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LONG SWINGS IN THE GROWTH OF GOVERNMENT EXPENDITURE: AN INTERNATIONAL HISTORICAL PERSPECTIVE

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Abstract

Using an international historical perspective this study aims at identifying the main empirical regularities, as well as countries' similarities and differences, in the long-run growth pattern of government expenditure, in order to distinguish among existing, competing theories of government growth. The application of nonparametric and parametric analyses to a sample of developed countries observed over the 1880-2011 period allows us to detect two main findings. The first is that, beyond the long-term growth of government expenditures in absolute terms, there is evidence of three expansionary long waves corresponding to the pre-armament booms in the WWs periods and the "golden age of public sector intervention". The latter is the reduction of the degree of cross-country heterogeneity in the pattern of relative and absolute growth of government expenditure since the 1960s. The "ratchet phenomenon" in the pre-WWII period and the change of the prevailing ideology from market failures to government failures in the more recent period provide explanations complementary to Wagner's law and consistent with the observed long-term evolution of the relative (and absolute) growth of government expenditure.

KEYWORDS: Wagner's Law, Government expenditures, Local nonparametric regression analysis, Cointegration test.

JEL codes: C14; C22; E60; H50; H60; N40.

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LONG SWINGS IN THE GROWTH OF GOVERNMENT EXPENDITURE: AN INTERNATIONAL HISTORICAL PERSPECTIVE

Marco Gallegati, Massimo Tamberi

1. Introduction

The increase in size of the public sector relative to the whole economy in developed economies during the past century is well documented in the literature on public sector economics (see Hindriks and Myles, 2006, Fig. 1, p. 90). A wide range of explanations, and associated variables, has been proposed for explaining the historical evolution of the role of government, and especially its absolute and relative growth along the development process. Wagner's (1883, 1893, 1911) formulation of the 'Law of Increasing State Activity',[1] was followed by Peacock and Wiseman's (1961) displacement effect and Bird's ratchet effect (1971, 1972), Baumol's law (Baumol, 1967), Musgrave's (1969) stages of development approach, Peltzman's (1980) model on the redistributive aspect of government activity, and, more recently, Rodrik's (1998) theory of trade openness, Alesina and Wacziarg (1998) theory of country size, the theories on the role of political organization in terms of electoral rules, type of government, and degree of political participation (Persson and Tabellini, 1999, and Milesi-Ferretti et al., 2002), and Acemoglu and Verdier (2000) on the trade-off between market and government failure.[2]

The role of the state in the historical process of economic development is also at the heart of the growing literature in historical political economy which is based on the concept of state

[1] Wagner's law is based on the simple regularities detected in the growth of government along the developmental path, represented by an absolute and relative expansion of the public sector within the national economy.
[2] Another strand of literature, somehow related to Wagner's and Rodrik's hypothesis, is the idea of North (1985, 1991, 2003) of the transition from informal to formal institutions.

capacity (Besley and Persson, 2009). According to the state capacity literature, state capacity, measuring the fiscal and administrative power of states, should be distinguished from either the size or the scope of the state. In this sense, the ability of collecting taxes and enforcing law and order is considered a key determinant of economic development (Dincecco, 2015, and Johnson and Koyama, 2017).[3]

The plethora of theories used to explain variation in the secular pattern of government expenditure is symptomatic of the complexity of providing an adequate explanation of long-term growth in government spending. Public expenditures decisions derive from a complex process (interplay) in which supply and demand[4] economic factors are mediated through political decision-making processes (Gerber and Jackson, 1993). Moreover, the observed long-run pattern of public expenditure growth includes periods of increased economic growth, expanded public activities, war or social upheavals, different phase of economic development and several waves of technological changes.

Over the last decades we have assisted to a proliferation of studies aiming at testing the existence of a long-run relationship between government expenditure and economic activity by means of advanced statistical and econometric techniques. Following Henrekson's (1993) study, unit root tests and cointegration analysis, with and without structural breaks, have become the standard tools in this literature because of their ability to take into account the non-stationarity features of the data. However, within these sophisticated techniques the complexity of the process at the basis of the growth of public sector economic activities is simplified to a

[3] These theories provide explanations at odds with the causal relationship implicitly assumed in Wagner's law.
[4] The demand of public goods during the development process has changed: from the 'traditional' services of defence, law and order, (early stage of development) to the demand for education, infrastructure, social security, health systems, up to new regulatory and protective functions associated with the evolution in the structure of the economy.

linear long-run relationship, with or without structural breaks (e.g. Peacock and Scott, 2000, Wagner and Weber, 1977, Durevall and Henrekson, 2011).

In face of this complex and diversified set of explanations, in this study we take an international historical perspective and search for regularities, as well as countries' similarities and differences, in the long-run growth pattern of government expenditure in a sample of 17 developed countries observed from the late XIXth century to date, with the goal of distinguishing among existing, competing theories of government growth. A "chaise longue" pattern of the expansion of government size along the developmental path is clearly evidenced by nonparametric local regression analysis. Two periods of growth in government size are detected, the first moderate the latter much stronger, followed by a flattening of the growth pattern at the highest level of per capita income. The results of the cointegration tests for the whole period show the existence of a long-run relationship government expenditure and GDP, with the GDP coefficient being large and significant, even after allowing for a structural break at an unknown date and for different sub-periods (pre- and post-WWII years). However, after the mid-1970s the estimated coefficients provide clear evidence of the attenuation of strength of the long-run relationship, with the sign of the coefficients turning to negative in many cases.

The rolling regression analysis of the long-run relationship allows detecting three periods of acceleration in government spending growth: the first two corresponding to periods of large-scale social upheavals (pre- armament booms), the third to the golden age of public sector intervention, followed by a quite uniform declining pattern across countries after the 1970s. In addition, while substantial cross-country heterogeneity is evident in the pre-1960s period, increased synchronization in the historical trends of absolute and relative government expenditures emerges after the 1960s, with a striking convergence of these long-term growth rates to values between 0 and 2% towards the end of the sample. We also show that the long run evolution of the growth rate pattern of government expenditure is a common feature of both relative and absolute measures of government spending.

Taken together, our findings suggest that the combination of alternative theories, rather than each of them taken in isolation, may provide a comprehensive explanation of long-run government expenditure. Specifically, two different explanations complementary to Wagner's law are suggested for interpreting the evolution of the long-term growth pattern of absolute and relative government spending: the "ratchet phenomenon" in the pre-WWII period, and the growing negative effects of government size on the efficiency of the economy and the related changes of the prevailing ideology on the role of the state in the economy in the post-WWII period.

The paper is structured as follows. Section 2 presents the data and the literature. Section 3 and 4 show the results of this study using nonparametric local regression analysis and cointegration analysis at the individual country level, respectively. Section 5 checks the time-varying patter of the long-term relationship, while Section 6 examines whether and how the pattern of the long-term evolution of the growth in government spending *differ between* absolute and relative measures. The interpretation of results is provided in Section 7, while section 8 concludes.

2. Dataset and literature

Researchers have usually explained the long-run evolution of government activity by looking at the ratio of public expenditure to gross domestic product (GDP).[5] Empirical works studying the relationship between government expenditure and economic activity employ both a time series and cross-section framework. While the country dimension may vary from several units to more than one hundred, the temporal dimension is generally quite low, with the time span being mostly limited to a sub-sample of the post-WWII period (e.g. Ram, 1987, Wahab, 2004,

[5] For an international comparison of the major trends of industrial countries, see Tanzi (1986).

Akitoby et al., 2006, Kolluri and Wahab, 2007, Lamartina and Zaghini, 2011, Bruckner et al., 2012, Afonso and Jalles, 2014, Magazzino et al. 2015). Very few studies use a longer time period, e.g. Henkerson (1993) for Sweden, Durevall and Henkerson (2011) for Sweden and the UK, Paparas et al. (2018) for the UK, Kuckuck (2012) for five western European countries. Even Oxley (1994) and Thornton (1999) use historical data from mid XIXth century for Britain and six European countries, respectively, but limiting their analysis to the pre-WWI period. The only paper that adequately exploits both the country and time dimensions is that by Easterly and Rebelo (1993) with historical data covering 26 countries from the late XIXth century.

In this study we use IMF data on government expenditure and GDP for several developed countries observed over a long time period, 1880-2011. The two dimensions are equally important for evaluating alternative explanations for the growth in the size of the public sector. A time span of a few decades may be unfit to allow identification of any significant structural change to be interpreted in terms of economic development, because of the long-run nature of the government expenditure-income relationship. Indeed, studies on economic development/growth generally takes the longest possible time span in order to be able to identify different phases of economic development (Maddison, 1982). The inclusion of countries at similar stages of economic development that differ for demographics, trade openness, political organization (i.e. electoral rules, size, type of government, degree of political participation), in principle allows to distinguish between alternative theories of government size such as demand-driven and supply-driven.

Using a very long-time span may be problematic in that historical time series are likely to exhibit short-lived transient components typical of war or crisis episodes, like abrupt changes, jumps and volatility clustering. The treatment of war years may results in inclusion of war interval dummies or, with missing values, in corrections applied to original data through interpolation (Metz 1992) or a priori elimination of their impact on the assumption that such shocks can be seen as disturbances in the normal structure of data (Korotayev and Tsirel 2010).[6] As shown in Table A1 of the Appendix 1, 11 countries out of 17 present missing data; in some cases missing is limited to very short period (2-3 years), but they are longer in other cases, concentrated either at the beginning of the sample, or in coincidence with war periods (for several European countries). For countries with missing data within the sample, i.e. Belgium, Germany, Spain, France, Netherlands and Norway, cointegration analysis is performed using a wavelet-based approximation of the aggregate series which yields unbiased and consistent estimates for the intercept and slope parameters, e.g. Ramsey et al. (2010), Gencay and Gradejovic (2011) and Gallegati and Ramsey (2012). In particular, wavelet multiresolution decomposition analysis, by returning at each step a set of averages (along with a set of differences between adjacent averages) based on different window length, allows to obtain a collection of approximations of the original signal, from finer (S_1) to coarser (S_4) resolution levels.[7]

The measure of public sector expenditure depends on the definition of government. Since countries differ a lot in terms of their processes and levels of fiscal decentralization, the ratio between the data for the alternative definitions of government expenditures, our variable of interest, may display a high degree of variability over time and across space. IMF data refer to general government, whereas, for example, data on public expenditures in Jordà et al. (2017) dataset refer to central government. However, since the countries included in our dataset (see appendix 1 for a complete list) differ substantially in terms of fiscal decentralization, we decide

[6] This practice, according to Freeman and Louçã (2001), would be equivalent to studying history by erasing part of the history.
[7] A brief introduction to wavelet analysis and detail of multiresolution decomposition and approximation analyses are provided in the Appendix 2.

to use IMF data for the main analysis and Jorda et al. (2017) macro-historical dataset for robustness purposes.[8]

3. Nonparametric local regression analysis

Given the ambiguity in Wagner's (1883) formulation of the 'Law of Increasing State Activity', at least 6 different parametric versions have been proposed in the literature, e.g. Peacock and Wiseman (1961), Gupta (1967), Goffman (1968), Pryor (1969), Musgrave (1969), Goffman and Mahar (1971) and Mann (1980). In what follows we use the probably most common formulation of Wagner's hypothesis as suggested by Musgrave (1969),

$$\textit{Government share} = \alpha + \beta\ \textit{(per-capita GDP)}$$

where the government share is measured as the log of the ratio between government expenditure and GDP in nominal terms, and the log per-capita GDP is measured in real terms.

Given the problems that arise from testing hypotheses about the growth in public expenditures, estimating a rigorously specified model using parametric analysis may appear to be an ambitious task because parametric regression analysis makes some strong assumptions[9] and because of its rigidity in terms of fixed parameters and functional forms. By contrast, nonparametric analysis, not imposing any specific form to the regression function in advance of examination of the data, avoids the risk of playing with the specification. Indeed, nonparametric regression analysis can capture the shape of a relationship between variables without us prejudging the issue, as they estimate the regression function *f(.)* linking the dependent to the independent variables directly, without providing any parameters estimate.

[8] In order to assess the robustness of our results, we have replicated all steps of our empirical analysis using Jordà et al. (2017) dataset measure about central government (see Appendix 1 for details). The results, not reported here to save space but available upon request from the authors, show that using different definitions of government expenditure does not affect the qualitative nature of our findings.

[9] Parametric regression analysis makes several strong assumptions: i) linear relationship between variables, ii) a conditional distribution of the dependent variable that, except for its mean, is everywhere the same, (and that this distribution is a normal distribution), and iii) observations independently sampled.

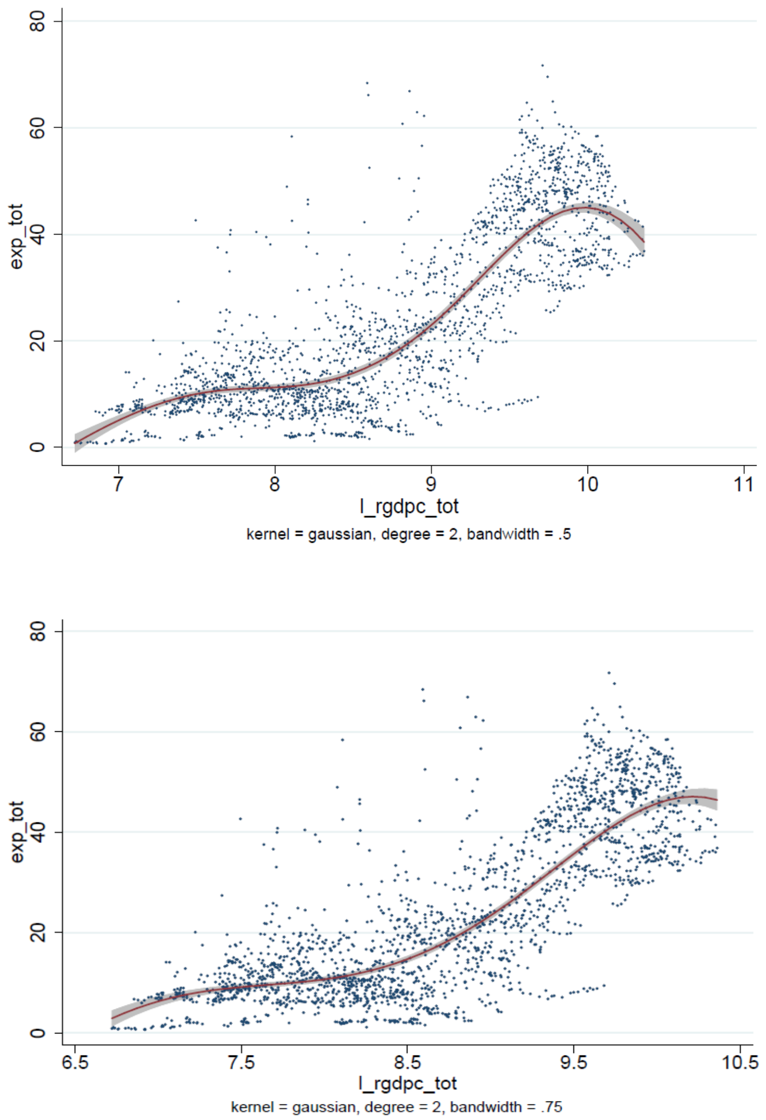
Therefore, we start estimating the long-term relationship between government share and per-capita GDP using non-parametric methods. There are several approaches available to estimate nonparametric regression models, and most of these methods assume that the nonlinear functions of the independent variables to be estimated by the procedures are *smooth* continuous functions. One of the most commonly available method of nonparametric regression called *lowess* or *loess* (Cleveland, 1979), an implementation of local polynomial regression. In the loess method the regression function is evaluated at each particular value of the independent variable, x_i, using a local neighborhood of each point and the fitted values are connected in a nonparametric regression curve. In fitting such a local regression, a fixed proportion of the data is included in each given local neighborhood, called the *span* of the local regression smoother (or the smoothing parameter), and the data points within each window are weighted by a smooth function whose weights decrease as the distance from the center of the window increases.

The nonparametric fitted curves in Figure 1 are produced by estimating a local polynomial regression of the government expenditure GDP ratio on the log of real per-capita GDP for all observations, and using a second order polynomial, a gaussian weighting scheme, and two different bandwidths, 75% and 50%, in the top and bottom panels of Figure 1, respectively.[10] Both panels of Figure 1 show a clear positive nonlinear relationship between the two variables. At low levels of income the positive relation displays a moderate trending pattern, while at intermediate levels, that is at values between 8 and 10, a considerable increase becomes evident followed by a flattening of the slope at the highest levels, values above 10. When the span is

[10] The application of a span equal to 25% of the data yields results similar to those obtained using a bandwidth of 0.50.

reduced from 0.75 to 0.5 the nonlinearity of the relationship is more evident with the final part of the curve becoming downward sloped.

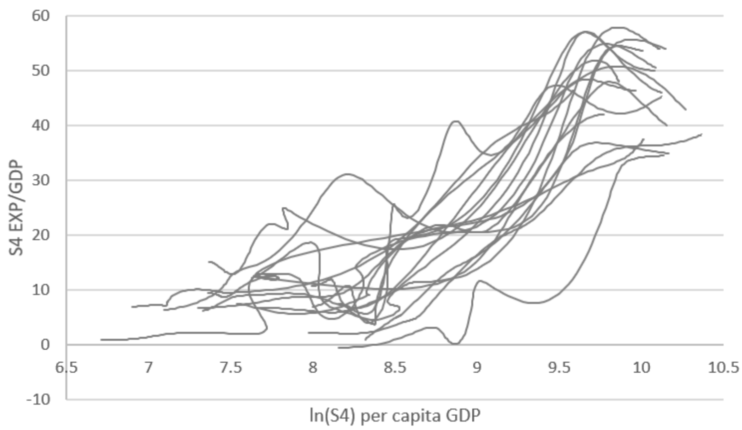
Figure 1 - Nonparametric relationship between government expenditure (% of GDP) and per capita income using different bandwidths: 0.75 (upper panel) and 0.5 (lower panel)



This approach implicitly hypothesizes the equivalence between all pairs of observations, regardless of any country characteristic (e.g. De Benedictis et al., 2008, 2009), and, in this sense, may reflect the relationship between government size and GDP for an average country. The pattern of the relationship along the developmental path may be used to test some of the theories proposed to explain the growth of public expenditure during the past century, such as the development models of public sector growth and Wagner's law. Indeed, a country's development stage is a crucial consideration for the validity of Wagner's law. Our findings seem to be consistent with Wagner's law, since an expanding government accompanies social progress and rising incomes.

In order to assess the relevance of the average-country hypothesis in Figure 2 we provide evidence of the Wagner's law for each individual country using the long-term components (S_4) of the expenditure GDP ratio and per-capita GDP, extracted from wavelet multilevel decomposition. Two distinct periods may be clearly distinguished, with the break occurring at a log value of per-capita GDP around 8.5: in the first, the expenditure GDP ratio grows moderately, with countries displaying a heterogeneous pattern; in the latter, countries show a strong homogeneous pattern with the ratio first increasing and then decreasing at the highest values of income.

Figure 2 – "Wagner's law" for each country using the long-term components of the expenditure GDP ratio and per-capita GDP (1881-2011)



4. Cointegration analysis

Since it is generally assumed that Wagner was referring to the trend in the ratio of government expenditure to GDP,[11] this specification is commonly interpreted as a long-run relationship. Thus, in the recent literature cointegration analysis has become the standard approach to investigate the existence of a long-run relationship between economic development and government activity by testing both the existence of a long-run relationship, as well as the sign, size and statistical significance of the coefficient of interest (Shelton, 2007, Durevall and Henrekson, 2011, Magazzino, 2012).

Based on the findings presented in Figure 2 we decide not to run panel data cointegration analysis but cointegration tests for each country.[12] Given that we only have two variables, and thus there cannot be more than one cointegrating vector, we apply the Engle-Granger (1987) procedure to test for cointegration.[13] Moreover, since nonparametric analysis indicates several shifts in the relationship between government expenditure and real per capita GDP we also use the Gregory-Hansen (1996) procedure that allows testing for the presence of a structural break at an unknown date. Both tests use the null hypothesis of no-cointegration, but the alternative in the Gregory-Hansen test is cointegration with or without a structural break. When both tests reject the null hypothesis the presence of cointegration without a structural break is assessed, but if only the Gregory-Hansen test rejects the null we conclude that there is cointegration with one structural break.

Table 1 reports for each country the estimates of the Engle-Granger cointegrating regression (column β) and cointegration ADF test statistics (column ADF) along with the Gregory-Hansen

[11] As stated by Musgrave (1969, p.73, fn.1) Wagner never made explicit whether he was thinking to a trend relationship.
[12] Repeating time-series estimates for each country in the sample allows to retain country-specific information, but at the cost of reducing the power of the statistical tests.
[13] Cointegrating regression analysis is performed on the natural logarithms of government expenditure and per capita GDP series to permit an interpretation of regression coefficients in terms of elasticities.

test (column GH) for the whole sample, 1880-2011. The cointegrating regression shows that the estimate of the elasticity, β, is always positive, with most of the coefficient values being close to the unity; exceptions are Italy and Spain, with values clearly below unity, and Switzerland whose value is more than double.

Table 1 - Engle-Granger cointegrating regression for the 1880-2011 period

Government/GDP = $\alpha + \beta$ (per capita GDP)

1880-2011	β	*ADF*	*GH*	*Break date*
Australia	1.500**	-2.586	-4.33	1948
Belgium	1.080**	-1.691	-5.23**	1930
Canada	0.945**	-3.263***	-4.18	1910
Suisse	2.237**	-2.109	-3.01	1928
Germany	0.721**	-3.092***	-4.01	1951
Denmark	1.110**	-2.257	-5.51**	1968
Spain	0.552**	-1.641	-3.36	1984
Finland	0.622**	-2.618	-5.16**	1949
France	0.754**	-2.428	-3.58	1926
Great Britain	0.986**	-2.095	-5.69**	1911
Italy	0.480**	-2.051	-4.31	1950
Japan	1.103**	-1.767	-4.90***	1946
Netherland	1.029**	-2.578	-3.68	1924
Norway	0.787**	-2.537	-4.67	1947
Portugal	0.729**	-1.591	-6.60*	1971
Sweden	0.832**	-2.818	-5.63**	1937
USA	1.508**	-3.984**	-6.42*	1933

Note: S_1 approximation is used for Spain, S_3 for Belgium, Netherlands and Norway, S_4 for Germany and France. ADF is the Engle-Granger cointegration test with lag 1 without war dummies. GH is the Gregory-Hansen cointegration test allowing for a regime shift. Significance levels: *1%, **5%, ***10%.

The Engle-Granger cointegration test statistics reject the null hypothesis of no cointegration only for Canada, Germany (both at the 10% level) and the US (at the 5% level). The break dates are of interest when the ADF test is insignificant and the GH test is significant, that is for Portugal (at the 1% level), for Belgium, Denmark, Finland, Great Britain and Sweden (at the 5% level), and for Italy (at the 10% level). All in all, even when allowing for a shift of the relationship, the full sample estimate seems to be supportive of Wagner's law for a limited number of countries only.

Since the time span covered by our dataset includes several war years and crisis periods it could be useful to split the sample into two sub-samples corresponding to pre- and post-WWII year periods, respectively. Moreover, we put in evidence the post-1975 subsample, which roughly corresponds to the time span mostly used in the recent empirical literature.

Table 2 reports the estimates of the Engle-Granger cointegrating regression and the cointegration ADF test statistics using three different sub-samples: 1880-1939, 1951-2011, and 1975-2011. Individual countries' coefficients differ significantly between pre- and post-WWII periods. Thus, even when similar over the whole sample, they are the "result" of very different stories.

Consider, for example, the coefficients for Denmark and Japan which are both 1.1 in the 1880-2011 period, but while Denmark has a low value (0.4) for the pre-WWII period and higher (1.3) for post-WWII period, Japan display the opposite behavior, high in the pre-WWII period (1.3) and low after WWII (0.3).

Table 2 - Cointegration test for different sub-periods

	1880-1939		**1950-2011**		**1975-2011**	
	β	ADF	β	ADF	β	*ADF*
Australia	1.741**	*-2.370*	0 .842**	*-2.353*	0.017	*-3.109****
Belgium	1.309**	*-2.751*	0 .754**	*-0.412*	-0.179**	*-2.656*
Canada	0.299*	*-2.371*	0 .855**	*-1.636*	-0.171*	*-3.028***
Suisse	2.683*	*-1.938*	2.062**	*-2.024*	0.387**	*-3.303***
Germany	-0.839**	*-2.665*	0 .432**	*-4.157**	-0.101*	*-4.105**
Denmark	0.440**	*-2.500*	1.339**	*-0.525*	0.161*	*-1.757*
Spain	0.820**	*-1.759*	0 .846**	*-0.464*	1.387**	*-2.364*
Finland	0.976**	*-2.992*	0 .691**	*-1.856*	0.269**	*-3.261****
France	0.328*	*-2.496*	0 .831**	*-0.743*	0.991**	*-3.435***
Great Britain	2.428**	*-3.067****	0.256**	*-3.674***	-0.141*	*-3.402***
Italy	0.500**	*-2.554*	0 .774**	*-0.030*	0.345**	*-0.812*
Japan	1.361**	*-2.756*	0.341**	*-0.817*	0.270**	*-1.253*
Netherland	-0 .395**	*-2.467*	0.725**	*-2.001*	-0.335**	*-3.451***
Norway	0.185*	*-2.332*	0.627**	*-0.094*	-0.082	*-1.940*
Portugal	0 .542**	*-1.975*	0.876**	*-0.311*	0.427**	*-1.529*
Sweden	0.432**	*-3.242****	0 .909**	*-1.944*	0.539**	*-2.148*
USA	1.289**	*-2.356*	0 .629**	*-2.070*	0.143**	*-2.601*
Mean Group	*0.929*		*0.603*		*-0.032*	
Std. Dev.	*0.909*		*0.242*		*0.070*	

Note: see Table 1. Mean Group reports the average value of individual countries' estimates.

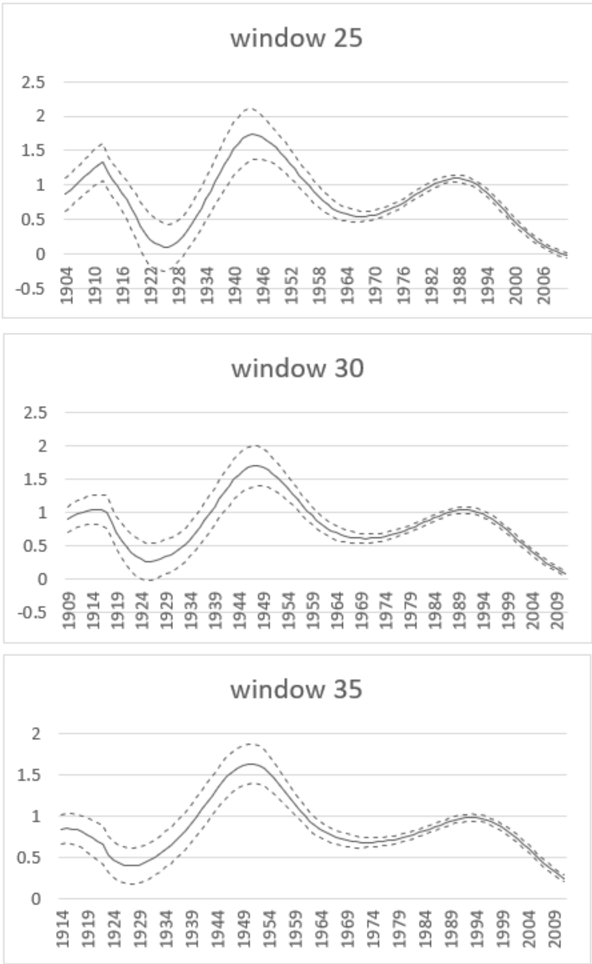
Interestingly, and consistently with the results reported using nonparametric analysis, there is clear evidence of a significant reduction in the size and variability of the estimate of the elasticity β from the pre- to the post-WWII period: the average magnitude decreases from .93 to .60 and the variability from 0.91 to 0.24. Moreover, the Engle-Granger cointegration test statistic is generally insignificant, the only exceptions being Sweden in the pre-WWII period, Germany in the post-WWII period, and the UK in both sub-samples.

Consistently with the results reported in the recent empirical literature on Wagner's law, after the 1970s the estimated coefficients suggest a completely different picture. The estimates of the elasticity β decrease, becoming negative in many cases, thus implying that the expenditure share decreases as GDP grows, with the positive parameters being generally very low (exceptions are Spain and France, with values close to unity, and Portugal and Sweden with values between 0.4 and 0.5). This is in striking contrast with what expected on the basis of Wagner's hypothesis. Moreover, the value of the standard deviation of the estimated coefficients indicates a strong common pattern across countries from the 1970s onward.

5. The time-varying pattern of the long-term relationship

In light of the evidence provided by both nonparametric and parametric analysis on the long-term pattern between government expenditure and per capita GDP provided, in what follows we investigate the time-varying pattern of the relationship using a rolling panel regression framework. This parametric method consists in estimating a rolling or centered moving regression using a shifting fixed window size and allows to identify the time-varying pattern of the long-term relationship. between government expenditures and output. Figure 3 shows the evolution over time of the rolling panel fixed-effect regression coefficient between the long-term components of the government GDP ratio and per capita GDP, estimated with different fixed window sizes set to 25, 30 and 35 years. The windows are moved forward by one observation so that we can plot a continuous line of estimated coefficients for the government/GDP - per capita GDP relationship. The coefficient lines are surrounded by a confidence band set at 1.96 times the standard error.

Figure 3 - Rolling panel fixed effects regression coefficient of the government expenditure ratio on per-capita GDP using fixed window sizes of 25, 30, 35 years



The main results are threefold. First, the long-term elasticity changes continuously and substantially over the period considered. Three periods of increase in the estimated coefficient are clearly detected, before WWs and in the 60s, followed by a considerable decrease of the long-term elasticity since the 1980s.[18] Second, the up and down swings tend to occur around an average elasticity not far from in the first part of the sample, value that almost halves after the 1960s. Third, the confidence bands shrink progressively over time, a result consistent with the estimates reported in Table 2 using different sub-samples.

6. Absolute or relative long-term growth rate pattern?

Government spending as a share (percent) of GDP is the measure that researchers usually look at when studying Wagner's law. However, the same long-term evolution of the government expenditure GDP ratio may be produced by two completely opposite patterns: in fact, government size can grow in relative terms either because the government growth has accelerated relative to GDP growth or due to a decline in the rate of growth of GDP relative to government expenditure. Therefore, in Figures 4 and 5 we present, respectively, the first two moments of the growth rate pattern of the government expenditure GDP ratio as well as its components, that is absolute government expenditure and GDP both in real terms.[14]

The upper panel of Figure 4 shows country differences in the long-term component of the growth rate of the government expenditure and GDP in real terms for each country from 1871 to 2011.[15] This difference provides us with evidence on the evolution (variation) of the long-term growth rate pattern of the relative government spending for each country.[16] Two main findings emerge: the wave-like pattern of the long-term growth rate of the government expenditure/GDP ratio (Crowley, 1970), and the clear break in the cross-country pattern. In particular, the substantial cross-country heterogeneity typical of the pre-1960s period is followed by a period of increasing synchronization among countries in the long-term pattern of the growth rate of relative government spending, in coincidence with the rapid growth of the

[14] Although the government size is expressed as the ratio between current values we use their real growth rates because of the different incidence of the inflation rate across periods in the post-WWII era.
[15] Long-term components are extracted through the MODWT applying the LA(8) Daubechies wavelet filter with reflecting boundary conditions and represent fluctuations greater than 32 years.
[16] Recall that the growth rate of a ratio is approximately equal to the difference of the growth rates of the numerator and denominator.

public sector that occurred during the so-called ”golden age of public sector intervention” (from 1960 to 1980).

The high degree of cross-country heterogeneity of the long-term patterns in the first part of the sample makes the detection of a common pattern a very difficult task. Thus, in each panel of Figure 4 we draw a thick line representing the cross-country average. Three long swings are now clearly detected with peaks occurring in 1910s, late 1930s, and early 1970s. The first two expansionary waves precede each of the two WWs and correspond to periods of pre-war armament booms. The 3rd, covering a period of 30-35 years after WWII, coincides with the stimulus given to the economy by governments involved in the reconstruction effort after WWII and culminated in the boom of the 1960s. Finally, following the peak of the early 1970s we assist to a reduction in the public expenditure growth relative to GDP that culminates in the trough of the early 1990s and is followed by a small increase in the first decade of the XXIst century.

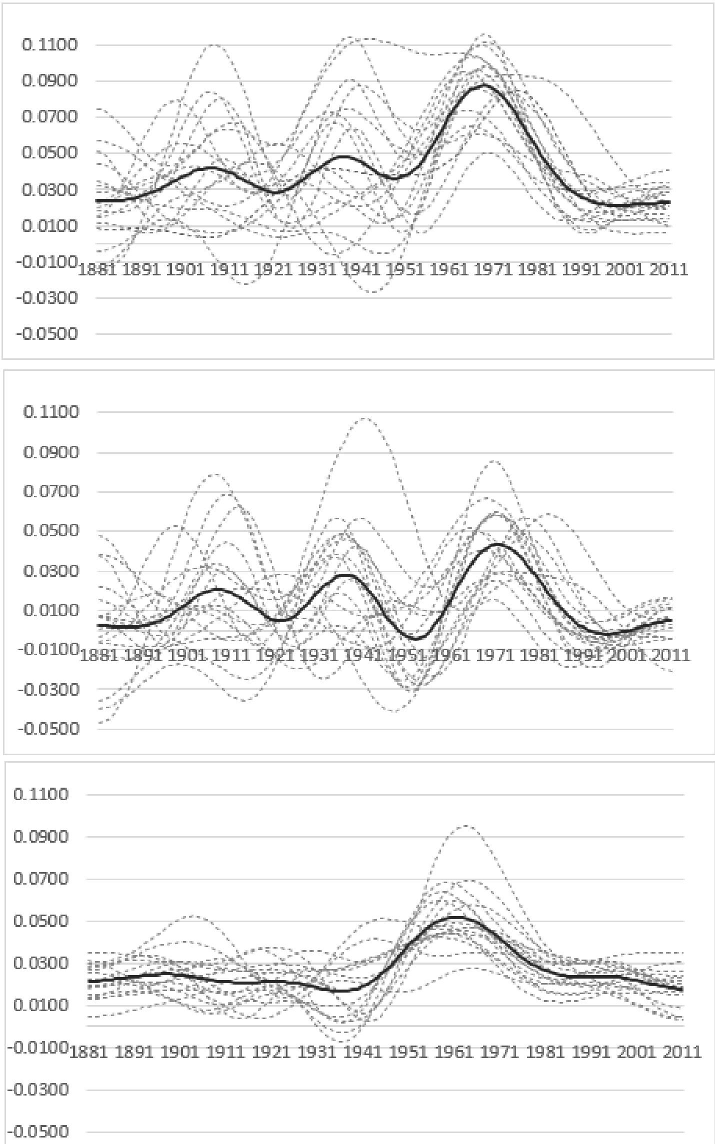
Furthermore, we split and observe separately the growth rates of two components of the expenditure/income ratio. The middle and bottom panel of Figure 4 show, for each country, the long-term pattern of the real growth rate of government expenditures and GDP, respectively, with the thick line representing the cross-country average historical trend of the two variables.

Two things are worth noting. The first, is the strong resemblance between the long swings in the growth rate of public expenditure in absolute and relative terms.[17] By contrast, the long-term pattern of the GDP growth rate displays a quite uniform pattern over the whole sample with a unique large peak in the early 1960s the growth rate of government expenditure in

[17] The value of the contemporaneous correlation between the long-term growth rates of government expenditure in relative and absolute terms is 0.61.

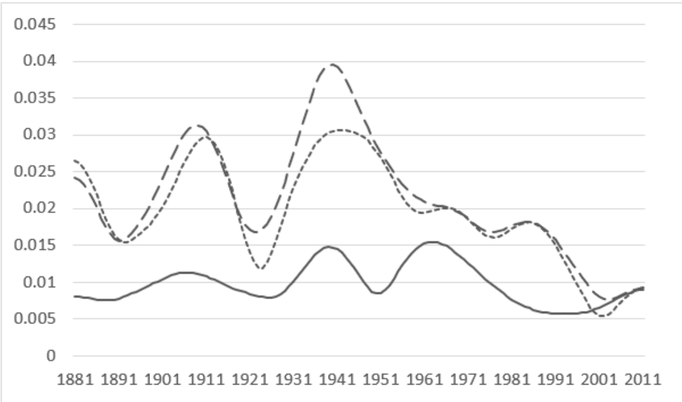
absolute terms may be considered the key variable for capturing long-term variations in the public sector share of GDP.

Figure 4 – Long-term growth rates in government expenditure-GDP ratio (top panel), real government expenditure (middle panel) and real GDP (bottom panel) for each country along with average values (solid lines) from 1881 to 2011



The second, measured by the standard deviation of the long-term growth rate components and displayed in Figure 5, is the striking reduction of the cross-country heterogeneity of the long-term trend patterns, more evident for absolute government expenditure than for GDP, which starts in the post-WWII period for government spending and after the 1960s for the real GDP. Other differences are the greater variability of the long-term growth rate pattern of government spending with respect to its GDP counterpart,[18] and a tendency to increasing divergency among GDP growth trends across countries in the new century. This last finding represents an inversion of the tendency to higher synchronization that characterized GDP patterns in the 2nd half of the XXth century, with a timing that seems to be consistent with different effects of globalization on individual countries' economic growth.

Figure 5 – Cross-country standard deviation of the long-term growth rates in government expenditure-GDP ratio (dashed line), real government expenditure (dotted line) and real GDP (solid line) from 1881 to 2011



To summarize, the visual inspection of the long-term pattern of the growth rate of government expenditures, both individually and in relation to GDP, provides several interesting findings: i) there is evidence consistent with the Wagner's law in absolute terms; ii) notwithstanding substantial cross-country heterogeneity until late 1950s, several expansionary waves are clearly detectable in coincidence with the two pre-WWs armament booms and the golden age of government expenditure; iii) the cross-country variation tends to decrease dramatically after WWII when increased synchronization becomes clearly evident.

[18] Real GDP growth rate values are mostly concentrated in the 0-5% range.

7. Results interpretation: from market failures to State failures

The empirical evidence provided in this paper has interesting implications for reconciling the contrast between the variety and plurality of theoretical approaches and the consensus that seems to emerge using the most recent statistical and econometric methods.

Two things are worth noting looking at the observed complex evolution of the long-run relative government spending. The first is the pattern of instability of the long-term relationship between relative government expenditure and output over time. The latter is the degree of cross-country heterogeneity in the trending pattern of absolute and relative spending, high in the pre-WWII period and low in the post-WWII period, as evidenced by the reduction of the confidence band over time in Figure 3 and the vertical line displayed in each panel of Figure 4. Theories aiming at explaining the long-term evolution of relative government spending are required to capture both the long swings in government expenditures and the common tendency observed in all countries in the post-WWII period.

The pre-WWII evidence, characterized by expansionary long waves corresponding to pre-armament boom periods and heterogeneity in the pattern of government spending, is consistent with the "ratchet phenomenon", that is the view that government expenditure tends to evolve in a steplike pattern represented by an acceleration of the growth rate occurring around periods of social upheavel, such as the two WWs period (Peacock and Wiseman, 1961; Bird, 1971, 1972; Higgs, 1985; Besley and Persson, 2008).

For the interpretation of the post-WWII evidence, the high degree of synchronization patterns combined with the characteristics of our sample[19] suggest that theories that emphasize

[19] As evidenced in section 2 our sample include developed countries differing for a number of country-specific features.

individual country differences are unlikely to provide an adequate explanation for the kind of general tendency observed in almost all countries from the 1960s. By contrast, theories that refer to the general motivation for the public intervention in the economy are natural candidates. Theories of market failures and State failures are general in that they reflect the main opposite views on the benefits and costs associated to government activity in market economies. Market failure policies, in the form of regulatory policies and public financing of services, are implemented by governments with the aim of improving the allocation of resources and redistribution of income.[26] Government failure theorists raise doubts about the efficacy of optimal public policies to correct market failures on the ground that such policies may create costs and inefficiencies considerably greater than that of market failure.

The combination of theories of market and State failures is also at the heart of the explanation of the nonlinear, inverted U-shaped relationship between GDP growth and government size, i.e. the Armey curve, within the approach on the optimal government size (Grossman, 1987, Vedder and Gallaway, 1998, Facchini and Melki, 2013). An interesting result of this literature is the country specificity of the optimal government size, measured by public expenditure GDP ratio. This optimal ratio seems to have been overcome during the second half of the XXth century by all countries with the consequence of exacerbating the negative effects of an excessive and growing burden of the government size on the efficiency of the economy.

For example, the emergence of the public choice school (Stigler, 1968, Buchanan, 1975) reflected the increasing recognition of the limitations of government in correcting the failures of the market and favored a rethinking of the appropriate role of government, with regulatory activities replacing direct production of services and price controls within the objectives of public policy. After such a shift in ideological belief on the role of government (Tanzi, 2011), the new prevailing view is clearly reflected in the slowing down of absolute and relative growth in public spending and GDP observed in all countries. Since mid-1970s,[20] government expenditure kept growing at lower pace compared with the 1970s for almost any country, with the growth rate of public spending now stabilized within the 0-2% range and still slowing.

The shift in the prevailing ideology in relation to the market/state relationship[21] has been also examined from the socio-political perspective. From the one side, both countries with majoritarian electoral rules and those with a proportional representation exhibited a leftward swing in the 50s and 60s, followed by a rightward shift in the 90s (Pickering and Rockey, 2011). From the other side, it has been observed even a growing market orientation ("neo-liberalism") of left or centre-left (political) elites (Mudge, 2008).

All in all, the evidence presented in this section supports the view that each theory, taken in isolation, cannot explain the observed long-run complex evolution of relative government spending. However, a combination of some of different theories associated with the Wagner's law referred to the absolute growth of government expenditures can. Specifically, we believe that the most plausible explanation for the actual general downward tendency of government expenditure growth is the shift from market to government failures.

8. Conclusions

In the recent literature on Wagner's law cointegration analysis has become the standard approach to investigate the existence of a long-run relationship between economic development and government activity because of its ability to manage the non-stationary features of the data. However, the simplicity of the theoretical relationship assumed by cointegration analysis

[20] The rapid expansion in the absolute and relative size of the public sectors between 1950 and 1970 may be considered the result of the positive attitude towards an interventionist role for government and the introduction and/or expansion of welfare state (and Keynesian) policies (Tanzi and Schuknecht, 2000).

[21] Pickering and Rockey (2011) have shown that the prevailing ideology can be a relevant determinant of goverment size.

contrasts with the complexity of the relationship to be estimated, at least over a long time span. Indeed, unless one takes as estimation period the years from 1960s onwards, when a striking convergence pattern across countries is observed, econometric models expressed in terms of fixed parameters and fixed structural equations are unlikely to capture the complex pattern displayed by the long-term pattern of the absolute and relative growth of government spending.

In this study we take a historical perspective on the long-run evolution of the relationship between the government GDP ratio and per capita GDP. Two main findings are in evidence. The first is that, beyond the growth of government expenditures in absolute terms, there is evidence of three expansionary long waves corresponding to the pre-armament booms in the WWs periods and the "golden age of public sector intervention". The latter is the reduction of the degree of cross-country heterogeneity in the pattern of relative and absolute growth of government expenditure in the post-WWII period.

Taken together our results suggest that the combination of theories, rather than each of them taken in isolation, may provide a comprehensive explanation of the long-run pattern of absolute and relative growth of government expenditure. Specifically, two different explanations complementary to Wagner's law are suggested for interpreting the evolution of the long-term growth pattern of absolute and relative government spending: the "displacement or ratched effect" in the pre-WWII period, and the change in the 2nd half of the previous century from the "market failure view" to the "limited government intervention view" because of the increased recognition of government failures.

Two kinds of market failure have emerged in the last decades: the upsurge of income inequality, also linked to the global financial crisis, and global warming. Together with the present pandemics, they represent challenges that demand a global collective response, because of the huge social costs associated to them. It is thus possible that those global events may pave the way for another change in the prevailing ideology, i.e. a turn to government, in the sense of a "shift or toleration for a wider scope of effective governmental authority over economic decision-making" (Higgs, 1985).

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Appendix 1: Dataset

Data for government expenditure and per capita income come from different sources. Per capita income data are from the Maddison Project Database 2018 (available at https://www.rug.nl/ggdc/historicaldevelopment/maddison/releases/maddison-project-database-2018). For government expenditure data there are several different sources of historical data available. We relied on the long-term data IMF dataset (based on Mauro et al., 2015), available at this address: http://www.imf.org/external/datamapper/index.php The dataset refers to "general" government expenditures and potentially covers all world countries for the period 1800-2011, but only a few of them have data in the 19th century. Since data are really scarce before, we use 1880 as the initial date. From 1880 on data for 21 countries are available, but we select a subset of 17 countries, the same included in the Jordà-Schularick-Taylor Macrohistory Database, available at: http://www.macrohistory.net/data/ , for robustness purposes as data on public expenditure in this dataset refer to "central" government expenditure. The complete list of countries is: Australia (AUS), Belgium (BEL), Canada (CAN), Deutschland (DEU), Denmark (DNK), Finland (FIN), France (FRA), Great Britain (GBR), Italy (ITA), Japan (JPN), Netherland (NLD), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE) and the United States (US).

Historical datasets contain missing data, mainly concentrated in war years. Table 1 provides the list of countries and associated missing data in our dataset. The treatment of missing data is explained in the last part of section 2 in the main text.

Table A1: Presence of missing data by country

Australia	1880-1900
Belgium	1914-1923; 1925-1926; 1928-1929;1931-1932;1940-1942; 1944-1945
Deutschland	1914-1924; 1935-1949
Finland	1880-1881
France	1914-1924; 1939-1945
Japan	1944-1946
Netherland	1914-1922; 1940-1948
Norway	1880
Portugal	1914-1916
Spain	1936-1939
Switzerland	1880-1898; 1914-1928

Appendix 2: Some basic concepts on wavelet analysis

The wavelet transform provides a time-frequency representation of the signal using a set of orthogonal basis functions, named wavelets. The base of the wavelet transform, the wavelet, is designed upon some desired properties associated with that function, the admissibility and regularity conditions. According to the admissible condition, the wavelet must oscillate to have its mean value equal to zero

$$\int_{-\infty}^{\infty} \psi(u)du = 0$$

According to the regularity condition, the wavelet has exponential decay so that the wavelet must oscillate and is localized in the sense that it decreases rapidly to zero as t tends to infinity

$$\int_{-\infty}^{\infty} \psi(u)^2 du = 1$$

All the wavelet functions used in the transformation are generated from a basic wavelet function $\psi(u)$, called mother wavelet, through translation (shifting) and scaling (dilation or compression) of the mother wavelet. The mother wavelet, defined as

$$\psi_{s,u}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-u}{s}\right)$$

is a function of two parameters s and u. The translation or location parameter u indicates where the wavelet is centered along the signal and it is shifted through the signal. Thus, it corresponds to the time information in the wavelet transform. The scaling or dilation parameter s controls the length of the wavelet and is defined as the inverse of frequency and corresponds to frequency information. Scaling either dilates (expands) or compresses a signal. Large scales (low frequencies) dilate the signal and provide detailed information hidden in the signal, while small scales (high frequencies) compress the signal and provide global information about the signal.

A parsimonious representation of the evolution over time of the periodic components of a signal is provided by the discrete wavelet transform (DWT) which uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal in a way that the information contained in the signal can be summarized in a minimum number of wavelet coefficients. The DWT proceeds by the pyramid algorithm. Applying the DWT reduces the original N data points into two series of length N/2: one of these contains the smoothed information[22] and the other contains the detail information. Keeping the details and doing an additional transform of the smoothed series we can produce two series of length N/4, with smoothed and detail information, and so on. If the original time series was some power of 2, $N=2^J$, then the number of coefficients at the end would total N, and would contain all of the information in the original time series, organized according to scale and location, the number of coefficients at each scale being:

$$N = N/2^J + N/2^J + N/2^{J-1} + \ldots N/4 + N/2$$

The deconstruction of the function f(t) is, therefore:

$$f(t) \approx \Sigma_k\, s_{J,k}\varphi_{J,k}(t) + \Sigma_k\, d_{J,k}\psi_{J,k}(t) + \Sigma_k\, d_{J-1,k}\psi_{J-1,k}(t) + \ldots + \Sigma_k\, d_{1,k}\psi_{1,k}(t)$$

with $N/2^J$ $s_{J,k}$ coefficients, $N/2^J$ $d_{J,k}$ coefficients, $N/2^{J-1}$ $d_{J-1,k}$ coefficients ... and N/2 $d_{1,k}$ coefficients. Further, the approximation can be re-written in terms of collections of coefficients at given scales as:

$$f(t) \approx S_J + D_J + D_{J-1} + \ldots D_2 + D_1$$

where S_J contains the "smooth component" of the signal, and the D_j , j = 1, 2, ..J, the detail signal components at ever increasing levels of detail. S_J provides the large scale road map, D_1 shows the pot holes. The previous equation indicates what is termed the multiresolution decomposition, where a signal is decomposed into several components, each associated with a different frequency band, with a resolution matched to its scale. Specifically, using multi-resolution technique by which different frequencies are analyzed with different resolutions, the wavelet transform gives good time resolution and poor frequency resolution at high frequencies, and good frequency resolution and poor time resolution at low frequencies.

By sequentially adding the detail level components D_4, D_3, D_2 to the lower "smooth" component S_4 we get three additional levels of approximation, S_3, S_2 and S_1. The higher the index, the smoother is the function: S_1 captures fluctuations greater than 4 years, S_2 greater than 8 years, and S_3 greater than 16 years. Table A1 presents the frequency domain interpretation in terms of periods for each detail and approximation level when annual data are used.

Table A2: Frequency interpretation of detail and approximation levels

Detail level, D_j	**Years**	**Approximation level, S_J**	**Years**
D_1	*2-4*	***S_1***	*from 4 to ∞*
D_2	*4-8*	***S_2***	*from 8 to ∞*
D_3	*8-16*	***S_3***	*from 16 to ∞*
D_4	*16-32*	***S_4***	*from 32 to ∞*

In practical application the maximal overlap discrete wavelet transform (MODWT) is used instead of the DWT. The MODWT is a compromise between the continuous wavelet transform (CWT), with continuous variations in scale, and the DWT which uses only a limited number of translated and dilated versions of the mother wavelet to decompose the original signal. The MODWT is highly redundant so that the transformations at each scale are not orthogonal, but the offsetting gain is that applying the transform leaves the phase invariant, a very useful property in analyzing transformations, and the transform is not restricted to limitations imposed by the dyadic expansion used by the DWT. Indeed, because of the practical limitations of the DWT, wavelet analysis is generally performed by applying the MODWT, a non-orthogonal variant of the classical discrete wavelet transform that, unlike the DWT, is i) translation invariant, as shifts in the signal do not change the pattern of coefficients, ii) can be applied to data sets of length not divisible by 2^J and iii) returns at each scale a number of coefficients equal to the length of the original series.