_\theta, \tau_\theta^{-1})$ and manager i observes a private signal $s_i = \theta + \epsilon_i$ ($\epsilon_i \sim N(0, \tau_\epsilon^{-1})$). It is worth noting that the prior mean μ_θ and θ can be understood as a public signal of precision τ_θ.

In the following, we present the bank system model that allows the micro-foundation of the decision of each manager.

3.2 The bank balance sheet and the resistance function

Suppose that there is a continuum of banks with at their head a manager. Banks are homogeneous. We assume that the bank's market is not in perfect competition. Each bank has fund E which is fixed, and it has access to the desired uninsured short-term debt D, whereas the active side of the balance sheet is entirely composed by credit C. In addition, we assume that shareholders can control neither before nor after the decision of the manager. Each manager observes his private signal about the future state of the world (θ) which is *the probability of default* on credit. As we will see, the manager faces a dichotomous choice. He has to decide between a high debt to equity leverage (l_h) or a lower one (l_l) taking into account the signal on θ. The manager achieves a premium, in the form of reputation, if he decides to set the high leverage and the bank has at least the same profit of a bank that it does not. On the other hand, managers are aware that if the bank has an high leverage, then it can be the object of an inspection by the CB. This inspection causes, on average, monetary cost V. The number of inspections Ψ is decided by the CB and it is public.

Finally, let us define some quantities that will be used in the following: $l = \frac{D}{E}$ is the debt to equity leverage ratio -from now on for sake of brevity *leverage*-, and $v \equiv \frac{V}{E}$ the share between the cost due to the inspection and the equity.

Thinking the problem in term of balance-sheet variation is easier. Since the equity is fix, the increase of credit has to be fully funded by debt ($\Delta C =$

[4] The incomplete information game is referred to in the literature (e.g. Carlsson and Van Damme 1993) as *global game*.

ΔD). Therefore, given $\Psi < y < 1$, the expected profit variation is:

$$\Delta\Pi_i^e = \Delta C(1+r_c)(1-\theta) + \Delta C\theta(1-LGD) - \Delta D(1+r_d) - V\frac{\Psi}{y}. \quad (1)$$

Where r_c is the interest rate on credit pay by borrowers, LGD is the average lost given default, r_d is the interest rate that the bank pays on debt and $\frac{\Psi}{y}$ is the probability to be inspected. As $\Delta C = \Delta D$ and $\frac{\Delta D}{E} = l_h - l_l$, dividing by E both side:

$$\frac{\Delta\Pi_i^e}{E} = (l_h - l_l)(1+r_c)(1-\theta) + (l_h - l_l)\theta(1-LGD) - (l_h - l_l)(1+r_d) - v\frac{\Psi}{y}.$$

where l_h is the high leverage, while l_l is the low leverage.

The manager chooses the leverage to maximize the expected profit. As proof in Appendix A, the optimal decision is either to set $l_l = \underline{l}$, namely the maximum leverage below which the authority does not consider a more punctilious inspection necessary, or to set $l_h = \bar{l}$, which is the maximum leverage permitted by the authority.

$$\Delta r_{e,i}^e = (\bar{l} - \underline{l})(1+r_c)(1-\theta) + (\bar{l} - \underline{l})\theta(1-LGD) - (\bar{l} - \underline{l})(1+r_d) - v\frac{\Psi}{y}. \quad (2)$$

The manager increases his reputation if he decides for $\bar{l}$ and the bank obtains unless the same profit, and thus shareholders the same return:

$$(\bar{l} - \underline{l})(1+r_c)(1-\theta) + (\bar{l} - \underline{l})\theta(1-LGD) - (\bar{l} - \underline{l})(1+r_d) - v\frac{\Psi}{y} = 0. \quad (3)$$

Therefore, if the probability of the extra profit of the bank is higher than γ, then the best strategy of the manager is playing $y_i = 1$, namely set $\bar{l}$.

The hunch is that the manager increases his prestige if he is in charge of a big bank and the bank has at least the same economic result of a small bank. It is worth noting that the strategic complementarity stems from the decreasing probability to be inspected due to the increase in the mass of banks with a high leverage. This fact connects the manager's decision to the decisions of all other managers and to the rigidity of supervisory authority.

Suppose that all managers play $y_i = 1$ and therefore $y = 1$. In this case, the variation of the expected return of shareholders is 0 if the state of the world is

$$\tilde{\theta} = \frac{(\bar{l} - \underline{l})(r_c - r_d) - v\Psi}{(\bar{l} - \underline{l})(r_c + LGD)}. \quad (4)$$

So if $\theta > \tilde{\theta}$, the dominant strategy for all managers is playing $y_i = 0$.

Vice-versa, suppose that just a fraction of managers between 0 and Ψ plays $y_i = 1$, as a result all banks with an high leverage are inspected certainly. The state of the world for which the variation of the expected return of shareholders is 0 then is

$$\underset{\sim}{\theta} = \frac{(\bar{l} - \underline{l})(r_c - r_d) - v}{(\bar{l} - \underline{l})(r_c + LGD)}. \tag{5}$$

So if $\theta < \underset{\sim}{\theta}$, the dominant strategy is playing $y_i = 1$.

Finally, suppose that $\Psi < y < 1$, for each y, we compute the proper state of the world (θ) such that the variation of the expected return of shareholders is 0. This allows to define *the resistance function*. From Equation 3, with simple algebraic passages, we obtain that

$$y > h(\theta) = \frac{v\Psi}{(\bar{l} - \underline{l})[(1 + r_c)(1 - \theta) + \theta(1 - LGD) - (1 + r_d)]}. \tag{6}$$

This is the resistance function, namely the function that defines the minimum mass of managers that has to play $y_i = 1$ for each state of the world (between $\underset{\sim}{\theta}$ and $\tilde{\theta}$) above which it is convenient for a manager playing $y_i = 1$.

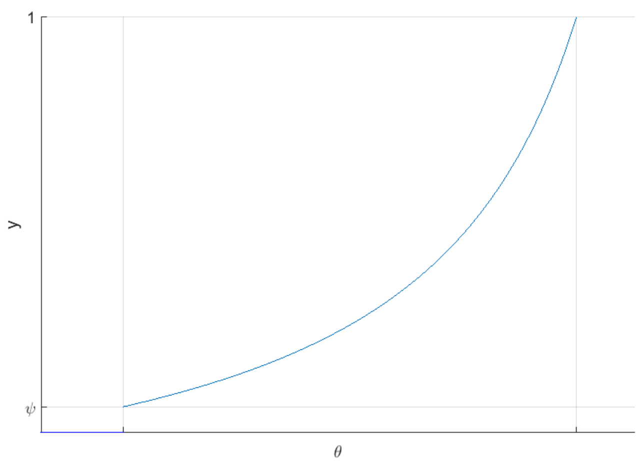


Figure 2: The resistance function $h(\theta)$. Above the blue line $y > h(\theta)$ and the best choice for managers is to play $y_i = 1$.

3.3 Equilibrium and strategic complementarity

We exploit the well-known properties of *monotone supermodular games* to find the condition whereby there is a single equilibrium and to define the equilibrium (Carlsson and Van Damme 1993). We seek a symmetric equilibrium between threshold strategies under which a manager will choose $y_i = 1$ if, and only if, he receives a signal whose value is below a certain threshold.

As explained by Vives (2014), it is useful to think the structure of the game as the best reply of a player $r(\cdot)$ to the common signal threshold $\hat{s}$ used by other players. Each equilibrium is characterized by a signal threshold s^* such that $s^* = r(s^*)$. This signal threshold will imply a critical world state $\theta^* \in [\underset{\sim}{\theta}, \tilde{\theta})$, below which the mass of managers, which play $y_i = 1$, is successful.

The best reply can be computed in two steps[5].

1. Given a signal threshold $\hat{s}$ at which other managers play $y_i = 1$, the manager computes the state of the world threshold $\hat{\theta}$ such that it is convenient playing $y_i = 1$ if, and only if $\theta < \hat{\theta}$, namely when $P(s < \hat{s}|\hat{\theta}) = h(\hat{\theta})$ if $\hat{\theta} > \underset{\sim}{\theta}$ and $\hat{\theta} = \underset{\sim}{\theta}$ otherwise. Therefore it locates the state of the world threshold $\hat{\theta} = \theta_F(\hat{s})$ for which the mass of managers playing $y_i = 1$ is successful, which is increasing in $\hat{s}$. In other words, on this curve, whenever $\hat{\theta} > \underset{\sim}{\theta}$, the fraction of managers playing $y_i = 1$ is $y = P(s < \hat{s}|\hat{\theta})$ and it equals the critical fraction above which it is convenient playing $y_i = 1$.

$$s_F(\theta) = \theta + \frac{1}{\sqrt{\tau_\epsilon}}\Phi^{-1}(h(\theta)) \tag{7}$$

2. Given the state of the world threshold $\hat{\theta}$, the manager computes the signal threshold $\hat{s}$ below which it is optimal play $y_i = 1$, namely $P(\theta < \hat{\theta}|\hat{s}) = \gamma$. This condition gives the signal threshold curve $\hat{s} = s_T(\hat{\theta})$. Therefore, it locates the threshold $\hat{s} = s_T(\hat{\theta})$ for which the expected manager payoff is the same whatever decision he takes, or in other words, he is indifferent between $y_i = 1$ and $y_i = 0$.

$$s_T(\hat{\theta}) = \frac{\tau_\theta + \tau_\epsilon}{\tau_\epsilon}\hat{\theta} - \frac{\tau_\theta}{\tau_\epsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\epsilon}}{\tau_\epsilon}\Phi^{-1}(\gamma) \tag{8}$$

The best response function is the composition of the two thresholds: $r(\hat{s}) = s_T(\theta_F(\hat{s}))$,

$$r(\hat{s}) = \frac{\tau_\theta + \tau_\epsilon}{\tau_\epsilon}\theta_F(\hat{s}) - \frac{\tau_\theta}{\tau_\epsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\epsilon}}{\tau_\epsilon}\Phi^{-1}(\gamma) \tag{9}$$

As a higher threshold $\hat{s}$ by others induces each manager to use a higher threshold too, this is certainly a strategic complementarities game. The intersection points of the two curves ($\hat{\theta} = \theta_F(\hat{s})$ and $\hat{s} = s_T(\hat{\theta})$) are equilibria, which can be defined even as the fixed points of the best response function.

We collect the properties of the equilibrium in Proposition 1.

Proposition 1 *Let h_1 denote the smallest slope of $h(\cdot)$ between $(\underset{\sim}{\theta}, \tilde{\theta})$.*

[5] The proof is given in Vives (2014) and reported in the Appendix B.1

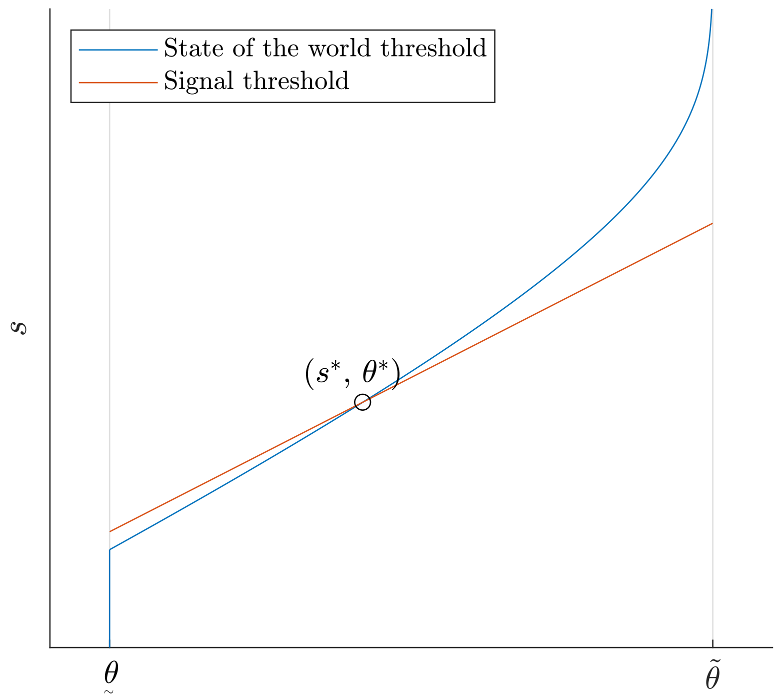


Figure 3: Model equilibrium. ($r_i = 0.07$, $LGD = 0.2$, $r_d = 0.02$, $\bar{l} = 10$, $\underline{l} = 5$ $v = 0.1$, $\gamma = 0,5$, $\mu_\theta = 0.135$, $\tau_\theta = 100$ e $\tau_\epsilon = 1000$)

1. *Each equilibrium is characterized by two threshold* (s^*, θ^*)*, where* s^* *is the signal threshold below which a manager chooses* $y_i = 1$ *and,* $\theta^* \in [\underset{\sim}{\theta}, \tilde{\theta})$ *is the state of the world below which the mass of managers, who have chosen* $y_i = 1$*, is successful.*

2. *There is a critical value of* $h(\underset{\sim}{\theta})$ *denoted by* $\bar{h}_0(\underset{\sim}{\theta})$ *such that:*

   - *if* $h(\underset{\sim}{\theta}) \geq \bar{h}_0(\underset{\sim}{\theta})$ *then there is at least the smallest equilibrium* $\theta^* = \underset{\sim}{\theta}$.
   - *if* $h(\underset{\sim}{\theta}) < \bar{h}_0(\underset{\sim}{\theta})$ *then* $\theta^* > \underset{\sim}{\theta}$ *for any equilibrium.*

3. *The equilibrium is unique if* $\frac{\tau_\theta}{\tau_\epsilon} \leq h_1\sqrt{2\pi}$.

The proof of this proposition is given in Vives (2014) but, for completeness, we report it in Appendix B.2. A peculiar feature of this type of game is that the extremal equilibria are in the threshold strategy and others equilibria are bound between the largest and smallest one. When the uniqueness condition reported in the third point of the *Proposition* 1 is verified, then the largest and the smallest equilibrium coincide, and therefore there is a unique equilibrium.

The extent of strategic complementarity among the managers' actions depends on the slope of the best response function. The maximal value of the slope is given by $\bar{r}' \equiv \frac{\tau_\theta+\tau_\epsilon}{\tau_\epsilon+h_1\sqrt{2\pi\tau_\epsilon}}$, which depends on $h(\theta)$ and, in particular, it is increasing in h_1^{-1}. The degree of strategic complementarity is higher whenever h is less sensitive to θ, τ_θ is higher (high precision of public signal), and τ_ϵ is lower (low precision of private signal). The *resistance function* is the Equation 6. The derivative of $h(\theta)$ respect to θ is the following

$$\frac{\partial h(\theta)}{\partial \theta} = h_\theta = \frac{v\Psi(\bar{l}-\underline{l})(r_c+LGD)}{\Big(((\bar{l}-\underline{l})[(1+r_c)(1-\theta)+\theta(1-LGD)-(1+r_d)]\Big)^2}.$$

The smallest slope of $h(\cdot)$ on $(\underset{\sim}{\theta},\tilde{\theta})$ is when θ tends to $\underset{\sim}{\theta}$ from right, so we compute the limit:

$$h_1 = \lim_{\theta\to\underset{\sim}{\theta}^+} h_\theta = \frac{(\bar{l}-\underline{l})(r_c+LGD)\Psi}{v}. \tag{10}$$

The minimum slope of *the resistance function* is increasing in Δl, r_c, LGD and Ψ whereas is decreasing in v. If for h_1 there is only an equilibrium, then the model, for whatever θ, has just an equilibrium (Proof in Appendix B.1).

Regarding the strength of the strategic complementarity among managers depends on the slope of the best response function. Specifically, it is increasing in Δl and r_c, while decreasing in the LGD, r_d, v, and in the amount of control done by the CB (Ψ).

3.4 Comparative statics

We confine ourselves to analyze the full comparative statics properties of unique equilibrium despite these properties hold both when it is unique and when there are multiple equilibria also for the extremal ones. Moreover, they hold for any equilibrium even if unstable or intermediate, if we suppose that out of equilibrium-adjustment is adaptive. The results are stated formally in Proposition 2. For the sake of brevity let us collect together r_d, $\underline{l}$, LGD, v and Ψ in α, while $\bar{l}$ and r_c in β.

Proposition 2 *Let $h(\underset{\sim}{\theta};\alpha;\beta) < \bar{h}_0$.*

1. *The thresholds θ^* and s^* and the probability that a manager observes signal for which the best strategy is to play $y_i = 1$ ($P(s < s^*)$) are all decreasing in γ, in the expected value of the state of the world μ_θ and in α, while it is increasing in β.*

2. *The change of the public signal μ_θ has a multiplier effect on the equilibrium thresholds which is enhanced when τ_θ is higher.*

3. *Let $\gamma < 0.5$. If μ_θ is low enough ($\mu_\theta \leq \underset{\sim}{\theta}$ is sufficient)[6], then a more precise signal (an higher τ_θ) increases θ^*, s^*, and $P(s < s^*)$; whereas*

[6]The condition is if for $\theta = \mu_\theta$, $s_T(\theta) > s_F(\theta)$ then an increase of τ_θ or a decrease of τ_ϵ increase θ^*, s^* and $P(s < s^*)$.

a more precise private signal reduces them. If μ_θ is high enough ($\mu_\theta > \tilde{\theta} - (\tau_\theta + \tau_\epsilon)^{-0.5}\Phi^{-1}(\gamma)$ is sufficient), then the results are reversed[7].

The proof of this proposition is given in Vives (2014) but, for completeness, we report it in Appendix B.3. It is worth noting that:

- as $y = P(s < s^*|\theta)$, then y is decreasing in both γ and μ_θ. Namely, given θ, both a worsening of the economic outlook (an increase of μ_θ) and an increase of the probability such that the manager is indifferent between the two choices (an increase of γ), lead to a reduction of the credit issues in the economy. Thus, if managers were risk lovers (risk adverse), they would be indifferent between the two choices even with a lower (upper) probability that enough managers play $y_i = 1$. Therefore, ceteris paribus, if we change the managers' risk appetite, we observe a variation of credit in the system.

- Likewise, we can read the effects of a variation of both the public and the private signal. In few words, given θ, any variation that causes a left shift of the threshold θ^* and a downward shift of the threshold s^* lead a reduction of the credit issues in the economy. Vice-versa, any variation that causes a right shift of the threshold θ^* and an upward shift of the threshold s^* lead an increase of the credit issues in the economy.

Let us now apply the results of the proposition to illustrating the comparative static results in the model. In the model $\bar{h}_0$ depends on the amount of inspection decided by the CB ($\Psi \in (0, 1)$). In each case exists a $\bar{\Psi}$ that if $\Psi > \bar{\Psi}$ then $\theta^* = \underset{\sim}{\theta}$; while, if $\Psi < \bar{\Psi}$ then $\theta^* > \underset{\sim}{\theta}$ at any equilibrium. The proof of the Proposition 3 is in the Appendix B.4.

Proposition 3 *Let $\Psi < \bar{\Psi}$. We analyze how $P(\theta < \underset{\sim}{\theta})$, $P(\theta > \tilde{\theta})$, and y change when a parameter varies.*

1. *we define $P(\theta < \underset{\sim}{\theta})$ as the probability of extreme credit expansion. This probability decreases if $\underset{\sim}{\theta}$ decreases and when μ_θ grows. Therefore $P(\theta < \underset{\sim}{\theta})$ decreases in the LGD, r_d, $\underset{\sim}{l}$, and v, while it increases in $\bar{l}$, and r_c. Moreover, if $\underset{\sim}{\theta} < \mu_\theta < \tilde{\theta}$, then it is increasing in the precision of public signal τ_θ. Finally, it is independent of the number of inspections decided by the CB Ψ, of the precision of private signal τ_ϵ and of the probability γ.*

2. *we define $P(\theta > \tilde{\theta})$ as the probability of a credit crunch. This probability decreases if $\tilde{\theta}$ increases and when μ_θ decreases. Therefore $P(\tilde{\theta} < \theta)$*

[7]The condition for the comparative statics results in Proposition 2.3 are sufficient but not necessary. When the equilibrium is unique it can be checked that it can be strengthened as follows: there are threshold $\hat{\mu}_\theta$ and $\dot{\mu}_\theta$ with $\hat{\mu}_\theta < \dot{\mu}_\theta$ such that θ^* increases with τ_θ if and only if $\mu_\theta < \dot{\mu}_\theta$, and θ^* increases with τ_ϵ if and only if $\mu_\theta > \hat{\mu}_\theta$.

Table 1: Summary of the results reported in the Proposition 3

	$P(\theta < \underaccent{\tilde}{\theta})$ (Credit expansion)	$P(\theta > \tilde{\theta})$ (Credit crunch)	$P(s < s^*\|\theta)$ (y)
r_c	$+$	$-$	$+$
LGD	$-$	$+$	$-$
r_d	$-$	$+$	$-$
$\underline{l}$	$-$	$+$	$-$
v	$-$	$+$	$-$
Ψ	0	$+$	$-$
$\bar{l}$	$+$	$-$	$+$
μ_θ	$-$	$+$	$-$
τ_θ	$+$	$+$	if $\mu_\theta < \hat{\mu}_\theta$ $+$; else $-$
τ_ϵ	0	0	if $\mu_\theta < \dot{\mu}_\theta$ $-$; else $+$
γ	0	0	$-$

increases in the LGD, r_d, $\underline{l}$, v, and Ψ, while it decreases in $\bar{l}$, and r_c. Moreover, if $\underaccent{\tilde}{\theta} < \mu_\theta < \tilde{\theta}$, then it is increasing in the precision of public signal τ_θ. Finally, it is independent of the precision of private signal τ_ϵ, and of the probability γ.

3. *y is the fraction of managers which plays $y_i = 1$ and, accordingly, the fraction of banks with $\bar{l}$. $y = P(s < s^*|\theta)$ or in few words y is the fraction of managers which has a private signal below the signal threshold s^*. $P(s < s^*|\theta)$ decreases if s^* decreases and when μ_θ or γ grow. Therefore, it decreases in the LGD, r_d, $\underline{l}$, v, and Ψ, while it increases in $\bar{l}$, and r_c. If $\mu_\theta < \hat{\mu_\theta}$, then the probability is increasing in τ_θ and it is decreasing in τ_ϵ, whereas if $\mu_\theta > \dot{\mu_\theta}$ it is increasing in τ_ϵ and decreasing in τ_θ. (where $\hat{\mu_\theta} < \dot{\mu_\theta}$)[8].*

The results reported in the Proposition 3 and summaries in Table 1 enable some economic consideration, in particular, it is worth emphasizing as an increase of Ψ enhances the probability of a credit crunch, and reduces the amount of credit in the economy; whereas it has no effect on the probability of uncontrolled credit expansion. Therefore, the CB cannot influence the threshold below which there is a common convenience between banks to play $y_i = 1$ through an increase of supervisions Ψ.

The third point of the Proposition 2 indicates that realizing more public information could be not social optimum both when μ_θ is low (too good economic outlook), and is high (too bad economic outlook). In this case, the public signal is a coordinating device for managers that can potentially lead them to take the same decision, since each one knows that others will weight

[8]If $\gamma = \frac{1}{2}$ then $\hat{\mu}_\theta = \dot{\mu}_\theta = \theta^*$ and therefore if $\mu_\theta < \theta^*$ y is increasing in τ_θ and decreasing in τ_ϵ, otherwise in the other way round.

the public signal heavily. This effect is reinforced if public information is precise and the private signal is not, as the value of the public information increases.

4 The macro-prudential regulation and the structural effect of monetary policy

The crisis of 2007 has reiterated the need for a macro-prudential approach of banking supervision and regulation. A form of responsibility for financial stability is now widely regarded as an essential characteristic of central banking. The change of paradigm is essentially due to the central role that the banks' behaviour had in the recent crisis both in boost the bubble and in the credit crunch. Credit boom and credit crunch are somehow similar, indeed in both it is discernible a coordinated homogeneous behaviour of the majority of banks, as the Northen Rock case has shown (Shin 2009). To shut down this vicious cycle, many CBs have started what is called a macro-prudential policy through both monetary policy and new banking regulation tools (Stein 2012, Basil 2011).

The purpose of the macro-prudential policy is to address negative externalities by acting as a countervailing force to the natural decline in measured risks in a boom and the subsequent rise in measured risks in the downturn, and to mitigate risks linked to financial sector concentration and interconnectedness. As such, the macro-prudential policy has both a time dimension and a cross-sectional (or structural) dimension.

4.1 The CB problem

For simplicity, we assume that the CB is in charge of the macro-prudential mandate. The CB has to pursue the macro-prudential goal which consists of preserving the financial stability, but on the other hand, it must maximize the social welfare which depends by the amount of credit in the economy and by the cost of an expansionary monetary policy.

The macro-prudential regulation is nested in the following micro-prudential framework. The CB settles $\bar{l}$ and $\underline{l}$, respectively the maximum leverage permitted and the maximum leverage below which the CB considers safe a bank. Furthermore, given the budget constrain, the CB can inspect a fraction Ψ of banks whit a leverage above $\underline{l}$.

To simulate the countercyclical buffer that several CB are implementing, we suppose that the CB can decide a *reduction of the maximum level of leverage permitted* ($\bar{l}^b < \bar{l}$). Whereas, to mimic the monetary policy measures e.g. the TLTRO done by the ECB, we suppose that the CB can implement a *loan support program* which in turn reduce the cost of bank financing ($0 < r_d^C < r_d$). These measures are settled by the CB, under complete autonomy, to keep the probability of extreme events in a tolerance range.

Let us define P_{CE} and P_{CC} respectively the maximum probability tolerated by the CB of a credit boom and of a credit crunch. To archive the macro-prudential target, the CB has to keep $P(\theta < \underset{\sim}{\theta}) \leq P_{CE}$ and $P(\theta > \tilde{\theta}) \leq P_{CC}$. Therefore, the CB scope is to maximize the social welfare but keeping the probabilities of extreme events below certain thresholds.

The CB's maximization problem is the following:

$$\begin{aligned}
\max_{\bar{l}^b, r_d^C} \, & W^e = f(l(\bar{l}^b, r_d^C; \mu_\theta)) - g(r_d^C) \\
\text{s.t. } & \bar{l}^b \leq \bar{l} \\
& r_d^C \leq r_d \\
& P(\theta < \underset{\sim}{\theta}(\bar{l}^b, r_d^C); \mu_\theta) \leq P_{CE} \\
& P(\theta > \tilde{\theta}(\bar{l}^b, r_d^C); \mu_\theta) \leq P_{CC} \\
& l = \bar{l}^b y(\bar{l}^b, r_d^C; \mu_\theta) + \underline{l}(1 - y(\bar{l}^b, r_d^C; \mu_\theta))
\end{aligned}$$

Where W^e is the expected social welfare, it is made up by $f(\cdot)$ which represents the welfare due to loans, and $g(\cdot)$ which is the cost function of a expansionary monetary policy. The welfare increases respect the amount of credit in the economy l but the marginal benefit decreases ($f_l' > 0$, $f_l'' < 0$). Whereas the cost function $g(\cdot)$ has the following properties: $g(r_d) = 0$, $g'_{r_d^C} < 0$ and $g''_{r_d^C} > 0$, namely if $r_d^C = r_d$ there is not cost, while the cost increases more than in proportion with the gap between r_d and r_d^C. So higher is the reduction of the interest rate higher is the cost of the measure.

The expected welfare is strictly increasing in $\bar{l}^b$. Indeed, if $\bar{l}^b$ grows then increases even y, namely the fraction of banks with high leverage[9]. More complex is the effect of a reduction of r_d^C, on the one hand, a lower interest rate increases the amount of loans, but on the other hand, it generates costs. These costs are due to the active role of the CB which extends loans with an interest rate lower than the market interest rate. Since the CB has not funds, the cost lies on the society. At this early stage, let us assume that the expected welfare is strictly increasing ever respect to r_d^C, namely we suppose that $|f_l'(l(\bar{l}, r_d^C; \mu_\theta))l'_{r_d^C}| < |\, g'_{r_d^C}|$[10]. This means that in term of welfare the cost of the *loan support program* is higher than the benefit of the grow of loans in the economy.

Given a generic public signal μ_θ, with a CB's precision of $\tau_{\theta_{CB}}$, we can define a normal economic time any situation in which the CB does not need to use macro-prudential tools ($\bar{l}^b = \bar{l}$ and $r_d^C = r_d$). In Figure 4 is displayed an example. The thresholds of the CB (z_{CE}, z_{CC}) are within $\underset{\sim}{\theta}$ and $\tilde{\theta}$, the probability of extreme events are lower than the desired ones.

[9]See Table 1, third colon.

[10]We discuss later in the text this hypothesis.

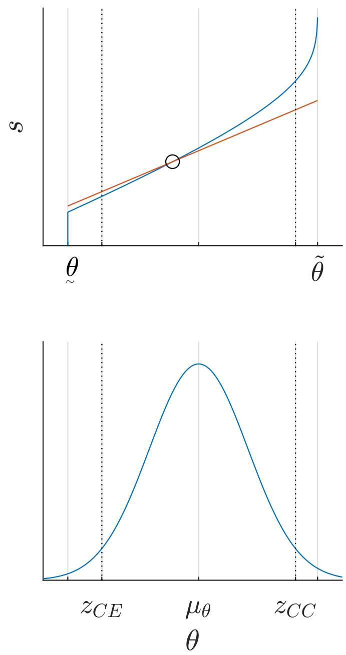


Figure 4: Normal time. ($r_c = 0.07$, $LGD = 0.2$, $r_d = 0.02$, $\bar{l} = 10$, $\underline{l} = 5$, $\Psi = 0.338$, $v = 0.05$, $\gamma = 0.5$, $\mu_\theta = 0.135$, $\tau_\theta = 100$, $\tau_\epsilon = 1000$ and $\tau_{\theta_{CB}} = 10000$. Finally we hypothesize the macro goal is to keep both probabilities of a uncontrolled credit expansion and of a credit crunch below 2,5% and z_{CE} and z_{CC} are respectively thresholds.) On average we obtain: $s^* = 0.135$, $y = 0.49$ and $W^e = \frac{ln(l)}{2} - b^{(r_d - r_d^C)} + 1 = \frac{ln(7.45)}{2} - (10^{10})^{(0.02-0.02)} + 1 = 1$.

4.2 The optimal macro-prudential policy with a positive shock

If there is a public positive shock, then the expectation of the future economic outlook suddenly improves. This is reflected in the model by a decrease of μ_θ up to μ_θ^b. If the shock is enough strong ($\mu_\theta^b < \underset{\sim}{\theta} + z_{\%}\sqrt{\tau_{\theta_{CB}}^{-1}} < \mu_\theta$)[11], then the probability of a state of the world in which there is a dominant strategy is higher than the maximum tolerated by the CB. On the left-hand side of Figure 5, $\underset{\sim}{\theta}$ is on the right of z_{CE}. The CB, setting the new *maximum leverage permitted* $\bar{l}^b$, can bring back the probability of uncontrolled credit boom to the maximum tolerated. The CB, with a stringent measure, could reduce the probability even further, but since it must maximize the welfare, it is not an optimal decision. On the right-hand side of Figure 5, we can observe as, after the reduction of the maximum leverage permitted, the $P(\theta < \underset{\sim}{\theta}^b)$ is equal to the maximum tolerated (P_{CE}), and $\underset{\sim}{\theta}^b$ coincides with z_{CE}.

[11] $z_{\%}$ is the value of the normal value table. It depends on the maximum probability of uncontrolled credit expansion tolerated by the CB.

To define which is the proper $\bar{l}^b$ to set, the CB has to compute the leverage for which

$$\underset{\sim}{\theta}^b + \sqrt{\tau_{\theta_{CB}}^{-1}} z_{\%} = \mu_\theta^b, \tag{11}$$

$$\frac{(\bar{l}^b - \underline{l})(r_c - r_d) - v}{(\bar{l}^b - \underline{l})(r_c + LGD)} + \sqrt{\tau_{\theta_{CB}}^{-1}} z_{\%} = \mu_\theta^b, \tag{12}$$

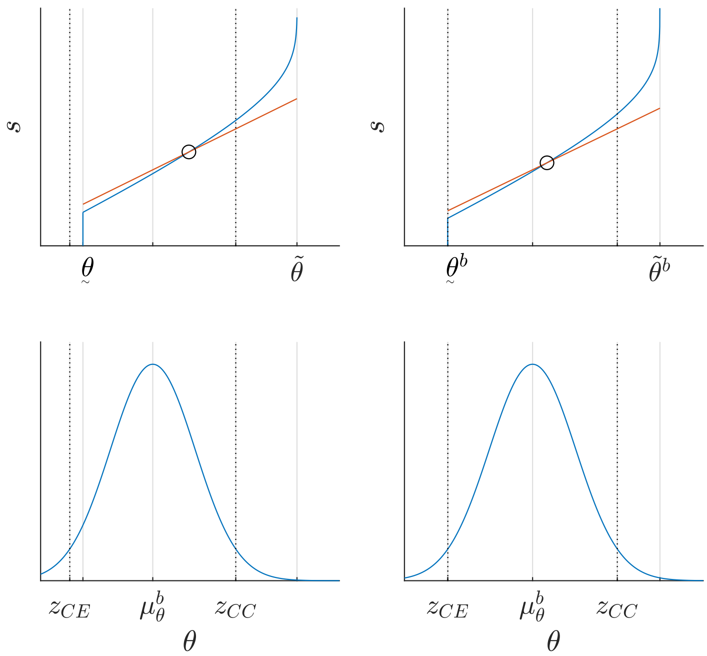
$$\bar{l}^b = \frac{v + \underline{l}[(r_c - r_d) - (\mu_\theta^b - \sqrt{\tau_{\theta_{CB}}^{-1}} z_{\%})(r_c + LGD)]}{(r_c - r_d) - (\mu_\theta^b - \sqrt{\tau_{\theta_{CB}}^{-1}} z_{\%})(r_c + LGD)}. \tag{13}$$

Where $z_{CE} = \sqrt{\tau_{\theta_{CB}}^{-1}} z_{\%}$ depends by the maximum probability of uncontrolled credit expansion that the CB tolerates. $\underset{\sim}{\theta}^b$ is the new threshold below which there is a mutual convenience to choose the high leverage. Reducing the *maximum leverage permitted* to $\bar{l}^b$ is the optimal macro-prudential policy. It allows the CB to maximize the social welfare but preserving the financial stability.

The macro-prudential measure decreases the number of banks which decide to increase their leverage. So the total amount of credit in the economy with the macro-prudential measure is lower than without, both as there is a reduction of banks that have a high leverage, and as the *maximum level of leverage permitted* is lower. Graphically, the reduction of the *maximum level of leverage permitted* moves the curve of *the state of the world threshold*[12] left. The macro-prudential measure affects the incentives that each manager takes into account in its decision, limiting the vicious cycle for which a manager's decision is driven by other's decisions.

Let us present the numerical example reported in Figure 5. We hypothesize that the average amount of credit default decrease of 1.5%. This causes that the probability of a credit boom is too high. The number of banks that on an average have a high leverage jumps from 49.6% to 72.5%, so there is 22.9% of banks more than before that choose the high leverage. It deserves to be underlined that of this 22.9% actually 4.3% of managers observe a signal worst then the minimum signal needed before. In other words, there is 4.3% of managers that, even if they observe a worse signal, they change their mind just because there are more managers that choose the high leverage. This fact can be interpreted as an overreaction of the banking system to the shock, and it highlights how in certain circumstances banks' decisions could be led by lemmings, losing the connection with the economic fundamentals. The CB, setting the *maximum leverage permitted* to $\bar{l}^b = 9.365$, is able to keep the probability of an extreme event equal the CB's tolerance, avoid that managers with a worst signal change their decisions, and in addiction the macro-prudential measure reduce y up to 62.5%. So, the CB, through

[12]See Section 3.3.



(a) *Boom*

(b) *Effect of the reduction of leverage permitted*

Figure 5: With a positive shock ($\mu_\theta^b = 0.12$). On the left-hand side, there is the boom situation without the macro-prudential policy ($s^* = 0.1389$, $y = 0.725$, $l = 8.625$ and $W^e = 1.077$). On the right-hand side there is the effect of the reduction of the *maximum leverage permitted*. The optimal macro-prudential policy is to set $\bar{l}^b = 9.365$ ($s^* = 0.13$, $y = 0.625$, $l = 7.725$ and $W^e = 1.022$). In the name of financial stability, the CB is available to lose part of the welfare's increase.

the control of the *maximum leverage permitted*, can influence incentives that all banks take into account in their decisions. Finally, it deserves to be highlighted that the CB, in the name of the financial stability, loses some social wellness. In particular, in the example the social wellness drops from 1.077 to 1.022.

4.3 The optimal macro-prudential policy with a negative shock

Suppose that the future economic outlook gets worse and the public signal becomes μ_θ^C ($\mu_\theta < \tilde{\theta} - \sqrt{\tau_{\theta_{CB}}^{-1}} z_\% < \mu_\theta^C$). On the left-hand side of Figure 6, we can observe that the probability of a credit crunch is excessively high. The threshold of the maximum credit crunch probability is on the right respect $\tilde{\theta}$ so $P(\theta > \tilde{\theta}) > P_{CC}$. The CB, starting a *loan support program*, can keep the probability of a credit crunch equal to P_{CC}. Moreover, it can damper the reduction of credit in the economy and therefore increase the heterogeneity in the economic system. Since we have assume that $|f_l'(l(\bar{l}^b, r_d^C; \mu_\theta))l'_{r_d^C}| < |\ g'_{r_d^C}|$ then the CB's optimal choice is to keep the probability of a credit crunch equal to P_{CC}. To define which is the proper cost of funding r_d^C that the CB has to set, it must compute the rate of interest for which

$$\tilde{\theta}^C - \sqrt{\tau^{-1}_{\theta_{CB}}} z_{\%} = \mu_\theta^C, \tag{14}$$

$$\frac{(\bar{l} - \underline{l})(r_c - r_d^C) - v\Psi}{(\bar{l} - \underline{l})(r_c + LGD)} - \sqrt{\tau^{-1}_{\theta_{CB}}} z_{\%} = \mu_\theta^C, \tag{15}$$

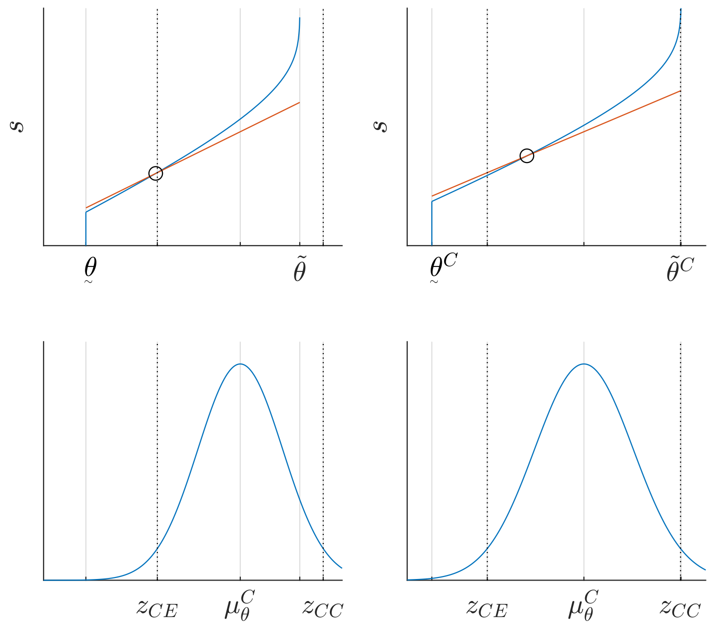
$$r_d^C = r_c - \frac{v\Psi +}{(\bar{l} - \underline{l})} - (r_c + LGD)(\mu_\theta^C + \sqrt{\tau^{-1}_{\theta_{CB}}} z_{\%}). \tag{16}$$

Where $z_{CC} = \sqrt{\tau^{-1}_{\theta_{CB}}} z_{\%}$, it depends by the maximum probability of a credit crunch that the CB tolerates. As reported in Figure 6, with the *loan support program*, the CB decreases the probability of a credit crunch and boosts the credit in the economy, however it is welfare costly. Graphically *the loan support program* moves the curve of *the state of the world threshold* (defined in Section 3.3) right. Therefore, ceteris paribus, the mass of banks that has to choose $y_i = 1$, for which a bank is indifferent between the two strategies, decreases for each θ.

Even in this case, through the numerical example reported in Figure 6, we can underline that the shock causes an overreaction of the market. The increase of 1% of the average default probability on credit reduces on average y up to 34.6%. Therefore there is a reduction of y respect the normal case of 14.4%, however, there is 3% of managers that, even if they observes a signal that before were enough, now they change their mind only because the number of banks with a high leverage is lower than before. The CB, reducing the cost of financing, can keep the probability of a credit crunch equal to the maximum tolerate and increases y up to 41.4%.

Thus we point out how the CB can use monetary policy measure with a structural goal as explained by Stein (2012). The *loan support program*, reducing the cost of banks funding, can decrease the probability of a credit crunch and the homogeneity in the bank system.

Given this hypothesis $|f_l'(l(\bar{l}^b, r_d^C; \mu_\theta))l'_{r_d^C}| < |\ g'_{r_d^C}|$, it is obvious that the W^e decreases. As long as this assumption is true, the CB is willing to use the *loan support program* strictly to preserve the financial stability, namely keep the credit crunch probability in an acceptable range. On the other hand, if we relax this hypothesis $|f_l'(l(\bar{l}^b, r_d^C; \mu_\theta))l'_{r_d^C}| < |\ g'_{r_d^C}|$, then the *loan support program* may be useful not only to preserve the financial stability but even to push credit up. Indeed in that case, the increase of welfare due to the credit grow is higher that the cost of a further reduction of r_d^C. For



(a) *Crisis*

(b) *Effect of loan support program*

Figure 6: On the left-hand side, there is the crisis situation in which μ_θ^C is equal to 0,145 and the CB does not react ($s^* = 0.1325$, $y = 0.3463$, $l = 6.73$ and $W^e = 0.9534$). The CB miss the macro-prudential goals. While on the right-hand side, it is reported the crisis situation but with the effects of the *loan support program* done by CB that reduces r_d to $r_d^C = 0,0188$. z_{CC} is equal to $\tilde{\theta}^C$, and the CB archives the macro-goal ($s^* = 0.1381$, $y = 0.414$, $l = 7.07$ and $W^e = 0.949$).

instance, if the parameter of the cost of the monetary policy in the numeric example is $b = 2 \cdot 10^8$, then in the crisis scenario with $r_d^C = 0.0188$, we would obtain $W^e = 0.9547 > 0.9534$, but with a $r_d^C = 0.0175$ we would maximize the welfare ($W^e = 0.9552$). Therefore in this case the CB does not limit itself to preserve the financial stability, but push credit up to maximize the social welfare.

5 Conclusion

The model presented in this paper shows how the rigidity of bank supervision can create frictions in the banking system that springs strategic complementarity behaviour among banks. More specifically, in a banking system in which the supervisory authority has a budget constraint on the resources that it can allocate to monitor, and supervision is costly for banks, we derive the conditions for strategic complementarities in the behaviour of banks.

The standard explanation of the crisis is the shift in risk perceptions[13]. However, with our model, we corroborate the view propose by Danielsson and Shin (2003), Adrian and Shin (2013) and Aikman et al. (2015) that the strategic complementarities within the financial system exacerbate the problem of the gap between economic agents decisions and economic fundamentals. Therefore, the shift on risk perception and strategic complementarities go hand in hand, with macro-prudential tools needed to mitigate both of them. As an offsetting force to strategic complementarity, macro-prudential measures should be thought and used to avoid situations in which the incentive to take an equal decision among banks is likely to prevail. As a corollary, the effectiveness of macro-prudential measures depends on the ability of the authority to affect the mutual incentives take into account by the majority of the economic players in their decisions. Reducing the incentive to take the prevailing decision in the banking system decreases the mass of banks that takes it, which in turn reduces the incentive to take that decision. Macro-prudential policy, reducing the strategic complementarity forces, narrows the gap between the economic fundamentals and the decision of economic agents, thus making the banking system more resilient. In this context, we show that the countercyclical buffer and a monetary policy measure that reduces the cost of banks funding (e.g. TLTRO) are useful tool to reduce and manage the strategic complementarity forces in the bank system. However, as clarified by Borio (2014), macro-prudential instruments do not give superpower to CBs. The effectiveness of macro-prudential measures depends on the financial and economic condition and the convergence to the same goal of others monetary and fiscal policy.

[13]The crisis of 2007 has shown that during crisis stage the financial accelerator theory is not enough to explain the credit fluctuation observed. To fill this gap, Borio and Zhu (2012) and Shin (2010) point out that during the economic booms banks and borrowers underestimate risks and, when the crash comes, they overestimate risks.

References

Adrian, Tobias and Hyun Song Shin (2013). "Procyclical leverage and value-at-risk". In: *The Review of Financial Studies* 27.2, pp. 373–403.

Aikman, David, Andrew G Haldane, and Benjamin D Nelson (2015). "Curbing the credit cycle". In: *The Economic Journal* 125.585, pp. 1072–1109.

Alvarez, Inmaculada et al. (2017). *The use of the Eurosystems monetary policy instruments and operational framework since 2012*. Tech. rep. European Central Bank.

Basil, Commission (2011). *Basilea 3 Schema di regolamentazione internazionale per il rafforzamento delle banche e dei sistemi bancari*. Tech. rep. Documentation Center. Bank International Settlements.

Bebchuk, Lucian A and Itay Goldstein (2011). "Self-fulfilling credit market freezes". In: *The Review of Financial Studies* 24.11, pp. 3519–3555.

BGFRS (2015a). *Bank Holding Company Supervision Manual*. Tech. rep. Board of Governors of the Federal Reserve System.

BIS (2015). *Report on the impact and accountability of banking supervision*. Tech. rep. Bank for International Settlements.

BoE (2011). "Instruments of macroprudential policy". In: *Bank of England Discussion Paper* 20.

Borio, Claudio (2014). "Macroprudential frameworks:(too) great expectations?" In: *Macroprudentialism*, pp. 29–45.

Borio, Claudio and Haibin Zhu (2012). "Capital regulation, risk-taking and monetary policy: a missing link in the transmission mechanism?" In: *Journal of Financial Stability* 8.4, pp. 236–251.

Borio, Claudio, Craig Furfine, Philip Lowe, et al. (Mar. 2001). "Procyclicality of the financial system and financial stability: issues and policy options". In: *BIS papers* 1, pp. 1–57.

Brunnermeier, Markus et al. (2009). *The fundamental principles of financial regulation*. Vol. 11. ICMB, Internat. Center for Monetary and Banking Studies.

Brunnermeir, Markus and Arvind Krishnamurthy (2014). *Risk Topography: Systemic Risk and Macro Modeling*. University of Chicago Press.

Bucalossi, Annalisa et al. (2016). *Basel III and recourse to Eurosystem monetary policy operations*. eng. ECB Occasional Paper 171. Frankfurt a. M.: European Central Bank.

Carlsson, Hans and Eric Van Damme (1993). "Global games and equilibrium selection". In: *Econometrica: Journal of the Econometric Society*, pp. 989–1018.

Claessens, Stijn (2014). *An overview of macroprudential policy tools*. 14-214. International Monetary Fund.

Crockett, Andrew (2000). *Marrying the micro- and macro-prudential dimensions of financial stability*. Tech. rep. BIS.

Danielsson, Jon and Hyun Song Shin (2003). “Endogenous risk”. In: *Modern risk management: A history*, pp. 297–316.
Dudley, William (2014). *Testimony on improving financial institution supervision: examining and addressing regulatory capture.* Tech. rep. Federal Reserve Bank of New York.
— (2016). *Opening remarks at the Conference on Supervising Large, Complex Financial Institutions: Defining Objectives and Measuring Effectiveness.* Tech. rep. Federal Reserve Bank of New York.
ECB (2014). *Guide to Banking Supervision.* Tech. rep. European Central Bank.
— (2017). *SSM thematic review on IFRS 9.* Tech. rep. European Central Bank.
Eisenbach, Thomas et al. (2017). “Supervising Large, Complex Financial Institutions: What Do Supervisors Do?” In: *Federal Reserve Bank of New York Economic Policy Review* 23.1, p. 57.
Eisenbach, Thomas M, David O Lucca, and Robert M Townsend (2016). *The economics of bank supervision.* Tech. rep. National Bureau of Economic Research.
Fawley, Brett W and Christopher J Neely (2013). “Four stories of quantitative easing”. In: *Federal Reserve Bank of St. Louis Review* 95.January/February 2013.
Freixas, Xavier, Luc Laeven, and José-Luis Peydró (2015). *Systemic risk, crises, and macroprudential regulation.* MIT Press.
Geanakoplos, John (2010). “The leverage cycle”. In: *NBER macroeconomics annual* 24.1, pp. 1–66.
Hanson, Samuel G, Anil K Kashyap, and Jeremy C Stein (2011). “A macroprudential approach to financial regulation”. In: *Journal of economic Perspectives* 25.1, pp. 3–28.
He, Zhiguo and Arvind Krishnamurthy (2013). “Intermediary asset pricing”. In: *American Economic Review* 103.2, pp. 732–70.
Hirtle, Beverly J, Anna Kovner, and Matthew Plosser (2016). *The impact of supervision on bank performance.* Tech. rep. Staff Report, Federal Reserve Bank of New York.
Ivanov, Ivan, Benjamin Ranish, and James Wang (2017). *Strategic Response to Supervisory Coverage: Evidence from the Syndicated Loan Market.* Tech. rep. Board of Governors of the Federal Reserve System (US).
Kandrac, John and Bernd Schlusche (2017). *The Effect of Bank Supervision on Risk Taking: Evidence from a Natural Experiment.* Tech. rep. Board of Governors of the Federal Reserve System (US).
Kiyotaki, Nobuhiro and John Moore (1997). “Credit cycles”. In: *Journal of political economy* 105.2, pp. 211–248.
Lowe, Philip, Claudio Borio, et al. (2002). *Asset prices, financial and monetary stability: exploring the nexus.* Tech. rep. Bank for International Settlements.

Morris, Stephen and Hyun Song Shin (1998). "Unique equilibrium in a model of self-fulfilling currency attacks". In: *American Economic Review*, pp. 587–597.
— (2004). "Coordination risk and the price of debt". In: *European Economic Review* 48.1, pp. 133–153.
Repullo, Rafael and Jesús Saurina (2011). *The Countercyclical Capital Buffer of Basel III: A Critical Assessment*. Tech. rep. CEPR Discussion Papers.
Rezende, Marcelo and Jason Wu (2014). "The effects of supervision on bank performance: Evidence from discontinuous examination frequencies". In: *Midwest Finance Association 2013 Annual Meeting Paper*.
Rochet, Jean-Charles and Xavier Vives (2004). "Coordination failures and the lender of last resort: was Bagehot right after all?" In: *Journal of the European Economic Association* 2.6, pp. 1116–1147.
Schoenmaker, Dirk (2014). *Macroprudentialism*. VoxEU eBook, CEPR.
Shin, Hyun Song (2009). "Reflections on Northern Rock: The bank run that heralded the global financial crisis". In: *Journal of economic perspectives* 23.1, pp. 101–19.
— (2010). *Risk and liquidity*. Oxford University Press.
Shin, Hyun Song et al. (2011). "Macroprudential policies beyond Basel III". In: *BIS papers* 1.5.
Stein, Jeremy C (2012). "Monetary policy as financial stability regulation". In: *The Quarterly Journal of Economics* 127.1, pp. 57–95.
Taylor, John B. (2010). "Commentary: monetary policy after the fall". In: *Proceedings - Economic Policy Symposium - Jackson Hole*, pp. 337–348.
Van Zandt, Timothy and Xavier Vives (2007). "Monotone equilibria in Bayesian games of strategic complementarities". In: *Journal of Economic Theory* 134.1, pp. 339–360.
Vives, Xavier (2014). "Strategic complementarity, fragility, and regulation". In: *The Review of Financial Studies* 27.12, pp. 3547–3592.
White, William R et al. (2009). "Should monetary policy lean or clean?" In: *Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper* 34.

A Managers' Leverage decision

Suppose a simple bank which the active side of the balance-sheet is fully compose by credit C, while in the passive side there are debt D and equity E, and equity is prefixed. The shareholders' profit is:

$$\Pi = C(1 + r_c) - D(1 + r_d) - E.$$

Dividing by E both side, we obtain:

$$\begin{aligned} 1 + r_e = & \quad \frac{C}{E}(1 + r_c) - \frac{D}{E}(1 + r_d) \\ = & \quad \frac{D + E}{E}(1 + r_c) - \frac{D}{E}(1 + r_d) \\ = & \quad (l + 1)(1 + r_c) - l(1 + r_d) \end{aligned}$$

Trivially, the goal of the bank's manager is to maximize the r_e taking in to account the constraint on the leverage.

$$\begin{aligned} & \max_{l} \ (l + 1)(1 + r_c) - l(1 + r_d) \\ & \text{s.t. } l \leq \bar{l} \end{aligned}$$

We solve the maximization problem:

$$\begin{aligned} (1 + r_c) - (1 + r_d) - \lambda = 0 \\ \bar{l} - l \geq 0 \\ \lambda(\bar{l} - l) = 0 \\ \lambda \geq 0 \end{aligned}$$

There are just two possibility. If $\lambda = 0$, then $r_e = r_d = r_e$, the leverage is indeterminate and the bank activity is not convenient. On the other hand, if $\lambda \neq 0$, then $l = \bar{l}$, and thus the manager sets the maximum leverage.

Therefore, assuming the second $\lambda > 0$, in the model there are just two possible level of leverage. $\underline{l}$ which is the maximum leverage below which the authority considers the bank safe and $\bar{l}$ the maximum leverage permitted. Specifically, each manager who receives a sufficiently good signal chooses $\bar{l}$, while others $\underline{l}$. In Section 3.3 we point out which is a sufficiently good signal.

B Proofs

B.1 Characterization of the best reply

From the equality $\Phi(\sqrt{\tau_\epsilon}(\hat{s} - \theta)) = h(\theta)$ in the range $(\underline{\theta}, \tilde{\theta})$ we can solve for $\hat{s}$ as a function of θ; thus we obtain $s_F(\theta) = \theta + \frac{1}{\sqrt{\tau_\epsilon}}\Phi^{-1}(h(\theta))$ with derivative $s'_F = 1 + \frac{1}{\sqrt{\tau_\epsilon}} h'(\phi(\Phi^{-1}(h(\theta))))^{-1} > 0$, where ϕ is the density of the standard normal distribution. The threshold $\hat{\theta} = \theta_F(\hat{s})$ is obtained as the inverse of $s_F(\cdot)$ whenever $\hat{\theta} \in (\underset{\sim}{\theta}, \tilde{\theta})$ and $\hat{\theta} = \underset{\sim}{\theta}$ otherwise. Since ϕ is bounded above by $\frac{1}{\sqrt{2\pi}}$ and since h_1 is the smallest slope of $h(\cdot)$, it follows that s_F' is bounded below: $s_F' \geq 1+\sqrt{\frac{2\pi}{\tau_\epsilon}}h_1$. Hence $\theta_F'(\hat{s}) \leq (1+\sqrt{\frac{2\pi}{\tau_\epsilon}}h_1)^{-1}$ with strict inequality except possibly when $h(\theta) = \frac{1}{2}$, because then $\Phi^{-1}(\frac{1}{2}) = 0$ and ϕ attains its maximum: $\phi(0) = \frac{1}{\sqrt{2\pi}}$.

At the critical signal threshold $\hat{s}$, for a given state of the world threshold $\hat{\theta}$ the expected payoffs for acting and for not acting should be the same:

$$E[\pi(1, y(\theta, s); \theta) - \pi(0, y(\theta, s); \theta)|s = \hat{s}]$$

$$= P(\theta < \hat{\theta}|\hat{s})B + P(\theta \geq \hat{\theta}|\hat{s})(-C) = 0$$

or

$$P(\theta < \hat{\theta}|\hat{s}) = \Phi\Big(\sqrt{\tau_\theta + \tau_\epsilon}(\hat{\theta} - \frac{\tau_\theta\mu_\theta + \tau_\epsilon\hat{s}}{\tau_\theta + \tau_\epsilon})\Big) = \gamma,$$

where $\gamma \equiv \frac{C}{C+B} < 1$. It follows that signal threshold curve is given by

$$s_T(\hat{\theta}) = \frac{\tau_\theta + \tau_\epsilon}{\tau_\epsilon}\hat{\theta} - \frac{\tau_\theta}{\tau_\epsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\epsilon}}{\tau_\epsilon}\Phi^{-1}(\gamma),$$

and

$$r(s)\hat{} = s_T(\theta_F(\hat{s})) = \frac{\tau_\theta + \tau_\epsilon}{\tau_\epsilon}\theta_F(\hat{s}) - \frac{\tau_\theta}{\tau_\epsilon}\mu_\theta - \frac{\sqrt{\tau_\theta + \tau_\epsilon}}{\tau_\epsilon}\Phi^{-1}(\gamma),$$

Therefore, the maximal value of the slope is

$$\bar{r}' \equiv \frac{\tau_\theta + \tau_\epsilon}{\tau_\epsilon + h_1\sqrt{2\pi\tau_\epsilon}}$$

which is increasing in h_1^{-1} and in τ_θ, while with respect to τ_ϵ it is first decreasing and then increasing. In particular It is trivial checked that $\frac{\partial\bar{r}'}{\partial\tau_\epsilon} < 0$ for $\tau_\epsilon < \tau_\theta$.

B.2 Proof Proposition 1

(1) The game is monotone supermodular as $\pi(y_i, y; \theta)$ has increasing differences in $(y_i, (y, -\theta)$, namely the differential payoff to act $(\pi^1 - \pi^0)$ is increasing in the aggregate action and in the negative of the state of the world $(y, -\theta)$ and signals are affiliated. This means that extremal equilibria exist, are symmetric (because the game is symmetric), and are in monotone (decreasing) strategies in type (Van Zandt and Vives 2007). Since there are only two possible actions, the strategies at an extremal equilibrium must be of the threshold form: $y_i = 1$ if and only if $s_i < \hat{s}$, where $\hat{s}$ is the threshold. It follows also that the extremal equilibrium thresholds, denoted $\bar{s}$ and

$\underline{s}$, bound the set of strategies that result from the iterated elimination of strictly dominated strategies. In sum, the extremal equilibria are in thresholds strategies and those bound any other possible equilibrium. If $\bar{s} = \underline{s}$ then the game is dominance solvable and the equilibrium is unique. Such an equilibrium is characterized by two thresholds: s^* denotes the signal threshold to act; and θ^* denotes the state-of-the-world critical threshold, below which the mass managers is successful and a manager, who plays $y_i = 1$, receives payoff B. In equilibrium, the fraction of managers

$$y(\theta^*, s^*) = P(s < s^*|\theta) \equiv \Phi(\sqrt{\tau_\epsilon}(s^* - \theta^*))$$

must be no larger than the critical fraction above which it pays to act, $h(\theta^*)$. Note that $\theta^* \in [\underset{\sim}{\theta}, \tilde{\theta})$ because $\lim_{\theta \to \underset{\sim}{\theta}} h_\theta = 0$, $h(\cdot)$ is strictly increasing on $(\underset{\sim}{\theta}, \tilde{\theta})$, and $h(\tilde{\theta}) = 1$. Furthermore, at the critical signal threshold, the expected payoffs for acting and for not acting should be the same:

$$P(\theta < \theta^*|s^*) \equiv \Phi\Big(\sqrt{\tau_\theta + \tau_\epsilon}(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\epsilon s^*}{\tau_\theta + \tau_\epsilon})\Big) = \gamma,$$

where $\gamma \equiv \frac{C}{B+C} < 1$.

(2) Consider an equilibrium (s^*, θ^*) . We show that if $\Phi(\sqrt{\tau_\epsilon}(s^* - \underset{\sim}{\theta})) \leq h(\underset{\sim}{\theta})$ then $\theta^* = \underset{\sim}{\theta}$. This will be so if $s_F(\underset{\sim}{\theta}) \geq s_T(\underset{\sim}{\theta})$. It is immediate then that $s_F(\underset{\sim}{\theta}) \geq s_T(\underset{\sim}{\theta})$ if $h(\underset{\sim}{\theta}) \geq \bar{h}_0(\underset{\sim}{\theta})$, where

$$\bar{h}_0(\underset{\sim}{\theta}) \equiv \Phi\Big(\frac{\tau_\theta}{\tau_\epsilon}(\underset{\sim}{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\epsilon}}\Phi^{-1}(\gamma)\Big) > 0.$$

For $h(\underset{\sim}{\theta}) < \bar{h}_0(\underset{\sim}{\theta})$ we have that $\theta^* > \underset{\sim}{\theta}$ at any equilibrium. In summary, there is a critical $\bar{h}_0(\underset{\sim}{\theta}) \in (0,1)$ such that for $h(\underset{\sim}{\theta}) < \bar{h}_0(\underset{\sim}{\theta})$ we have $\theta^* > \underset{\sim}{\theta}$ and for $h(\underset{\sim}{\theta}) > \bar{h}_0(\underset{\sim}{\theta})$ there is always unless one equilibrium with $\theta^* = \underset{\sim}{\theta}$.

(3) From B.1 we have that if $/\frac{\tau_\theta}{\tau_\epsilon} \leq h_1\sqrt{2\pi}$ then $r'(\hat{s}) \leq 1$ and the equilibrium is unique. In this case, the game is dominance solvable because $\bar{s} = \underline{s}$. Furthermore, it should be clear that the critical thresholds θ^* and s^* move together. Let $h(\underset{\sim}{\theta}) < \bar{h}_0(\underset{\sim}{\theta})$. Then we can combine

$$\Phi(\sqrt{\tau_\epsilon}(s^* - \theta^*)) = h(\theta^*) \tag{17}$$

and

$$\Phi\Big(\sqrt{\tau_\theta + \tau_\epsilon}(\theta^* - \frac{\tau_\theta \mu_\theta + \tau_\epsilon s^*}{\tau_\theta + \tau_\epsilon})\Big) = \gamma \tag{18}$$

to obtain:

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\theta}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\epsilon}\Phi^{-1}(\gamma) = 0 \tag{19}$$

Equation 19 may have multiple solutions in θ^*. As $\theta \to \tilde{\theta}$ we have that $\Phi^{-1}(h(\theta)) \to +\infty$ and $\varphi = -\infty$; as $\theta \to \underset{\sim}{\theta}$ we have that $h(\theta) \to h(\underset{\sim}{\theta})$ and

$\varphi \to \varphi(\underset{\sim}{\theta}) > 0$ whenever $h(\underset{\sim}{\theta}) < \bar{h}_0$. Hence there is at least one solution $\theta^* \in [\underset{\sim}{\theta}, \tilde{\theta})$. That solution will be unique if $\varphi' < 0$; while if $\varphi' > 0$ for a potential solution $\varphi(\theta) = 0$ then there will be multiple solutions. γ tends to 0 (resp., 1) we have that $\Phi^{-1}(\gamma)$ tends to $-\infty$ (resp., $+\infty$), s^* tends to $+\infty$ (resp., $-\infty$), and θ^* tends to $\tilde{\theta}$ (resp., $\underset{\sim}{\theta}$). These claims follow from Equation 18 and because $\theta^* \in [\underset{\sim}{\theta}, \tilde{\theta})$. There is indeed a unique solution if $\frac{\tau_\theta}{\sqrt{\tau_\epsilon}} \leq h_1\sqrt{2\pi}$. We have that $\varphi'(\theta) = \tau_\theta - \frac{\sqrt{\tau_\epsilon} h'}{\phi(\Phi^{-1}(h(\theta)))}$ and φ' is bounded above as: $\varphi' \leq \tau_\theta - h_1\sqrt{2\pi\tau_\epsilon}$ with strict inequality except possibly when $h(\theta) = \frac{1}{2}$. Therefore, if $\frac{\tau_\theta}{\sqrt{\tau_\epsilon}} \leq h_1\sqrt{2\pi}$ then $\varphi' \leq 0$.

B.3 Proof Proposition 2

If $h(\underset{\sim}{\theta}; \alpha; l) < \bar{h}_0$, then θ^* is determined by

$$\varphi(\theta^*) \equiv \tau_\theta(\theta^* - \mu_\theta) - \sqrt{\tau_\theta}\Phi^{-1}(h(\theta^*)) - \sqrt{\tau_\theta + \tau_\epsilon}\Phi^{-1}(\gamma) = 0$$

We study how change in the parameters effect $\varphi(\cdot)$.

(1) The result for θ^* follows because φ is decreasing in μ_θ, γ and α, while it is increasing in l (since $\frac{\partial\varphi}{\partial\alpha} < 0$ and $\frac{\partial\varphi}{\partial l} > 0$). The threshold s^* moves with θ^*.[14] The result for $P(s < s^*)$ is immediate for γ, α and β. While for μ_θ the effect is due to up shift of the s distribution.

(2) Suppose the equilibrium is unique. The equilibrium signal threshold is determined by $r(s^*; \mu_\theta) - s^* = 0$. From this it follows that, for a marginal change in μ_θ,

$$\left|\frac{ds^*}{d\mu_\theta}\right| = \frac{|\partial r/\partial \mu_\theta|}{1 - r'} > \left|\frac{\partial r}{\partial \mu_\theta}\right|$$

whenever $0 < r' < 1$. In consequence, an increase in μ_θ will have a larger effect on the equilibrium threshold s^* than does the direct impact on a players best response $\frac{\partial r}{\partial \mu_\theta} = -\frac{\tau_\theta}{\tau_\epsilon}$. The same is true for discrete changes even with multiple equilibria, provided we restrict our attention to extremal equilibria or allow for adaptive dynamics. The multiplier effect is greatest when r' is close to 1 at equilibrium (i.e., when strategic complementarities are strong) and we approach the region of multiple equilibria (and this happens when τ_θ is large because r' is increasing in τ_θ).

(3) First consider changes in τ_θ. We have the equality

$$\frac{\partial\varphi}{\partial\tau_\theta} = \theta^* - \mu_\theta - (\tau_\theta + \tau_\epsilon)^{-\frac{1}{2}}\frac{\Phi^{-1}(\gamma)}{2}$$

[14] The result for μ_θ follows also from a general argument in monotone supermodular games. We know that extremal equilibria of monotone supermodular games are increasing in the posteriors of the players (Van Zandt and Vives 2007). A sufficient statistic for the posterior of a player under normality is the conditional expectation $E[\theta|s] = \frac{\tau_\theta\mu_\theta + \tau_\epsilon s}{\tau_\theta + \tau_\epsilon}$, which is increasing in μ_θ. Thus it follow that extremal equilibrium thresholds $(-\theta, -s^*)$ are increasing in μ_θ.

and therefore $\frac{\partial\varphi}{\partial\tau_\theta} > 0$ when $\underset{\sim}{\theta} \geq \mu_\theta$, since $\theta^* > \underset{\sim}{\theta} > \mu_\theta$ and $\Phi^{-1}(\gamma) < 0$ for $\gamma < \frac{1}{2}$. Note that, for $\theta^* \geq \mu_\theta$, if τ_θ increases then $P(\theta < \theta^*) = \Phi(\sqrt{\tau_\epsilon}(\theta^* - \mu_\theta))$ also increases.

Consider now changes in τ_ϵ. With our current assumptions and using the equality $\varphi(\theta^*) = 0$ in $\frac{\partial\varphi}{\partial\tau_\epsilon}$, we obtain

$$\frac{\partial\varphi}{\partial\tau_\epsilon} = -(\theta^* - \mu_\theta - (\tau_\theta + \tau_\epsilon)^{-\frac{1}{2}}\Phi^{-1}(\gamma))\frac{\tau_\theta}{2\tau_\epsilon}$$

and $\frac{\partial\varphi}{\partial\tau_\epsilon} < 0$. The results follow. If $\mu_\theta \geq \tilde{\theta} - (\tau_\theta + \tau_\epsilon)^{-\frac{1}{2}}\Phi^{-1}(\gamma)$ then $\theta^* < \mu_\theta$ and the results are reversed because $\theta^* < \tilde{\theta} \leq \mu_\theta$ (since $\Phi^{-1}(\gamma) < 0$, we have $-\Phi^{-1}(\gamma) > -\frac{\Phi^{-1}(\gamma)}{2}$). In addiction, the conditions assumed are sufficient but not necessary. Indeed when the equilibrium is unique they can be strengthened as follows: there are thresholds $\hat{\mu}_\theta$ and $\dot{\mu}_\theta$ with $\hat{\mu}_\theta < \dot{\mu}_\theta$ such that (a) θ^* increases with τ_θ if and only if $\mu_\theta < \hat{\mu}_\theta$ and (b) θ^* increases with τ_ϵ if and only if $\mu_\theta > \dot{\mu}_\theta$. Indeed, let $\theta^*(\mu_\theta)$ be the unique equilibrium. We have that $\hat{\mu_\theta}$ is the unique solution in μ_θ to $\frac{\partial\varphi}{\partial\tau_\theta}(\theta^*(\mu_\theta), \mu_\theta) = 0$ or to $\theta^*(\mu_\theta) = \mu_\theta + (\tau_\theta + \tau_\epsilon)^{-\frac{1}{2}}\frac{\Phi^{-1}(\gamma)}{2}$ and that $\dot{\mu_\theta}$ is the unique solution in μ_θ to $\frac{\partial\varphi}{\partial\tau_\epsilon}(\theta^*(\mu_\theta), \mu_\theta) = 0$ or to $\theta^*(\mu_\theta) = \mu_\theta + (\tau_\theta + \tau_\epsilon)^{-\frac{1}{2}}\Phi^{-1}(\gamma)$. The solution are unique since $\theta^*(\mu_\theta)$ is decreasing in μ_θ. This means also that $\hat{\mu_\theta} < \dot{\mu_\theta}$ as $\Phi^{-1}(\gamma) < 0$ because $\gamma < \frac{1}{2}$. Therefore there are thresholds $\hat{\mu}_\theta$ and $\dot{\mu}_\theta$ with $\hat{\mu}_\theta < \dot{\mu}_\theta$ such that (a) θ^* increases with τ_θ if and only if $\mu_\theta < \hat{\mu}_\theta$ and (b) θ^* increases with τ_ϵ if and only if $\mu_\theta > \dot{\mu}_\theta$.

B.4 Proof Proposition 3

The Proposition 3 presents the comparative static results in the bank credit market. In sum: there is a critical $\bar{\Psi} \in (0, 1)$ such that when $\Psi \geq \bar{\Psi}$ there is always an equilibrium in which $\theta^* = \underset{\sim}{\theta}$, while for $\Psi < \bar{\Psi}$ then $\theta^* > \underset{\sim}{\theta}$ at any equilibrium. If $\Psi < \bar{\Psi}$ and there is a unique equilibrium then the comparative static results are those reported in Table 1.

From the proof of Proposition 1 it follows that the critical ratio $\bar{\Psi}$ satisfies

$$\bar{\Psi} = \Phi\Big(\frac{\tau_\theta}{\sqrt{\tau_\epsilon}}(\underset{\sim}{\theta} - \mu_\theta) - \sqrt{1 + \frac{\tau_\theta}{\tau_\epsilon}}\Phi^{-1}(\gamma)\Big),$$

where :

$$\underset{\sim}{\theta} = \frac{(\bar{l} - \underline{l})(r_c - r_d) - v}{(\bar{l} - \underline{l})(r_c + LGD)}.$$

Note that $\bar{\Psi}$ depends on the parameters of the model and in particular it decreases in r_d, LGD, $\underline{l}$ and v whereas it increases in $\bar{l}$ and r_c. Finally it is independent by γ and Ψ. The other results follow from the prior Proposition 2 or by direct inspection of the equilibrium condition $\varphi(\theta) = 0$. Let us just underline that firstly, each variation of $\underset{\sim}{\theta}$ causes a variation of the same sign of $P(\theta < \underset{\sim}{\theta})$, whereas each variation of $\tilde{\theta}$ causes a variation of the opposite sign of the $P(\theta > \tilde{\theta})$ and secondly, that s^* moves accordingly with θ^* and this is useful to a fast understanding of the third column of the Table 1[15].

[15] Any variation of s^* causes a variation of the same sign of the $P(s < s^*|\theta)$.