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Does Interbank Market Matter for Business Cycle Fluctuation? An Estimated DSGE Model with Financial Frictions for the Euro Area

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Abstract

The aim of this paper is to assess the impact of the interbank market on the business cycle fluctuations. We build a DSGE model with heterogeneous households and banks. Two kind of banks are in the model: Deficit banks which are net borrowers on the interbank market and they provide credit to the real economy. The surplus bank are net lender and they could choose to provide interbank lending or purchase government bonds.

The portfolio choice of the surplus bank is affected by an exogenous shock that modifies the riskiness of the interbank lending thus allowing us to capture the collapse of the interbank market and the fly to quality mechanism underlying the 2007 financial crisis.

The main result is that an interbank riskiness shock seems to explain part of the 2007 downturn and the rise of the interest rate on the credit market just after the financial turmoil.

JEL Class.: E30, E44, E51.

Keywords: DSGE model, financial frictions, interbank market, Bayesian estimation

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1 Introduction

The interbank market is the primary source for banks that want to gather liquidity for the creation of new loans. Shocks that affect this market could have important repercussions both on the entire financial market and on the real side of the economy.

Figure 1 shows the spread between the three month Euribor and the overnight index swap on the EONIA interest rate. The OIS spread is considered the most important indicator to evaluate the degree of health of the liquidity market. The higher the spread, the higher is the risk perceived by the financial intermediaries that daily operate on the interbank market.

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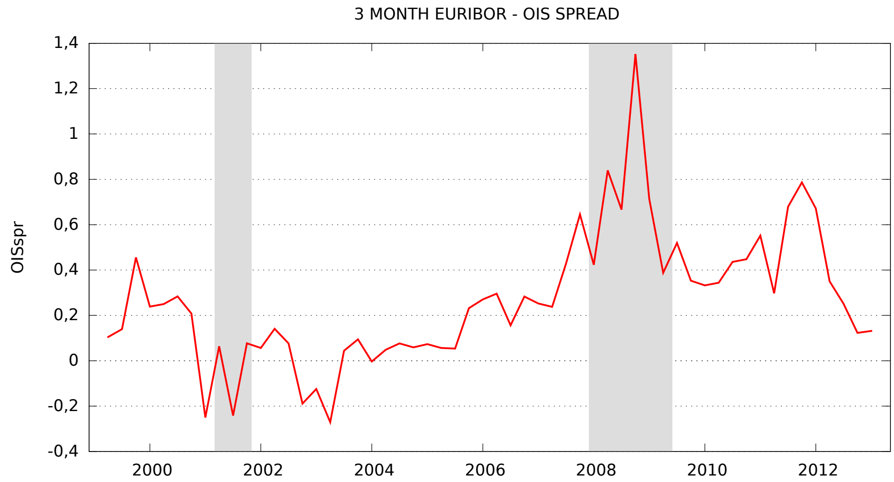
Figure 1: Overnight index swap - 3 month euribor spread



Source: Authors elaboration of Datastream data

After the collapse of Lehman Brothers banks do not trust each other anymore and the OIS spread at the end of 2008 rose of almost 100 basis point with respect to the beginning of the year. The fear that in the balance sheet of a counterpart could be hidden an unquantifiable amount of toxic assets caused a sudden and extended drainage of liquidity from the interbank market. Banks looked for a "safe heaven" choosing low but safe returns like government bonds. Socio (2011) provides an extensive empirical analysis about the causes of the rise of the OIS spread in the time span between the end of the 2008 and the entire 2010.

Our first objective is to understand the role of the interbank market in the widespread of the financial crisis using a dynamic stochastic general equilibrium framework. We find that a) an interbank market riskiness shock could generate a decrease of the loans provided by the bank to the real economy and, as a consequence, this could generate a fall of the GDP mainly driven by a fall of the investment made by the firms and the impatient households, b) a riskiness shock played a crucial role in the 2008 rise of the interest rates in the credit market.

After the 2007 financial crisis a growing number of works focused their attention on the supply side of the credit market. Until then, the prevailing literature framework to model financial frictions within dynamic models referred to Bernanke et al. (1999) financial accelerator. This kind of literature, and its further extension like Christiano et al. (2010), focused its attention mainly on the demand side of the credit market neglecting the role of financial intermediaries in the business cycle. An excellent review of this literature can be found in Brazdik et al. (2011).

Since the financial turmoil was mainly due to the sudden collapse of the credit supply provide by financial intermediaries to the households and the firms, models based on the financial accelerator framework missed a key channel in the transmission of the financial crisis. Inserting an active banking sector became a first order priority amongst all the macroeconomists.

The model proposed by Gerali et al. (2010) represents a step ahead in this sense. They were able to insert a complete banking system into a classical DSGE model like, Smets and Wouters (2007) and Christiano et al. (2005), in which the role played by financial intermediaries is crucial in the credit allocation to firms and households. They successfully inserted a certain degree of heterogeneity between the households populating the economy. In fact, the presence of a bank requires the distinctions of the households in net savers, the households providing the deposits, and in borrowers, the receivers of the credit supply. However there is no role for an explicit interbank market.

Models like those proposed by Dib (2010), deWalque et al. (2010) and Goodhart et al. (2009) set up a framework in which the interbank market plays a decisive role in the choice of the amount of funds to place in the real economy. Goodhart et al. (2009) first attempt to include an active interbank market. They place in their model two kinds of banks. A surplus bank which obtains funds from the household and allocates these resources in the deficit bank through the interbank market channel. A deficit bank receives loans from the surplus one to finance the corporate lending to the Yeoman farmer. The central bank is able to influence the interest rate only through the deficit bank.

deWalque et al. (2010) built up a RBC model very close to Goodhart's but they specified a complete real business cycle framework for the rest of the agents. The banking sector is divided in two different intermediaries: a deposit bank which collects savings from the households and invests them into the interbank market and a merchant bank that is a net debtor which collect interbank funds and using them to finance the firms. All banks operate in a competitive setting. In the conclusions, they underlined how, a relatively simple model, captures some stylized facts of the interest rate term structure and on defaults rate on interbank market. Moreover, the introduction of a capital requirement like the one proposed by the Basel I agreement reduces the long run growth but it improves the resistance of the system to shocks, while the Basel II accords enhances the business cycle fluctuation.

Carrera and Vega (2012) developed a hierarchical bank system in which the exchanges between the central bank and the retailers are managed by another subject called the narrow bank. The role of the narrow bank is to manage the liquidity provided by the central bank and to allocate these resources into the interbank market. The retail bank has the goal of obtaining savings and issuing loans to the real economy. Similar in spirit, Hilberg and Hollmayr (2011) developed a model in which an investment bank provides the liquidity to the retail banks. The retail bank is subject to a borrowing constraint (See Kiyotaki and Moore (1997) and Iacoviello (2005)) in the amount of funds it can obtain from the investment bank and it can collateralize only a fraction of its detained assets. Both Carrera and Vega (2012) and Hilberg and Hollmayr (2011) conclude that the presence of the interbank market dampens the effect of the financial accelerator.

Dib (2010) constructed a very complex model in which the saving bank, financed by households deposits, plays a crucial role in the allocation of the resources between interbank lending and the risk free government bond. The central bank can alter the composition of the saving bank balance sheet when it intervenes to stem the inflation growth or the output gap. The key result is that under a capital requirement regime the presence of a banking sector attenuates the effects of different shocks. Moreover, many stylized facts of the US business cycle are capture by the model making it particular suited to be used to analyze the impacts of the financial sector on the rest of the economy.

Gertler and Kiyotaki (2010) build a DSGE model with an interbank market in which all banks borrow from and lend to firms. In their model the interbank market arises because banks are subject to idiosyncratic liquidity shock. Limited pleadgeability gives place to an endogenous leverage constraint where bankers need to use their own equity in order to attract external creditors. The model proposed by Gertler and Kiyotaki (2010) tends to generate a stronger financial accelerator effect with respect to models where the capital market is not microfunded and, as a consequence, it gives a very important role for assets prices.

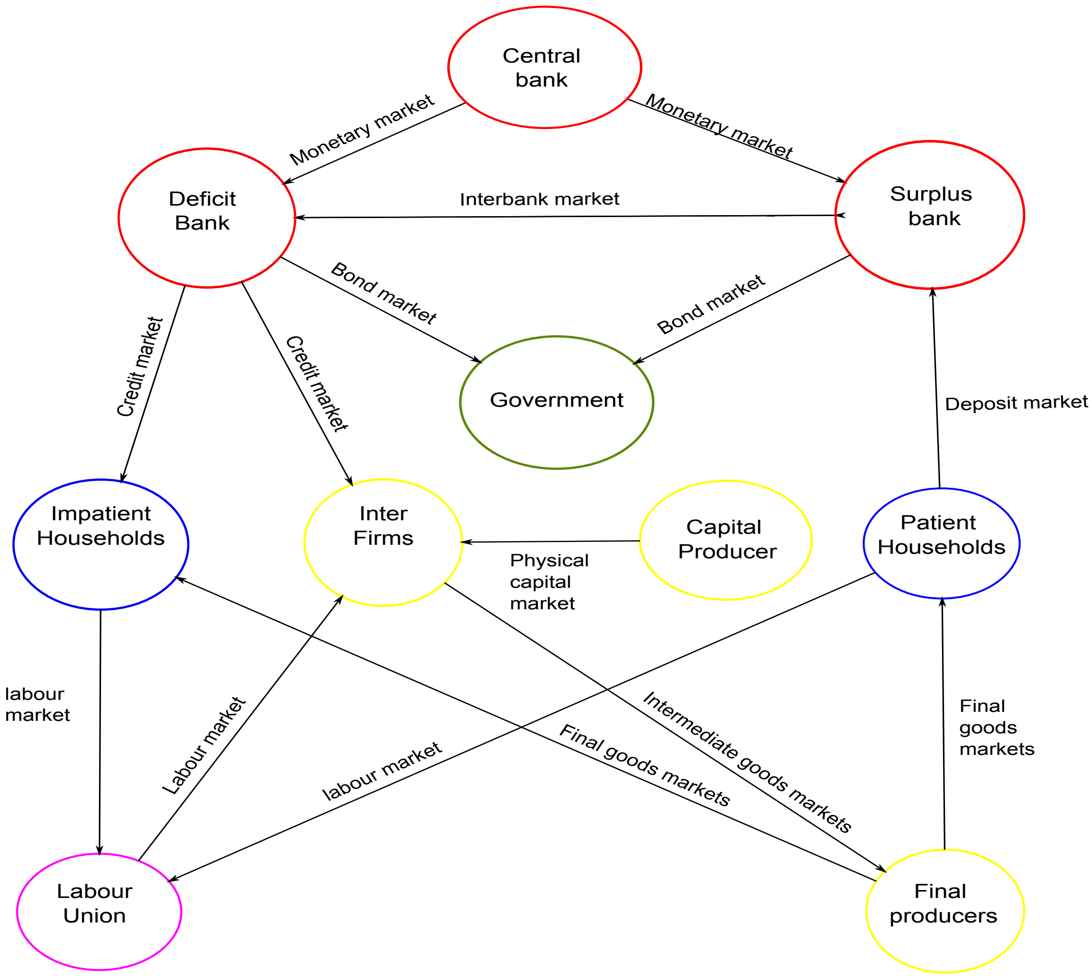
We decide to extend the model proposed by Gerali et al. (2010) including an interbank market like in Dib (2010) in order to analyze the effect of an unexpected turmoil of the interbank landing introducing an exogenous riskiness shock.

The paper is organized as follow. Section 2 explains in details the model and the relative equations. Sections 3 and 4 deal with the solution methods and the estimation techniques. Section 5 and 6 focus their attention on the dynamical properties of the model and the contribution of each shocks to the business cycle. Finally, Section 7 summarizes the main findings and the possible extensions.

2 The model

Our model (See figure 2) is an extension of the one proposed by Gerali et al. (2010) (henceforth, GNSS). The whole economy is made of several representative agents each of them maximizes his objective function under a budget constraint. Two kind of households, Patient and Impatient, live in the model. Patient households have a higher intertemporal discount factor than the Impatients households. Therefore, Patient households are net savers and they decide how much to consume, to work and the amount of deposits to allocate at the surplus bank. Impatient households are net borrowers and they choose how much to consume and to work. They finance part of

Figure 2: Model scheme



their spending obtaining loans from the retail branch of the deficit bank. Both Patient and Impatient households sell their work to a union that sells a composite labor factor to the intermediate firm.

The rest of the real economy has a standard setting like in Smets and Wouters (2007) and Christiano et al. (2005). There are two kind of firms, the intermediate producers and the final goods producers. The intermediate firms operate under monopolistic competition and they are able to fix prices. They rent physical capital from the producers of capital goods and sell their intermediate goods to the producers of final goods. Final goods producers operate under perfect competition but with sticky prices. They buy the intermediate products, they pack them into an undifferentiated final good that they sell back to Patient and Impatient households. Intermediate firms could finance a fraction of their investment obtaining loans from the retail branch of the surplus bank.

The bank system of the model is an extension of Gerali et al. (2010) and Dib (2010). The deficit banks is modeled like in Gerali et al. (2010). We have a retail branch that is directly connected with the firms and the households. The retail branches operate under monopolistic competition and they could set the interest rate on loans provided to the impatient households and the firms. A wholesale branch of the deficit banks has to manage the capital position of the holding. We slightly modify the original framework like in Vlcek and Roger (2011) by introducing a portfolio choice between loans and government bond. While in Vlcek and Roger (2011) the portfolio choice is set exogenously by the monetary authority, we endogenize the choice between entrepreneurial lending and government bonds purchase. Deficit banks may finance themself through the interbank channel from the surplus bank like in Dib (2010).

The surplus banks collect loans from Patient households and invest part of their deposits either in the interbank market or in government bond. Monetary policy is conducted by the central bank which follows a Taylor rule. We close our model specifying a very stylized government sector that obeys to an intertemporal budget constraint.

2.1 Patient households

Patient households choose $c(i)^P$, $h(i)^P$, and $d(i)^P$ (respectively, consumption, house services, and the amount of deposits) in order to maximize their utility function under the budget constraint. The utility function depends positively on consumption and houses services and negatively on the hours worked.

$$E_0 \sum_{t=0}^{\infty} \beta_P^t \left[(1-a^P)\epsilon_t^z log(c_t^P(i) - a^P c_{t-1}^P) + \epsilon_t^h log(h_t^P(i)) - \frac{l_t^P(i)^{(1+\phi)}}{1+\phi} \right] \quad (1)$$

where β_p is the intertemporal discount factor of Patient households while a_p represents the external habit formation in consumption with respect to the whole Patient households consumption. The exogenous variables ϵ_t^z and ϵ_t^h are two stochastic disturbances affecting consumption preferences and the house services demand. The budget constraint for the patient households is described by the following equation

$$c_t^P(i) + q_t^h \Delta h_t^P(i) + d_t(i)^P = w_t(i)^P l_t(i)^P + \frac{(1+r_{t-1}^d}{\pi_t} d_{t-1}^P(i) + Tr_t - T_t^P \quad (2)$$

The left hand side is the flow of expenses. It is composed by consumption, variation of the market value of housing services, where q^h is the real houses price, and the amount of deposit allocated at the surplus bank. The right hand side of equation 2 represents the resource owned by the patient households. w^P is the hourly wage , r^d is the net interest rate on deposits, π is the net inflation and T is a lump sum tax. All variables are expressed in real terms. Tr_t are the transfers from the economy to the Patient households. We assume that final goods producer firms are completely owned by the Patient households and they transfer to them their profits J^r while the deficit banks redistribute only a fraction $(1-\Omega)$ of their profits to the households. T_t^P is a lump sum tax used to finance the government expenditures. Patients households are net savers and they decide to allocate a fraction of their income in bank deposits at the surplus bank. Using the standard procedure of dynamic optimization we set up the Lagrangian function

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta_P^t \left[(1-a^P)\epsilon_t^z log(c_t^P(i) - a^P c_t^P) + \epsilon_t^h log(h_t^P(i)) - \frac{l_t^P(i)^{(1+\phi)}}{1+\phi} \right] +$$
$$\beta_P^t \lambda_t^P \Bigg[w_t^P(i) l_t^P(i) + \frac{(1+r_{t-1}^d)}{\pi_t} d_{t-1}^P(i) + Tr_t -$$
$$T_t^P - c_t^P(i) - q_t^h \Delta h_t(i)^P - d_t^P(i) \Bigg] \tag{3}$$

and derive it with respect to $c(i)^P$, $h(i)^P$, $l(i)^P$ and d_t^P. The first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial\ c_t^p(i)}: \qquad \frac{(1-a^p)\epsilon_t^z}{c_t^p(i) - a^p c_{t-1}^p(i)} = \lambda_t^p \tag{4a}$$

$$\frac{\partial \mathcal{L}}{\partial\ d_t^p(i)}: \qquad \lambda_t^p = \beta_p E_t \left[\frac{\lambda_{t+1}^p (1+r_t^d)}{\pi_t} \right] \tag{4b}$$

$$\frac{\partial \mathcal{L}}{\partial\ h_t^p(i)}: \qquad \lambda_t^p q_t^h = \frac{\epsilon_t^h}{h_t^p(i)} + \beta_P E_t \left[\lambda_{t+1}^p q_{t+1}^h \right] \tag{4c}$$

Equation 4a is the marginal utility of consumption. Combined with equation 4b it gives us the Euler equation of consumption and its optimal intertemporal path. Equation 4c is the Euler equation for the house services.

2.2 Impatient Households

Impatient households choose $c(i)^I$, $h(i)^I$, and $b(i)^I$ in order to maximize their utility function under the budget constraint. They behave exactly like Patient households, but instead of being net savers they are net borrowers. Consequently, they finance a fraction of their spending by obtaining loans $b(i)^I$ from the retail branch of the deficit bank:

$$E_0 \sum_{t=0}^{\infty} \beta_I^t \left[(1-a^I)\epsilon_t^z log(c_t^I(i) - a^p c_{t-1}^p) + \epsilon_t^h log(h_t^I(i)) - \frac{l_t(i)^{I(1+\phi)}}{1+\phi} \right] \tag{5}$$

Their budget constraint is described by the following expression

$$c_t(i)^I + q_t^h \Delta h_t(i)^I + \frac{(1+r_{t-1}^{bh})}{\pi_t} b_{t-1}(i)^I = w(i)_t^I l_t(i)^I + b_t^I(i) \tag{6}$$

As in Iacoviello (2005), the amount of funds the Impatient households can receive from the deficit bank is limited by the following borrowing constraint:

$$(1 + r_t^{bh})b(i)_t^I \leq m_t^I E_t[q_{t+1}^h h_t^I \pi_{t+1}] \tag{7}$$

The total exposure toward the deficit banks of the Impatient households must be less or equal of the expected value of the collaterals (houses) owned by the households. m_t^I represents the stochastic loan-to-value-ratio[1](LTV henceforth). Iacoviello (2005) demonstrates that in the neighborhood of the steady state the constraint always binds [2]. This allows us to solve the problem with an equality constraint. Setting up the Lagrangian function and deriving with respect to the choice variables, we obtain the first order conditions for the Impatient households

$$\begin{aligned}\mathcal{L} =& E_t \sum_{t=0}^{\infty} \beta_I^t \left[(1 - a^I)\epsilon_t^z log(c(i)_t^I - a^I c_t^I) + \epsilon_t^h log(h(i)_t^I) - \frac{(l(i)_t^I)^{(1+\phi)}}{1+\phi} \right] + \\ & \beta_I^t \lambda_t^I \left[w(i)_t^I l(i)_t^I + b_t^I(i) + t_t^I - c(i)_t^I - q_t^I \Delta h(i)_t^I - \frac{(1 + r_{t-1}^{bh})}{\pi_t} b(i)_{t-1}^I \right] + \\ & \beta_I^t s_t^I \left[m_t^I E_t[q_{t+1}^h h_t^I \pi_{t+1}] - (1 + r_t^{bh}) b_t^I(i) \right]\end{aligned} \tag{8}$$

The Euler equations are quite similar to the first order conditions of the Patient households. The only difference is the presence of a second Lagrange multiplier, s_t^I, associated with the borrowing constraint.

$$\frac{\partial \mathcal{L}}{\partial\ c_t^I(i)} : \qquad \frac{(1 - a^I)\epsilon_t^z}{c_t^I(i) - a^I c_{t-1}^I} = \lambda_t^I \tag{9a}$$

$$\frac{\partial \mathcal{L}}{\partial\ b_t^I(i)} : \qquad \lambda_t^I = \beta^I E_t \left[\frac{\lambda_{t+1}^I (1 + r_t^{bh})}{\pi_{t+1}} \right] + s_t^I (1 + r_t^{bh}) \tag{9b}$$

$$\frac{\partial \mathcal{L}}{\partial\ h_t^I(i)} : \qquad \lambda_t^I q_t^h = \frac{\epsilon^h}{h_t^I(i)} + \beta E_t \left[\lambda_{t+1}^I q_{t+1}^h \right] + s_t^I m_t^I E_t \left[q_{t+1}^h \pi_{t+1} \right] \tag{9c}$$

[1]LTV is usually defined as the ratio between the mortgage amount and the value of the property acquired with the mortgage. Higher LTV ratio implies a higher risk for the bank and an higher interest rate on the loan for the household

[2]Solving for occasionally binding constraints introduce some issues in the simulation of the model. See for further reference Holden and Paetz (2012)

Following Iacoviello (2005) the interpretation of the Lagrange multiplier s_t^I is straightforward: it represents the increase in utility that the Impatient households can obtain borrowing $(1 + r_t^{bh})$ from deficit banks consuming it and reducing consumption at time $t + 1$ by an appropriate amount.

2.3 Entrepreneurs

The Entrepreneurs are self employed intermediate goods producers. Entrepreneurs choose $c(i)^E$, $k(i)^E$, $l(i)^{E,P}$, $l(i)^{E,I}$, $b(i)^E$, $u(i)^E$, where each variable represents respectively consumption, capital used to produce intermediate goods, labor from patient and impatient household, the amount of loans obtained by the retail branch of the deficit bank and the degree of utilization of capital. Differently from Patient and Impatient households, the utility function depends only on entrepreneur's consumption:

$$E_0 \sum_{t=0}^{\infty} \beta_E^t \left[(1 - a^E)log(c_t^E(i) - a^E c_{t-1}^E)\right] \tag{10}$$

The budget constraint of the entrepreneurs is described by the following expression:

$$\begin{aligned} & c_t^E(i) + w_t l_t^{E,P}(i) + w_t l_t^{E,I}(i) + \frac{(1 + r_{t-1}^{be})}{\pi_t} b_{t-1}^E(i) + q_t^k k_t^E + f(u_t(i))k_t^E(i) \\ = & \frac{y_t^E}{x_t} + b_t^E(i) + q_t^k(1 - \delta)k_{t-1}^E(i) \end{aligned} \tag{11}$$

We specify the functional form of $f()$ like in Schmitt-Grohé and Uribe (2006):

$$f(u_t(i)) = \xi_1(u_t(i) - 1) + \frac{\xi_2}{2}(u_t(i) - 1)^2 \tag{12}$$

The production function is a classical Cobb-Douglass where, A_t^E represents a stochastic total factor productivity shock.

$$y_t^E(i) = A_t^E \left[k_{t-1}^E(i)u_t(i)\right]^{\alpha} l_t^E(i)^{(1-\alpha)} \tag{13}$$

Entrepreneurs use a combination of the labor supplied by the Patient and Impatient households following the expression

$$l_t^E(i) = l_t^{E,P}(i)^{\mu} l_t^{E,I}(i)^{(1-\mu)} \tag{14}$$

Like the Impatient households, Entrepreneurs are also subject to a borrowing constraint

$$(1 + r_t^{be})b_t^E(i) \leq m_t^E E_t[q_{t+1}^k k_t^E(i)\pi_{t+1}] \tag{15}$$

While Impatient households use their amount of houses as collateral, entrepreneurs use the expected value of their endowment of physical capital. Substituting equation 14 into equation 13 and then equation 13 and 12 into the budget constraint we maximize the utility function under equations 11 and 15 . The Lagrangian function is

$$\begin{aligned}
\mathcal{L} =& E_t \sum_{t=0}^{\infty} \beta_E^t \Big[log(c_t^E(i) - a^E c_{t-1}^E) \Big] + \\
& \beta_E^t \lambda_t^E \Bigg\{ \frac{A^E \left[k_{t-1}^E(i) u_t(i)\right]^\alpha \left[l_t^{E,P}(i)^\mu l_t^{E,I}(i)^{(1-\mu)}\right]^{(1-\alpha)}}{x_t} + b_t^e(i) + \\
& q_t^k(1-\delta)k_{t-1}^e(i) - c_t^E(i) - w_t l_t^{E,P}(i) - w_t l_t^{E,I}(i) - \frac{(1 + r_{t-1}^{be})}{\pi_t} b_{t-1}^E(i) - \\
& q_t^k k_t^E(i) - [\xi_1(u_t(i) - 1) + \frac{\xi_2}{2}(u_t(i) - 1)^2] k_t^e(i) \Bigg\} + \\
& \beta_E^t s_t^E \Big[m_t^E E_t \left[q_{t+1}^k \pi_{t+1}(1-\delta) k_t^E(i) \right] - (1 + r^{be}) b_t^E(i) \Big]
\end{aligned} \tag{16}$$

The relative first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial\ c_t^E(i)}: \quad \frac{(1-a^E)}{c_t^E(i)-a^E c_{t-1}^E} = \lambda_t^E \tag{17a}$$

$$\frac{\partial \mathcal{L}}{\partial\ k_t^E(i)}: \quad \lambda_t^E q_t^k = s_t^E m_t^E q_{t+1}^k \pi_{t+1}(1-\delta) + \beta_E E_t \lambda_{t+1}^E \big[r_{t+1}^k u_{t+1}(i) + \tag{17b}$$

$$q_{t+1}^k(1-\delta) - [\xi_1(u_{t+1}(i)-1) + \frac{\xi_2}{2}(u_{t+1}(i)-1)^2]\big] \tag{17c}$$

$$\frac{\partial \mathcal{L}}{\partial\ u_t(i)}: \quad r^k = \frac{\alpha A_t^E (k_{t-1}^E(i) u_t(i))^{(\alpha-1)} (l_t^{E,P}(i)^{\mu} l_t^{E,I}(i)^{(1-\mu)})^{(1-\alpha)}}{x_t} \tag{17d}$$

$$\frac{\partial \mathcal{L}}{\partial\ l_t^{E,P}(i)}: \quad w_t^P(i) = \frac{(1-\alpha) y_t^E(i)\mu}{P_t l_t^{E,P}(i)} \tag{17e}$$

$$\frac{\partial \mathcal{L}}{\partial\ l_t^{E,I}(i)}: \quad w_t^I(i) = \frac{(1-\alpha) y_t^I(i)(1-\mu)}{P_t l_t^{E,I}(i)} \tag{17f}$$

$$\frac{\partial \mathcal{L}}{\partial\ b_t^E(i)}: \quad \lambda_t^E = s_t^E(1+r_t^{be}) + \beta^E E_t \left[\lambda_{t+1}^E \frac{(1+r_t^{be})}{\pi_{t+1}}\right] \tag{17g}$$

The interpretation of the Lagrange multiplier s_t^E is equivalent to that of s_t^I in the impatient households equations.

2.4 Loans and deposits demand

Impatient households and Entrepreneurs have to decide the amount of loans to demand to each retail branch. Since an infinity of retail branch banks exists, being each of them a monopolistic competitor, the problem could be set like a classical Dixit and Stiglitz (1977) aggregator.

$$\min \int_0^1 r_t^S(j) b_t^S(i,j) dj \tag{18}$$

subject to

$$\left[\int_0^1 b_t^S(i,j)^{\frac{\epsilon_t^{bs}-1}{\epsilon_t^{bs}}} dj\right]^{\frac{\epsilon_t^{bs}}{\epsilon_t^{bs}-1}} = b_t^S(i) \tag{19}$$

where $S = H, E$ and ϵ_t^{bs} is the elasticity of substitution between loans provide by different retail banks. The solution of this minimization problem is the demand function of loans for the households and the firms.

$$b_t^S(j) = \left(\frac{r_t^{bs}(j)}{r_t^{bs}}\right)^{-\epsilon_t^{bs}} b_t^S \tag{20}$$

The higher is the elasticity of substitution between different loans, the lower is the mark up that each retail branch can apply to its loans. Similarly, the Patient households have to maximize the profit from providing deposit to the retail branch of the surplus bank.

$$\max \int_0^1 r_t^d(j) b_t^P(i,j) dj \tag{21}$$

subject to

$$\left[\int_0^1 d_t^P(i,j)^{\frac{\epsilon_t^d - 1}{\epsilon_t^d}} dj \right]^{\frac{\epsilon_t^d}{\epsilon_t^d - 1}} = d_t^P(i). \tag{22}$$

The demand function of deposits can be written as

$$d_t^P(j) = \left(\frac{r_t^d(j)}{r_t^d} \right)^{-\epsilon_t^d} d_t^P \tag{23}$$

An higher value of ϵ_d implies higher earnings for patient household and at the same time a higher mark down for banks. The detailed calculation of all the maximization steps is illustrated in appendix A.1.

2.5 Labor market

We strictly follow Gerali et al. (2010) to model the labor market. Two agents operate in the labor market, unions and labor packers. Workers provide a differentiated labor factor to unions. Unions have to maximize with respect to $W(m)_t^s$ the following profit function:

$$E_0 \sum_{t=0}^{\infty} \beta_t^s \lambda_t^s \left[\frac{W_t^s(m)}{P_t} l_t^s(i,m) - \frac{k_w}{2} \left(\frac{W_t^s(m)}{W_{t-1}^s(m)} - \pi_{t-1}^{\iota^w} \pi^{1-\iota^w} \right)^2 \frac{W_t^s}{P_t} - \frac{l(i,m)_t^{1+\phi}}{1+\phi} \right] \tag{24}$$

Changing the wage is costly for the unions. k_w represents the adjustment cost that unions have to face in order to reset the wage with respect to both past and steady state wage inflation. The weight ι_w is the importance assigned by the unions to the past salary with respect to the steady state one.

A continuum m of labor packers acquire labor from the unions and they sell, through a CES aggregator, an homogeneous labor factor to the intermediate firms. The following equation represents the labor demand.

$$l_t^s(i,m) = \left(\frac{W_t^s(m)}{W_{t-1}^s(m)}\right)^{-\epsilon_t^l} l_t^s \tag{25}$$

Substituting the labor demand into the profit function, and deriving with respect to $W(m)$, we obtain a version of the New Keynesian Phillips Curve for the wage inflation represented by equation 26. ϵ_t^l is a stochastic labor demand shock.

$$\begin{aligned} & k_w\left(\pi_t^{ws} - \pi_{t-1}^{\iota^w}\pi^{1-\iota^w}\right)\pi^{ws} = \\ & \beta^p E_t\left[\frac{\lambda_{t+1}^p}{\lambda_t^p} k_w\left[(\pi_{t+1}^{ws} - \pi_t^{\iota^w}\pi^{1-\iota^w}\right]\frac{(\pi_{t+1}^{ws})^2}{\pi_{t+1}}\right] + (1-\epsilon_t^l)l_t^s + \frac{\epsilon_t^l l_t^{s1+\phi}}{\lambda_t^s w_t^s} \end{aligned} \tag{26}$$

where the wage inflation π_t^{ws} is defined as:

$$\pi_t^{w^s} = \frac{w_t^s}{w_{t-1}^s}\pi_t \tag{27}$$

and the index s represents both Patient and Impatient households.

2.6 Capital producers

As in Christiano et al. (2005) there is a competitive capital producer sector. The capital producer buys a fraction of undepreciated capital $k_t - (1-\delta k_{t-1})$ from the entrepreneurs at the real price q_t^k and a fraction i_t of final goods from the final goods producers. Using these two inputs the capital producer maximize the following objective function.

$$E_t \sum_{t=0}^{\infty} \beta_t^E \lambda_t^E \left[q_t^k i_t - \frac{k_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right) q_t^k i_t\right] \tag{28}$$

Setting up the Lagrangian and deriving it with respect to investment i_t we obtain the equation that define the fundamental price of capital q_t^k.

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial\, i_t} : & q_t^k\left[1 - \frac{k_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2 + k_i\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)\frac{i_t \epsilon_t^{qk}}{i_{t-1}}\right] \\ & + \beta^E E_t\left[\frac{\lambda_{t+1}^E}{\lambda_t^E} k_i \left(\frac{i_{t+1}\epsilon_{t+1}^{qk}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \frac{q_{t+1}^k}{q_t^k}\epsilon_{t+1}^{qk}\right] = 1 \end{aligned} \tag{29}$$

The parameter k_i represents the adjustment cost that producers of capital goods have to pay every time they decide to vary the amount of investment. The exogenous process ϵ_t^{qk} is an AR(1) process that affects the efficiency of investment. The production of new capital for Entrepreneurs evolves according to the following law of motion:

$$k_t = (1-\delta)k_{t-1} + \left[1 - \frac{k_i}{2}\left(\frac{i_t \epsilon_t^{qk}}{i_{t-1}} - 1\right)^2\right] i_t \tag{30}$$

The parameter δ is the depreciation rate of physical capital.

2.7 Final goods producers

The final goods Producers are perfect competitors. They buy the intermediate goods and they combine them, with no additional costs, in an undifferentiated homogeneous final goods. They maximize the following profit function with respect to $P_t(j)$:

$$E_t \sum_{t=0}^{\infty} \beta_P^t \lambda_t^P \left[P_t(j) y(j)_t - P^w y_t(j) - k_p \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1}^{\iota_p} \bar{\pi}^{(1-\iota_p)}\right)^2 P_t y_t\right] \tag{31}$$

In addition they have to face an adjustment cost k_p in order to change their price. The current price is anchored to both past and steady state inflation with ι_p that represents the weight of the past inflation with respect to the steady state. Finally, final goods producers are constrained by the following demand function for the final goods:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_t^y} y_t \tag{32}$$

Substituting equation 32 into equation 31 and deriving with respect to the price we obtain the New Keynesian Phillips curve for price inflation. ϵ_t^y is a stochastic mark up shock.

$$\begin{aligned} &(1-\epsilon_t^y) + \epsilon_t^y \frac{P^w}{P_t} - k_p \left(\pi_t - \pi_{t-1}^{\iota_p} \pi^{(1-\iota_p)}\right) \pi_t + \\ &\beta^p E_t \left[\frac{\lambda_{t+1}^p}{\lambda_t^p} k_p \left(\pi_{t+1} - \pi_t^{\iota_p} \pi^{(1-\iota_p)}\right) \pi_t^2 \frac{y_{t+1}}{y_t}\right] = 0 \end{aligned} \tag{33}$$

2.8 Bank system

As in Gerali et al. (2010), Deficit banks are composed of Wholesale and Retail branches. Wholesale branches operate under perfect competition and their aim is to manage the cash flow position of the group. Retailers have to provide loans to the households and to the firms. They can exploit their market power to set the interest rates on market loans. Following Dib (2010), we add a Surplus bank that has to decide how many resources to allocate in the interbank or in the government bonds market.

2.9 Deficit Banks

2.9.1 The wholesale branch

Table 1: Defict bank's balance sheet

Assets	Liabilities
L_t	IB_t
GB_t^{db}	K_t^b

We model the wholesale branch similar to Gerali et al. (2010), introducing a portfolio choice like in Vlcek and Roger (2011). We depart from their framework endogenizing the choice of the share of loans on the total amount of assets, η, which was instead set by the monetary authority. The problem the wholesale branch has to face is the maximization of the cash flow of the entire holding subject to the bank's balance sheet constraint (See Table 1):

$$\max E_0 \sum_{t=0}^{\infty} \beta_P^t \lambda_t^P \Big[(1+R_t)^b \eta_t(j) B_t(j)(1-\delta_t^{db}) + (1+r_t^g)(1-\eta_t(j))B_t(j) - (1+r_t^{ib})IB_t(j) - K_t^b(j) - Adj_t^{kb}(j) - Adj_t^{mc}(j) \Big] \tag{34}$$

where $\beta_P^t \lambda_t^P$ represents the stochastic discount factor for the Wholesale branch[3], R_t^b , r_t^g and r_t^{ib} are respectively the (net) interest rate on loans from the wholesale branch to each retailers, the (net) interest rate on government bonds, the (net) interest rate on the loans obtained on the interbank market. The term

$$Adj_t^{kb}(j) = \frac{k_{kb}}{2} \left(\frac{K_t^b(j)}{B_t(j)} - v_b \right)^2 K_t^b(j) \tag{35}$$

is the bank capital requirement. The lowest is the ratio between bank capital and the total asset the higher is the penalty cost of providing and additional unit of loans to the retail branch. v^b is fixed at the 8% in order to replicate the Basel II capital requirement constraint. Similar to Dib (2010), the term Adj_t^{mc} is an adjustment cost that can be interpreted as a monitoring cost the bank has to pay to control the loans made to the retail branches. Its interpretation is straightforward. The higher is the share of resources allocated in the private loan market the higher is the cost the holding has to face in order to provide an additional unit of loan. It can be formalized as a quadratic adjustment cost

$$Adj_t^{mc}(j) = \frac{\chi_{db}}{2} \left([\eta_t(j) - \bar{\eta}] B_t(j) \right)^2 \tag{36}$$

Since we decide to use a first order approximation to solve the model, the introduction of this friction is necessary in order to pin down the portfolio choice and to avoid perfect substitutability between different assets due to the certain equivalence problem (see among the others Zagaglia (2009)). Deficit bank has to obey in every period to the following balance sheet constraint

$$B_t(j) = IB_t(j) + K_t^b(j) \tag{37}$$

where B_t is the total amount of assets, which includes both government bonds GB_t and loans L_t.

$$B_t(j) = L_t(j) + GB_t^{db}(j) \tag{38}$$

IB_t are the resource the deficit banks borrow in the interbank market from the surplus ones and K_t^b represents the bank capital which obeys to the

[3]The stochastic discount factor is equal to the marginal utility of consumption of the patient households because we are assuming that they are the only owners of the bank

following rule:

$$K_t^b(j)\pi_t = (1-\delta_b)K_{t-1}^b(j) + \Omega J_{t-1}^{db}(j) \tag{39}$$

δ_b and Ω are respectively the quarterly depreciation rate of bank capital and the share of profits used to accumulate new bank capital [4]. η represents the share of loans on the total amount of assets in the balance sheet of the bank [5].

$$L_t(j) = \eta_t(j)B_t(j) \tag{40}$$

Substituting equation 38 into 34 and deriving with respect to B_t and η_t we get the first order conditions for the wholesale branch problem

$$\frac{\partial}{\partial\ B_t} \quad : \quad R_t^b\eta_t(1-\delta_t^{db}) = r_t^{ib} - r_t^g(1-\eta_t) \tag{41}$$

$$-k_{kb}\left(\frac{K_t^b}{B_t} - v_b\right)\left(\frac{K_t^b}{B_t}\right)^2 + \chi_{db}\Big([\eta_t - \bar{\eta}]^2 B_t\Big) \tag{42}$$

$$\frac{\partial}{\partial\ \eta_t} \quad : \quad \eta_t = \bar{\eta} + \frac{R_t^b(1-\delta_t^{db}) - r_t^g}{\chi_{db}B_t} \tag{43}$$

Equation 41 links the interest rate on loans to interbank market condition and to the adjustment costs the bank have to face. Equation 43 describes the evolution of the share of bank loans to Entrepreneurs and Impatient households. δ_t^{db} is the (stochastic) riskiness of credit market. An increase of δ_t^{db} modifies the portfolio allocation of the bank. The highest the riskiness the more the wholesale branch would shift resources from private lending to risk free government bond, replicating the credit crunch mechanism of the recent financial crisis.

2.9.2 The retail branch

The retail branch of the deficit bank has the task of providing loans to the households and the entrepreneurs. The retailer bankers operate under monopolistic competition and they have the power to set the interest rate on

[4]Consequently, $(1-\Omega)$ is the dividend pay-off ratio that is the quantity of bank profit distributed to the Patient households. Assuming $\Omega = 1$ bank is following a zero dividends policy and all profits are used to increase the bank capital.

[5]Consequently $1-\eta$ will be defined as the share of government bond on the total assets.

their loans. They have to maximize the following profits function

$$\max E_o \sum_{t=0}^{\infty} \left[r_t^{bh}(j) b_t^I(i) + r_t^{be}(j) b_t^E(i) - R_t^b L_t(j) - Adj_t^{kn} \right] \tag{44}$$

subject to the loans demand of impatient households and entrepreneurs which are

$$b_t^n(i) = \left(\frac{r_t^{bn}(j)}{r_t^{bn}} \right)^{-\epsilon_t^{bn}} b_t^I \tag{45}$$

The adjustment costs are defined as

$$Adj_t^{kn} = \frac{k_{bn}}{2} \left(\frac{r_t^{bn}(j)}{r_{t-1}^{bn}(j)} - 1 \right)^2 r_t^{bn} b_t^n \tag{46}$$

Every time the bank changes the interest rate it has to pay a cost in term of profit. This adjustment cost introduces stickiness in the setting of interest rates on loans. We can look at the first order conditions for the retail branch as a New Keynesian Phillips Curve for loan interest rates (see Aslam and Santoro (2008)). Substituting the loans demand into the objective function and deriving with respect to r_t^{bh} and r_t^{be} we obtain

$$\begin{aligned} \frac{\partial}{\partial\, r_t^{bn}} : 1 - \frac{\Lambda_t^{bn}}{\Lambda_t^{bn} - 1} + \frac{R_t^b}{r_t^{bn}} \frac{\Lambda_t^{bn}}{\Lambda_t^{bn} - 1} - k_{bn} \left(\frac{r_t^{bn}}{r_{t-1}^{bn}} - 1 \right) \frac{r_t^{bn}}{r_{t-1}^{bn}} + \\ \beta_P E_t \left[\frac{\lambda_{t+1}^p}{\lambda_t^p} k_{bn} \left(\frac{r_{t+1}^{bn}}{r_t^{bn}} - 1 \right) \left(\frac{r_{t+1}^{bn}}{r_t^{bn}} \right)^2 \frac{b_{t+1}^n}{b_t^n} \right] = 0 \end{aligned} \tag{47}$$

where $n = h, e$. We express the elasticity of substitution between loans provided by different retails branches as a function of the mark up Λ [6]. Higher values of ϵ (or equivalently lower values of Λ_t) implies a lower market power and a lower margin of intermediation for the bank.

2.9.3 Aggregate activity

The profits of the entire holding are defined as the revenues coming from all the business lines of the bank minus the intra group activities and the

[6] The elasticity of substitution could be express as a function of the mark up

$$\epsilon_t = \frac{\Lambda_t}{\Lambda_t - 1} \tag{48}$$

adjustment costs. We can define the variable J_t^{db} as the total profits of the deficit group as

$$J_t^{db} = r_t^{bh} b_t^I + r_t^{be} b_t^E + r_t^g GB_t^{db} - r_t^{ib} IB_t - \sum Adj_t^{db} \tag{49}$$

2.10 Surplus banks

The Surplus bank collects deposit from the Patient households and decide to invest these resources either in the interbank market or purchasing government bonds like in Dib (2010). The financial position of the bank is summarized in Table 2.

Table 2: Surplus bank's balance sheet

Assets	Liabilities
IB_t	D_t^P
GB_t^{sb}	

The objective function to be maximized can be represented by

$$\begin{aligned} \max E_0 \sum_{t=0}^{\infty} \beta_P^t \lambda_t^P \Bigg[& s_t(j) r_t^{ib} (1 - \delta_t^{sb}) d_t^P(j) + (1 - s_t(j)) r_t d_t^P(j) - r_t^d d_t^P(j) \\ & - \frac{k_d}{2} \left(\frac{r_t^d(j)}{r_{t-1}^d(j)} - 1 \right)^2 r_t^d d_t^P - Adj_t^s(j) \Bigg] \end{aligned} \tag{50}$$

subject to

$$d_t^P(j) = \left(\frac{r_t^d(j)}{r_t^d} \right)^{-\epsilon_t^d} d_t^P \tag{51}$$

that is the deposits demand of the Patient households and the balance sheet constraint of the bank

$$IB_t(j) + GB_t^{sb}(j) = D_t^P(j). \tag{52}$$

The variable s_t represents the share of bank liabilities invested in the interbank market defined as:

$$IB_t(j) = s_t(j) D_t(j) \tag{53}$$

The term Adj_t^s is an adjustment cost that can be defined as

$$Adj_t^s(j) = \frac{\chi_{sb}}{2}\left[(s_t(j) - \bar{s})d_t^P(j)\right]^2 \tag{54}$$

It has the same interpretation of η_t in the case of the Deficit bank. The highest is the exposition on the interbank market with respect to the steady state value the higher is the cost the Surplus bank has to pay. As in the case of the Deficit bank the market power in setting the interest rate of deposits allows us to interpret the derivative of the objective function with respect to r_t^d as a New Keynesian Phillips curve for deposit interest rate.

$$\frac{\partial}{\partial\, r_t^d} \quad : -1 + \left(\frac{\Lambda_t^d}{\Lambda_t^d - 1}\right)\left(1 - s_t\frac{r_t^{ib}}{r_t^d}(1 - \delta_t^{sb}) - (1 - s_t)\frac{r_t}{r_t^d}\right) \tag{55a}$$

$$- k_d\left(\frac{r_t^d(j)}{r_{t-1}^d(j)} - 1\right)\frac{r_t^d}{r_{t-1}^d} + \chi_{sb}\left(s_t - \bar{s}\right)^2 d_t^P\left(\frac{\Lambda_t^d}{\Lambda_t^d - 1}\right)\frac{1}{r_t^d} \tag{55b}$$

$$+ \beta^P E_t\left[\frac{\lambda_{t+1}^P}{\lambda_t}k_d\left(\frac{r_{t+1}^d(j)}{r_t^d(j)} - 1\right)\left(\frac{r_{t+1}^d}{r_t^d}\right)^2\left(\frac{d_{t+1}^P}{d_t^P}\right)\right] = 0 \tag{55c}$$

$$\frac{\partial}{\partial\, s_t} \quad : s_t = \bar{s} + \frac{r_t^{ib}(1 - \delta_t^d) - r_t}{\chi_{sb} d_t^P} \tag{55d}$$

Symmetrically to the Deficit bank, the second first order condition represents the evolution of the share of resources allocated into the interbank market by the surplus bank. A sudden increase in the interbank market riskiness δ_t^d alters the allocation of the activities between the interbank and the government bond market. Similar to the mechanism of the deficit banks, an increase of δ_t^d should be able to replicate the unexpected collapse of the interbank channel.

2.10.1 Aggregate activity for the surplus bank

In a similar manner with respect to the deficit bank, the aggregate activity of the surplus bank can be summarized by the following expression for the profits.

$$J_t^{sb} = r_t^{ib} IB_t + r_t GB_t^{sb} - r_t d_t^P - \sum Adj_t^{sb} \tag{56}$$

2.11 Central Bank

The central bank governs the short term interest rate following a non linear Taylor rule:

$$(1+r_t) = (1+\overline{r})^{(1-\phi_R)}(1+r_{t-1})^{\phi_R}\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi(1-\phi_R)}(\Delta y_t)^{\phi_Y(1-\phi_R)}(1+\epsilon_r^R) \quad (57)$$

where $\overline{r}$ is the steady state value of the interest rate, while ϕ_R, ϕ_π and ϕ_Y are respectively the weights assigned by the central bank to the past short term interest rate, the inflation target and the GDP growth.

2.12 The Government

The government sector has to obey to an intertemporal budget constraint

$$G_t + GB^{sb}_{t-1}\frac{(1+r^g_{t-1})}{\pi_t} + GB^{db}_{t-1}\frac{(1+r^g_{t-1})}{\pi_t} = GB^{sb}_t + GB^{db}_t + T_t \quad (58)$$

where G_t is the exogenous public expenditure and T_t a lump sum tax. We further assume that the supply of government bonds is fixed in the short run and set equal to one. The market clearing condition for the government bond market will be

$$GB^{db}_t + GB^{sb}_t = 1 \quad (59)$$

In order to close the model, we assume that the interest paid by the government bonds is equal to the interest rate set by the central bank $r_t = r^g_t$.

2.13 Market clearing conditions and autoregressive process

We close our model specifying fifteen exogenous shock that evolve like AR(1) process in the form

$$\log(X_t) = (1-\rho)\log(\bar{X}) + \rho\log(X_{t-1}) + e_t \quad (60)$$

With respect to the original model we add three additional stochastic disturbances: the public expenditure G_t, the riskiness of interbank market (δ_d) and the riskiness of credit market (δ_{db}).

The resource constraint for the economy is described by the following equation

$$
\begin{aligned}
y_t =& c_t + q_t^k \left[k_t - (1-\delta)k_{t-1}\right] + k_{t-1}\left[\xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t-1)^2\right] + \\
& \frac{\delta_b K_{t-1}^b}{\pi_t} + G_t + \sum_{j=0}^{\infty} Adj_t^j
\end{aligned} \tag{61}
$$

where c_t is defined as

$$c_t = c_t^P + c_t^I + c_t^E \tag{62}$$

and

$$h = h_t^P + h_t^I \tag{63}$$

Without an explicit supply sector for housing, we close the model fixing a positive net supply $h = 1$ of the real estate sector. Moreover, the term $\sum_{j=0}^{\infty} Adj_t^j$ includes all the adjustment costs of the models.

3 Solution of the model

The log linearized version of the model can be written in the form proposed by Klein (2000)

$$AE_t\left\{H_{t+1}\right\} = BH_t + CZ_t \tag{64}$$

where H_t is a vector containing the endogenous variables of the model and Z_t the autoregressive process. The matrices A,B and C contain all the deep parameters of the model. Through the use of the Schur decomposition we were able to find a solution of the model and to represent it in state space form. All the procedure is carried on using DYNARE [7], a MATLAB and OCTAVE toolbox capable of solving and simulate DSGE model. We find a numerical solution for the steady state using a technique suggested by Judd (1998) (Appendix A.2 it could be found a detailed description of the procedure). After finding the steady state we applied a first order Taylor expansion around the steady state to solve and simulate our dynamic model.

[7] We used DYNARE 4.2.5 version Adjemian et al. (2011).

Following Fernandez-Villaverde (2010), the related state space for of the log-linearized model could be written as

$$S_{t+1} = \Psi_{s,1} S_t + \Psi_{s,2} \epsilon_t \tag{65}$$

where S_{t+1} is the law of motion of the state variables of the system. The equation for the observables variables could be written as

$$Y_t = \Psi_{o,1} S_t \tag{66}$$

$\Psi_{s,1}$,$\Psi_{s,2}$ and $\Psi_{o,1}$ are matrices that are non linear functions of the deep parameters of the model. Since there are no measurement errors [8] in the observable variables, the last equation depends only on the state variables of the system.

4 Estimation

Using the state space form, through the Kalman filter, we could recover the likelihood function $\mathcal{L}(Y, \nabla)$ of our model, where ∇ is the vector containing all the estimated parameters, and Ξ are the observables variables. Using the Bayes theorem we could obtained the posterior $p(\nabla, Y)$ of our parameters of interest

$$p(\nabla, \Xi) =\propto \mathcal{L}(\Xi, \nabla) p(\nabla) \tag{67}$$

where $p(\nabla)$ is the prior information we have about deep parameters. In the next section we analyze in detail the estimation strategy.

4.1 Dataset

We employ fifteen observable variables on the Euro area in order to carry on the estimation. We use gross domestic product, investment, consumption, public expenditure, house price, inflation, wage inflation, deposits, loans to households and entrepreneurs, deposits interest rate, central bank interest

[8] Since we have a number a exogenous shock equal to the number of observables there is no need to introduce measurement errors to avoid stochastic singularity of the likelihood.

rate, interbank market interest rate, and interest rate on households and firms loans. All variables, with the exception of the interest rate, are expressed in real terms. We made stationary all the time series applying the HP filter[9] and subtracting their sample mean to the interest rates. The vector of observable variables used to perform the estimation could be represented by

$$\underbrace{\Xi_t^{obs}}_{15*1} = \Xi_t - \overline{\Xi} \tag{68}$$

that is, as deviations of the endogenous variables from the steady state values.

4.2 Bayesian estimation

Following Gerali et al. (2010) and DARRACQ PARIES et al. (2011), we use Bayesian techniques in order to estimate only a small subset of the parameters, focusing our attention only on those affecting the dynamic of the system. The steady state parameters are calibrated in line with the values of Gerali et al. (2010). We departed from the original calibration imposing a steady state ratio of $\frac{K^b}{B}$ equal to 8% , fixing the depreciation rate of bank capital to 0.072 a slightly lower value then the one proposed in Gerali et al. (2010). The steady state value of the interbank market riskiness plays a crucial role in the steady state asset allocation between government bonds and loans. We set a value for the first parameter of 0.0025 very closed to one proposed by Dib (2010) to obtain a steady state value for the share of interbank market on the total deposits equal $\bar{s}$ to 0.9. In a similar manner we fix the second parameter to match a steady state value of $\bar{\eta}$ of 0.9. We assigned a very small value according to Dib (2010) for the monitoring cost of both the surplus and deficit bank on their loans. In Table 4 we could find the implied state state value of several variables of interest. The model produce endogenously values for the ratio between, consumption, investment, public expenditure over output that are in line with the empirical evidence. The steady state values for interest rates are in line with the means of the observables variables over the sample (See for a comparison Table 7 in Appendix

[9]Since we used quarterly data we assumed that the smoothness parameters of the HP filter is set equal to 1600.

Table 3: Calibrated parameters

Parameter	Definition	Value
β^P	Patient households discount factor	0.99430
β^I	Impatient households discount factor	0.97500
β^E	Entrepreneurs discount factor	0.97500
α	Capital share	0.25000
δ	Depreciation rate of physical capital	0.02500
ϕ	Inverse of Frisch elasticity	1.00000
μ	Share of Patient workers	0.80000
$\bar{m}^I$	Steady State value of LTV for impatient households	0.70000
$\bar{m}^E$	Steady State value of LTV for Entrepreneurs	0.35000
$\bar{\pi}$	Net Steady State inflation	1.00000
$\bar{\zeta}^d$	Elasticity of substitution of deposit	-2.26025
$\bar{\zeta}^{bh}$	Elasticity of substitution of households loans	4.62017
$\bar{\zeta}^{be}$	Elasticity of substitution of entrepreneurs loans	4.29126
ξ_1	Coefficient associated with the degree of utilization of physical capital	0.04580
ξ_2	Coefficient associated with the degree of utilization of physical capital	0.00458
v_b	Basel II capital requirement	0.08000
δ_b	Depreciation rate of bank capital	0.04600
Ω	Profits invested in new bank capital	1.00000
$\bar{\delta}^{sb}$	Steady State value of interbank riskiness shock	0.00250
$\bar{\delta}^{db}$	Steady State value of Loans riskiness shock	0.08000

Table 4: Implied steady state values of the model

Variables	Steady state values	Meaning
$\frac{C}{Y}$	0.70	Consumption over GDP
$\frac{I}{Y}$	0.11	Investment over GDP
$\frac{G}{Y}$	0.17	Government spending over output
$\frac{K}{Y}$	4.5	Capital over output
$\bar{r}^{ib}$	3.32	Annual interbank interest rate
$\bar{r}$	3.31	Annual central bank interest rate
$\bar{r}^{bh}$	4.70	Annual interest rate on households loans
$\bar{r}^{be}$	4.6	Annual interest rate on entrepreneurs loans
$\bar{r}^{d}$	2.29	Annual interest rate on deposit
$\frac{K^b}{B}$	0.08	Bank capital over assets
$\frac{B}{Y}$	2.60	Assets over GDP
$\frac{D}{Y}$	3.00	Deposits over GDP
$\frac{GB}{Y}$	0.70	Debt over GDP

A.9). The model produce a ratio between bank own capital and the total assets that is equal to the 8% in line with the Basel II capital requirement.

4.2.1 The choice of the Priors

Given the similarity between our model and Gerali et al. (2010) a very natural starting point for the selection of prior distributions is to choose those proposed in their original work. We choose to bring only slightly modifications with respect to the original setting. We selected an Inverse gamma distribution for the standard deviation of the exogenous shock, a quite natural choice for this kind of parameters (See Lombardo and McAdam (2012)). No modifications are carried on the prior mean and standard deviation. A Beta distribution for the autoregressive coefficient seems a plausible choice in order to bound the estimation of the parameter between zero and one. We shifted the prior mean from 0.8 to 0.7 and we widened the standard deviation from 0.1 to 0.2 in order to let the data speaks as free as possible since the new autoregressive coefficients are hard to set a priori. The only notable exceptions is the autoregressive coefficient of the technological shock. Evidence from the literature Smets and Wouters (2007) suggest a very high degree of persistence. We include this presample information choosing a very tight prior for technological shock. The prior of the adjustment costs are Gamma distributions in order to ensure a positive values for the related estimated parameters. We introduced two significant modifications for the prior of the adjustment costs of prices and real wages fixing the prior mean at the estimated values of Gerali et al. (2010) and reducing significantly the standard deviation from an original value of 50 to the new one of 5. Moreover, we choose quite tight priors for the monitoring costs of both the surplus and the deficit bank. The prior mean for both the parameters is taken from Dib (2010) which assigned a calibrated value of 0.001. The detailed choice of the priors and their distributions could be find in Table 5.

4.2.2 Posterior distributions

We obtained the posterior distribution applying the classical procedure of Monte Carlo Markov Chain simulation (See for a detailed explanation Fernandez-Villaverde (2010)). The posterior distribution of our model could be defined as:

$$\ln \mathcal{K}(\nabla|\Xi_T) = \ln \mathcal{L}(\Xi_T|\nabla) + \ln p(\nabla) \tag{69}$$

The first step is to find the mode of equation 69 using a Monte Carlo optimization procedure [10]. We use the mode and the inverse of the Hessian evaluated at the posterior mode as starting point for the mean and the variance of the proposal density to initialized the Metropolis Hastings algorithm. We choose a random walk proposal density defined as:

$$\nabla^* = \nabla_{i-1} + \epsilon_t \tag{70}$$

[10] See http://www.dynare.org/DynareWiki/MonteCarloOptimization.

The term ∇^* is a draw from the proposal density and the error term is distributed as :

$$\epsilon \sim \mathcal{N}(\nabla^{i-1}, c\Sigma) \tag{71}$$

The scale factor c for the covariance matrix is set equal to 0.3 in order to obtain an acceptance rate around 0.3%. We launched 2 chains each ones composed of 1.000.000 draws and we explore the posterior distribution finding the posterior mean and the posterior standard deviation. The CUSUM statistics[11] in Appendix A.7 ensure the convergence of the chains after discarding the 60% of the draws. The Iskrev's test (See Iskrev (2010)) is implemented in order to verify the local identification of the estimated parameters at the posterior mode. Moreover, in order to check the robustness of our results to a variation of our prior distributions we run a second estimation with the standard deviations of the priors enlarge by +50% with respect to the original specification. Further details can be found in Appendix A.8. In Appendix A.4 we plot the prior and the posterior density of the estimated parameters. The results are in line with the previous contributions. Moreover, the parameters χ_s and χ_{sb} are both small and in line with the calibrated value found in Dib (2010).

5 Quantitative experiment

The impulse response functions are reported in Appendix A.5. We analyzed the responses of total output, consumption, investment, deposits, inflation, illiquid asset of deficit bank, deficit bank capital, interbank landing, deficit bank government bond, deficit bank loans, policy rate, interbank interest rate, interest rate on households loans, interest rate on entrepreneurial loans, deficit bank profits, surplus bank profits, surplus bank government bond, share of interbank loans on deposits, final goods producers profits. All the impulse response functions are calculated using the posterior means of the

[11] To obtain the COSUM statistics we exploit the DSGEBaseyianToolbox provid by Ambrogio Cesa Bianchi. It can be downloaded from https://sites.google.com/site/ambropo/DSGEBayesianToolbox.zip?attredirects=0

Table 5: RESULTS FROM BAYESIAN ESTIMATION

parameters	Prior shape	Prior mean	Post dev	Post mode	Post mean	Conf interval
k_p	Γ	30.00	5.0000	53.947	54.112	42.627 65.325
k_{bh}	Γ	6.000	2.5000	8.1133	9.3041	5.9263 12.5688
k_{be}	Γ	3.000	2.5000	7.3961	9.3278	5.5188 13.0069
k_d	Γ	10.00	2.5000	6.4865	7.4698	4.7687 10.2041
k_i	Γ	2.500	1.0000	6.1990	6.4695	4.5961 8.2563
k_w	Γ	100.0	5.0000	104.93	105.25	96.711 113.75
k_{kb}	Γ	10.00	5.0000	5.6057	7.4613	1.4994 13.4703
χ_s	Γ	0.001	0.0050	0.0002	0.0003	0.0001 0.0006
χ_{sb}	Γ	0.001	0.0050	0.0014	0.0018	0.0008 0.0027
a^H	$\mathcal{B}$	0.500	0.1000	0.6007	0.6380	0.4768 0.8013
ι_w	$\mathcal{B}$	0.500	0.1000	0.3775	0.3900	0.2355 0.5403
ι_p	$\mathcal{B}$	0.300	0.1000	0.1944	0.2263	0.0864 0.3593
ϕ_R	$\mathcal{B}$	0.500	0.1000	0.8127	0.8147	0.7718 0.8611
ϕ_π	Γ	2.000	0.5000	2.0698	2.1502	1.7227 2.5606
ϕ_y	$\mathcal{N}$	0.100	0.1500	0.5088	0.4866	0.2967 0.6813

estimated parameters in percentage deviation from the steady state.

5.1 Technological shock

The response to a technological shock (See Figure 9) presents all the empirical evidence observed in the literature like in Smets and Wouters (2007) or Christiano et al. (2005). Total output, consumption, investment and the stock of physical capital react positively to an increase of technological efficiency, while the inflation rate decreases. Besides, the whole structure of the interest rates coherently decrease. The total amount of lending provided by the deficit bank to the economy increase for almost six quarters confirming the evidence found in Dib (2010) and Carrera and Vega (2012), even if the effect of the shock takes a longer time to disappear (about 20 quarters in both models), pushed by the rise of interbank lending injected by the surplus bank. The behavior of interbank loans is quite similar to Carrera and Vega (2012) but it differs significantly from the results of Dib (2010). In our model a positive technological shock impacts negatively both on the central bank interest rate and on interbank rate having a stronger influence on the second. The growing spread between the rate implies an increase in interbank lending over the government bond investment. If we think about a negative technological shock instead of a positive one we could see a flight to quality scenario. A sudden deterioration of the economic conditions encourages the surplus bank to shrink risky interbank lending. On the other side, the deficit bank, in front of the adverse economic scenario, decides prudently to increase the amount of government bonds in his portfolio as safe heaven. Finally, the profits of the final goods producers goes up due to the increment of output.

5.2 Monetary policy shock

Figure 10 represents the response to an increase of the central bank interest rate. The total output, consumption, investment, the stock of physical capital and inflation fall, while the entire structure of the interest rates rises sharply. The increase of the central bank interest rate influence the asset allocation of the surplus bank, diverting resources from the interbank market to government bond purchase. At the same time, the deficit bank reduces the loans provided to both the real economy and risk free market because both the interbank lending and the bank capital are negatively affected by the increase of the interest rate. The fall of bank capital for the surplus bank is mainly due to the contraction of its own profits which are lowered by the higher cost of external financing. Instead, the profits of the surplus rise above the steady state level. The highest price paid over the liabilities is more than compensated by the higher earnings due to the rise of interest rate on interbank lending and risk free bonds. An increase of the policy rate seems to penalize more the net debtor on the market. The deficit bank suffers more the monetary restriction of the central bank, while the surplus bank could exploit the advantage of interbank lending at a higher price.

The inclusion of an interbank market seems to dampen the negative effect of an increase of the policy rate on the output, consumption, investment, inflation, deposits and total loans. This result is coherent with Hilberg and Hollmayr (2011). Two notably exceptions are represented by the dynamic behavior of bank capital and the profits of deficits banks. The introduction of an interbank market seems to penalize more the weakest player on the market in favor to surplus banks. Instead, it is not clear if a technological shock is dampened by the presence of the interbank market.

5.3 Interbank riskiness shock

Figure 11 represents the impulse response functions of the model of an interbank riskiness shock. An increase of interbank riskiness causes a decline of the total output, investment and the stock of capital owned by the intermediate firms. The interbank lending provide by the surplus bank decrease causing a sudden rise of the whole structure of the interest rate. The results are coherent with other recent contribution in the literature like Boissay et al. (2013). At the same time the surplus bank increase the quantity of government bonds detained in his portfolio. On the deficit side of the credit market, the total lending provided by the deficit bank to the real economy diminishes producing a recession in the economy through a reduction of the investment and, as a consequence, a minor endowment of physical capital for the intermediate firms. The less the capital the less is the value of the collateral the Entrepreneurs could use to obtain credit from the bank, exacerbating the crisis on the credit market.

6 Historical variance decomposition

In this section we focus our attention on the historical decomposition of the observable variables in order to understand the contribution of each shock to the business cycle especially the role of the interbank riskiness shock. In Figures 12 and 13 the historical decompositions of eight main variables are reported. As we expected, the interbank market shock seems to explain most of the rise of the interest rate on the credit market during the 2008 financial crisis confirming that the model is able to explained one of the transmission mechanism we described in the introduction of the paper. Even after the 2008 the tensions on the interbank market could be explained through the interbank riskiness shock. What changed was the behavior of the monetary policy through the huge entries of liquidity in the banking system. The ECB indeed steered down the interest rate on the credit market drastically cutting the policy rate of over 300 basis point after the 2008 (See Figure 22).

The other important feature revealed by the historical decomposition is the fact that, like in Gerali et al. (2010), almost all the variations of the variables are driven by financial shocks after the 2007. This is a very nice feature of the model since it is quite implausible that the recent financial crisis could be explained in term of a negative total factor productivity shock.

7 Concluding remarks

In this paper we highlighted the role of the interbank market as an important component of the business cycle. We extended the model proposed by Gerali et al. (2010) including an interbank market like in Dib (2010) and we took it to the data of the Euro area using Bayesian estimation. The results suggest that our model could be able to replicate a turmoil on the interbank market replicating some features of the 2007 financial crisis. A liquidity crisis could divert resources from the risky interbank lending to a safer government bond holding, ending up with higher interest rate on entrepreneurial loans, less credit provided by the bank to the real economy, causing a recession driven by the fall of the investment. The historical decomposition we implemented shows how part of the rise of interest rate during the financial crisis could be explained by the introduction of a interbank riskiness shock.

Same questions remain unresolved and need to be clarified: a better specification of the bank capital market could improve the realism of the model and could help to study another aspect of the financial markets (See for a possible modeling strategy of the bank capital market Hollander and Liu (2013)).

Nevertheless, the story told by our model seems to be plausible and coherent with the 2007 financial crisis: the interbank market suddenly collapse and the contagion spreads to the real economy.

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8 Appendix

8.1 Appendix A.1

The i_{th} Impatient Household chooses the optimal amounts of loan to demand from the j_{th} retail bank in order to minimize his expenditure. Following Dixit and Stiglitz (1977)

$$\min \int_0^1 r_t^H(j) b_t^I(i,j) dj \tag{72}$$

subject to

$$\left[\int_0^1 b_t^I(i,j)^{\frac{\epsilon_t^{bh}-1}{\epsilon_t^{bh}}} dj \right]^{\frac{\epsilon_t^{bh}}{\epsilon_t^{bh}-1}} = b_t^I(i) \tag{73}$$

ϵ_t^{bh} is the elasticity of substitution between loans provided by different retail banks. Setting up the Lagrangian

$$\mathcal{L} = \int_0^1 r_t^H(j) b_t^I(i,j) dj + \lambda \left[b_t^I(i) - \left(\int_0^1 b_t^I(i,j)^{\frac{\epsilon_t^{bh}-1}{\epsilon_t^{bh}}} dj \right) \right] \tag{74}$$

Deriving with respect to $b_t^I(i)$

$$\frac{\partial \mathcal{L}}{\partial\, b_t^I(i)} : \int_0^1 r_t^{bh} dj - \lambda \left[\frac{\epsilon_t^{bh}}{\epsilon_t^{bh}-1} \left[\int_0^1 b_t^I(i,j)^{\frac{\epsilon_t^{bh}-1}{\epsilon_t^{bh}}} \right]^{\frac{1}{\epsilon_t^{bh}-1}} \frac{\epsilon_t^{bh}-1}{\epsilon_t^{bh}} b_t^I(i,j)^{-\frac{1}{\epsilon_t^{bh}}} \right] = 0 \tag{75}$$

Substituting the constraint into 75 we obtain

$$r_t^{bh}(j) - \lambda b_t^I(i)^{\frac{1}{\epsilon_t^{bh}}} b_t^I(i,j)^{-\frac{1}{\epsilon_t^{bh}}} = 0 \tag{76}$$

Rearranging the term

$$b_t^I(i,j) = \left[\frac{r_t^{bh}(j)}{\lambda} \right]^{-\epsilon_t^{bh}} b_t^I(i) \tag{77}$$

Substituting the last equation into the constraint

$$b_t^I(i) = \left[\int_0^1 \left[\left(\frac{r_t^{bh}(j)}{\lambda} \right)^{-\epsilon_t^{bh}} b_t^I(i) \right]^{\frac{\epsilon_t^{bh}-1}{\epsilon_t^{bh}}} \right]^{\frac{\epsilon_t^{bh}}{\epsilon_t^{bh}-1}} \tag{78}$$

After some algebra, we could obtain an expression for the Lagrange multiplier λ

$$\lambda = \underbrace{\left[\int_0^1 r_t^{bh}(j)^{1-\epsilon_t^{bh}}\right]^{\frac{1}{\epsilon_t^{bh}-1}}}_{r_t^{bh}} \tag{79}$$

Substituting the expression of the Lagrange multiplier into the first order condition

$$r_t^{bh}(j) - r_t^{bh} b_t^{\frac{1}{\epsilon_t^{bh}}} b_t^{-\frac{1}{\epsilon_t^{bh}}} = 0 \tag{80}$$

Rearranging the term we obtain the following expression

$$b_t^I(j) = \left(\frac{r_t^{bh}(j)}{r_t^{bh}}\right)^{-\epsilon_t^{bh}} b_t^I \tag{81}$$

which is the i_{th} Impatient Household loan demand provided by the j_{th} bank. The entrepreneurs face an identical problem while the patient households a slightly different one. They maximize the revenues of total saving solving the following problem

$$\max \int_0^1 r_t^d(j) b_t^P(i,j) dj \tag{82}$$

subject to

$$\left[\int_0^1 d_t^P(i,j)^{\frac{\epsilon_t^d-1}{\epsilon_t^d}} dj\right]^{\frac{\epsilon_t^d}{\epsilon_t^d-1}} = d_t^P(i) \tag{83}$$

8.2 Appendix A.2

8.3 Some theory beyond the resolution of non linear system

Following Judd (1998), the resolution of a non linear system

$$F(x) = 0 \tag{84}$$

could be seen as an equivalent minimization problem

$$\min \sum_{i=1}^{n} f^i(x)^2. \tag{85}$$

Judd (1998) underlined several problems beyond this approach but he stressed the fact that it could be a useful method to find educated initial guess for a local optimizer. In particular, this method is able to find the global minimum but it ignores local optimum. In case of standard DSGE model this issue does not matter. If the problem is well posed, only one solution exists around which the system is linearized (see Blanchard and Kahn (1980)).

8.4 Steady state procedure

We developed a simple procedure to exploit the equivalence between minimization problem and the resolution of a linear system. Since we already know the calibrated values of the parameters from Gerali et al. (2010), we used their calibration as starting point for our procedure.

- Step I: we extract n random vectors from a χ^2 distribution with one degree of freedom. Each vector is made of m elements where m is the number of unknown variables in the system. The choice of the χ^2 is motivated by the fact that the steady state of a DSGE model is usually a vector of non negative quantity[12]. We end up with a matrix x_0 of random initial guesses made of n columns and the number of rows m equal to the variables of the system.

[12] There are of course exceptions: in an open economy model the foreign net position of a country could be negative

- Step II: we transform our original system exploiting equation 85. We simply transform a problem of solving a system of k equations in k unknowns into a problem of finding the minimum of a function of several variables.

- Step III: we evaluate the transformed function for every columns of the matrix x_0. If we choose a large enough numbers of columns we would be able to find a vector of educated initial guesses.

- Step IV: knowing that the minimum of the transformed function is zero we pick up the random vector that return the lowest value of the function. Taking x_i as starting point for local optimizer we should be close enough to find the solution of the original system.

- Step V: we repeat the procedure until we obtain convergence of the local optimizer.

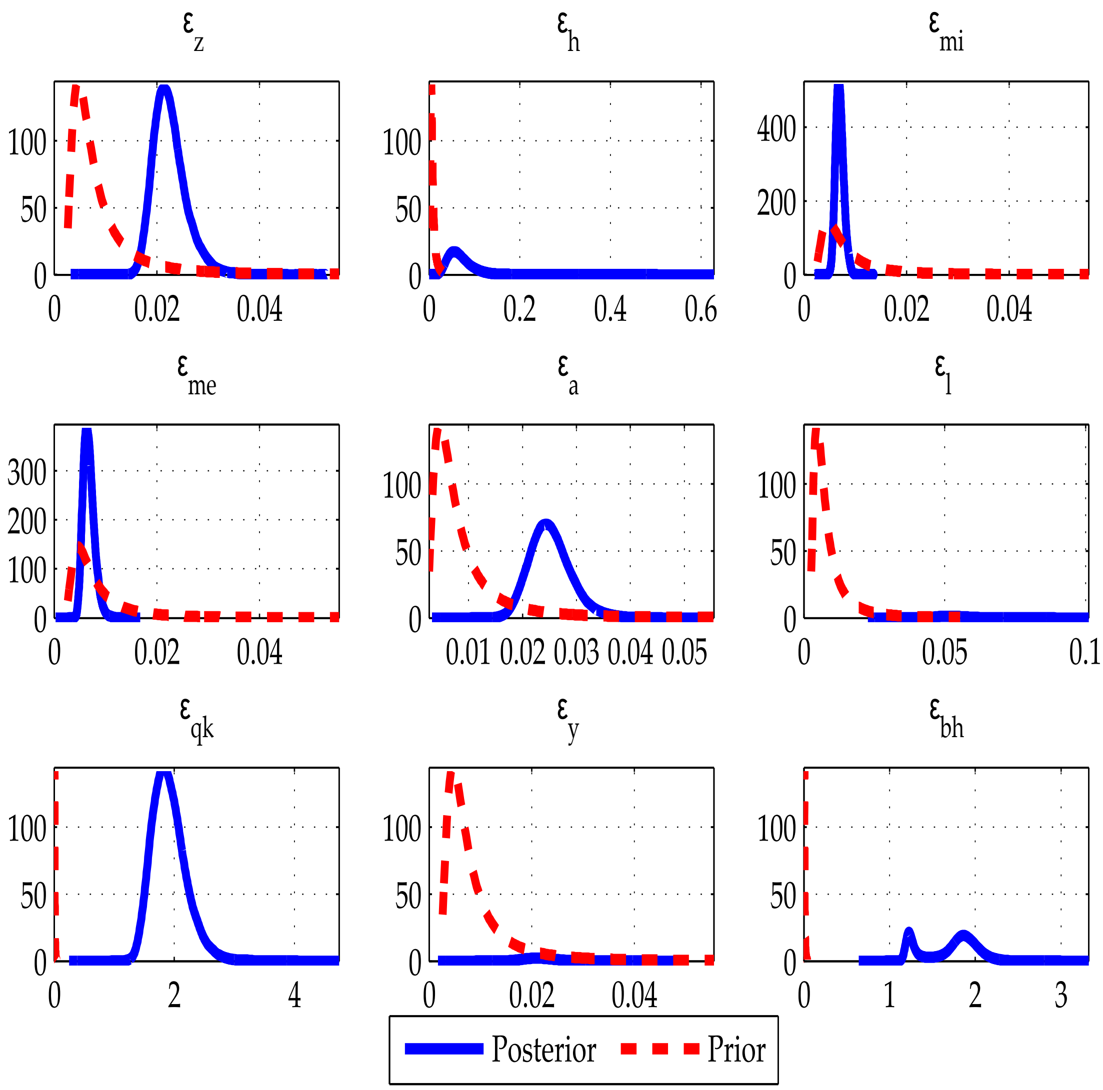
8.5 Appendix A.3

Table 6: RESULTS FROM BAYESIAN ESTIMATION

parameters	Prior shape	Prior mean	Post st. dev	Post mode	Post mean	Conf interval	
ρ_z	$\mathcal{B}$	0.700	0.1000	0.5296	0.5397	0.3970	0.6819
ρ_h	$\mathcal{B}$	0.700	0.1000	0.9814	0.9706	0.9517	0.9911
ρ_{mi}	$\mathcal{B}$	0.700	0.1000	0.9079	0.8986	0.8473	0.9517
ρ_{me}	$\mathcal{B}$	0.700	0.1000	0.9162	0.9064	0.8616	0.9528
ρ_{ae}	$\mathcal{B}$	0.900	0.0100	0.8472	0.8458	0.8243	0.8665
ρ_{el}	$\mathcal{B}$	0.700	0.1000	0.8246	0.7784	0.6808	0.8804
ρ_{qk}	$\mathcal{B}$	0.700	0.1000	0.4831	0.4922	0.3646	0.6190
ρ_{ey}	$\mathcal{B}$	0.700	0.1000	0.8764	0.8783	0.8470	0.9091
ρ_{bh}	$\mathcal{B}$	0.700	0.1000	0.8205	0.7695	0.6340	0.9038
ρ_{be}	$\mathcal{B}$	0.700	0.1000	0.7226	0.7413	0.5814	0.8994
ρ_d	$\mathcal{B}$	0.700	0.1000	0.7847	0.7415	0.6215	0.8605
ρ_G	$\mathcal{B}$	0.700	0.1000	0.5675	0.5743	0.4144	0.7292
ρ_{δ^d}	$\mathcal{B}$	0.700	0.1000	0.7739	0.7705	0.6749	0.8700
$\rho_{\delta^{db}}$	$\mathcal{B}$	0.700	0.1000	0.8308	0.7956	0.6887	0.9077
ρ_{kb}	$\mathcal{B}$	0.700	0.1000	0.7305	0.7256	0.6103	0.8434
σ_z	Γ^{-1}	0.010	0.0500	0.0202	0.0225	0.0175	0.0274
σ_h	Γ^{-1}	0.010	0.0500	0.0473	0.0719	0.0277	0.1116
σ_{mi}	Γ^{-1}	0.010	0.0500	0.0067	0.0070	0.0057	0.0082
σ_{me}	Γ^{-1}	0.010	0.0500	0.0061	0.0067	0.0050	0.0084
σ_{ae}	Γ^{-1}	0.010	0.0500	0.0559	0.0572	0.0476	0.0668
σ_l	Γ^{-1}	0.010	0.0500	1.7873	2.0096	1.4940	2.5070
σ_{qk}	Γ^{-1}	0.010	0.0500	0.0181	0.0192	0.0144	0.0237
σ_y	Γ^{-1}	0.010	0.0500	1.7176	1.7779	1.4694	2.0747
σ_{bh}	Γ^{-1}	0.010	0.0500	0.0529	0.0529	0.0027	0.0867
σ_{be}	Γ^{-1}	0.010	0.0500	0.0047	0.0356	0.0023	0.0808
σ_d	Γ^{-1}	0.010	0.0500	0.0653	0.0768	0.0544	0.0986
σ_{rt}	Γ^{-1}	0.010	0.0500	0.0015	0.0016	0.0013	0.0018
σ_G	Γ^{-1}	0.010	0.0500	0.0044	0.0045	0.0037	0.0054
σ_{δ_d}	Γ^{-1}	0.010	0.0500	0.0778	0.0832	0.0663	0.0990
$\sigma_{\delta_{db}}$	Γ^{-1}	0.010	0.0500	0.0802	0.0900	0.0675	0.1112
σ_{kb}	Γ^{-1}	0.010	0.0500	0.0420	0.0433	0.0356	0.0503

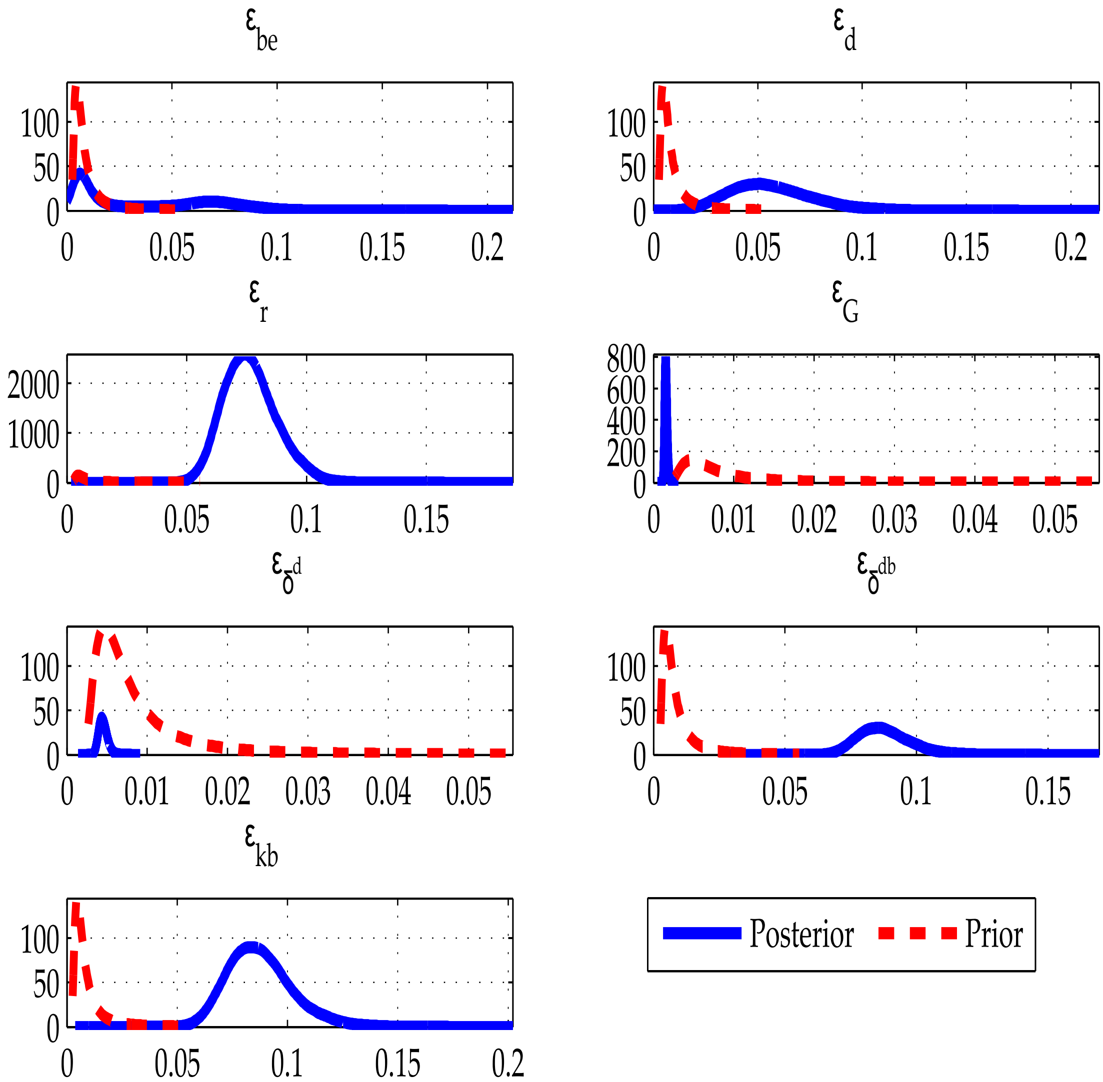
8.6 Appendix A.4

Figure 3: Prior and Posterior density



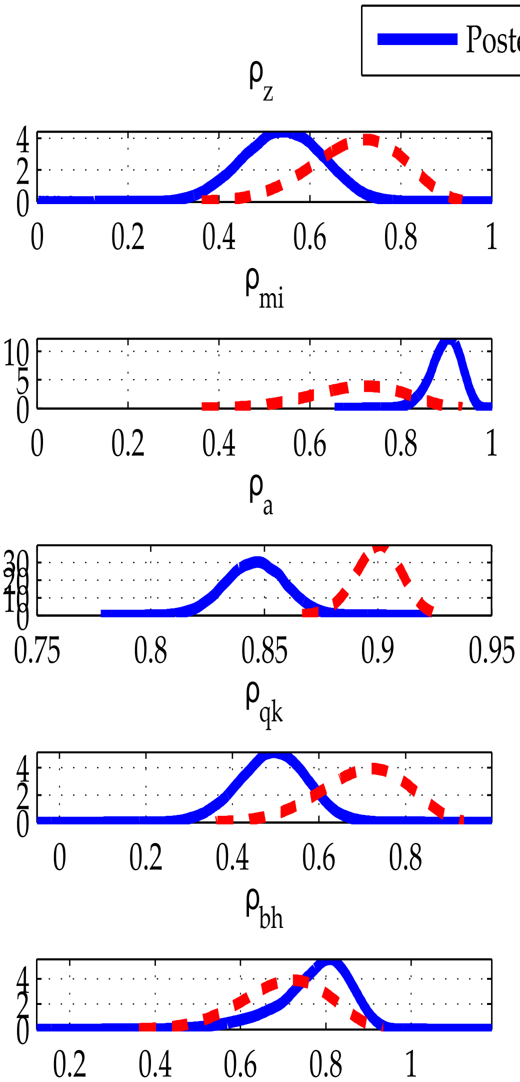
The red dot line represents the prior density, the solid blue line represents the posterior density.

Figure 4: Prior and Posterior density



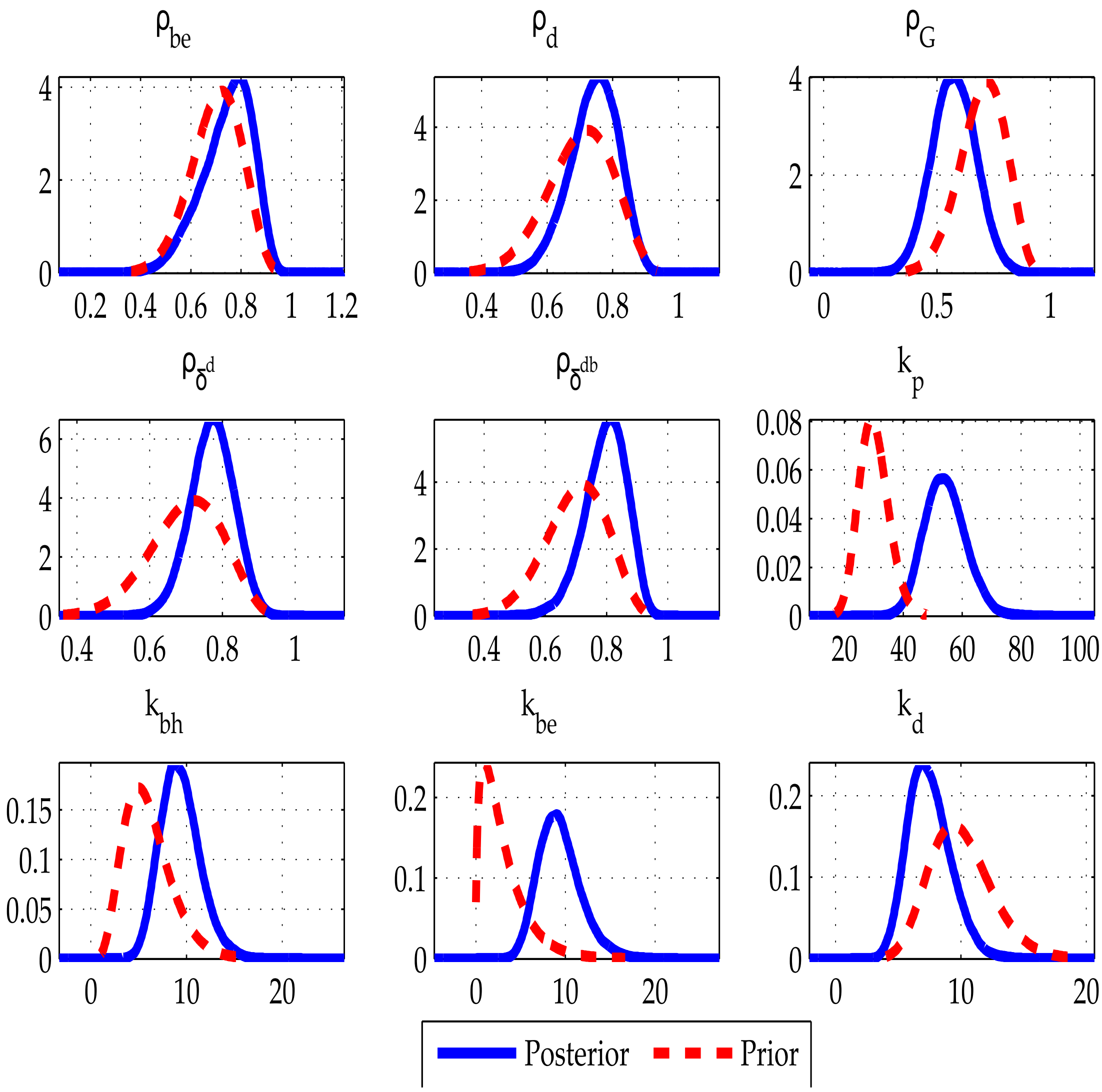
The red dot line represents the prior density, the solid blue line represents the posterior density.

Figure 5: Prior and Posterior density



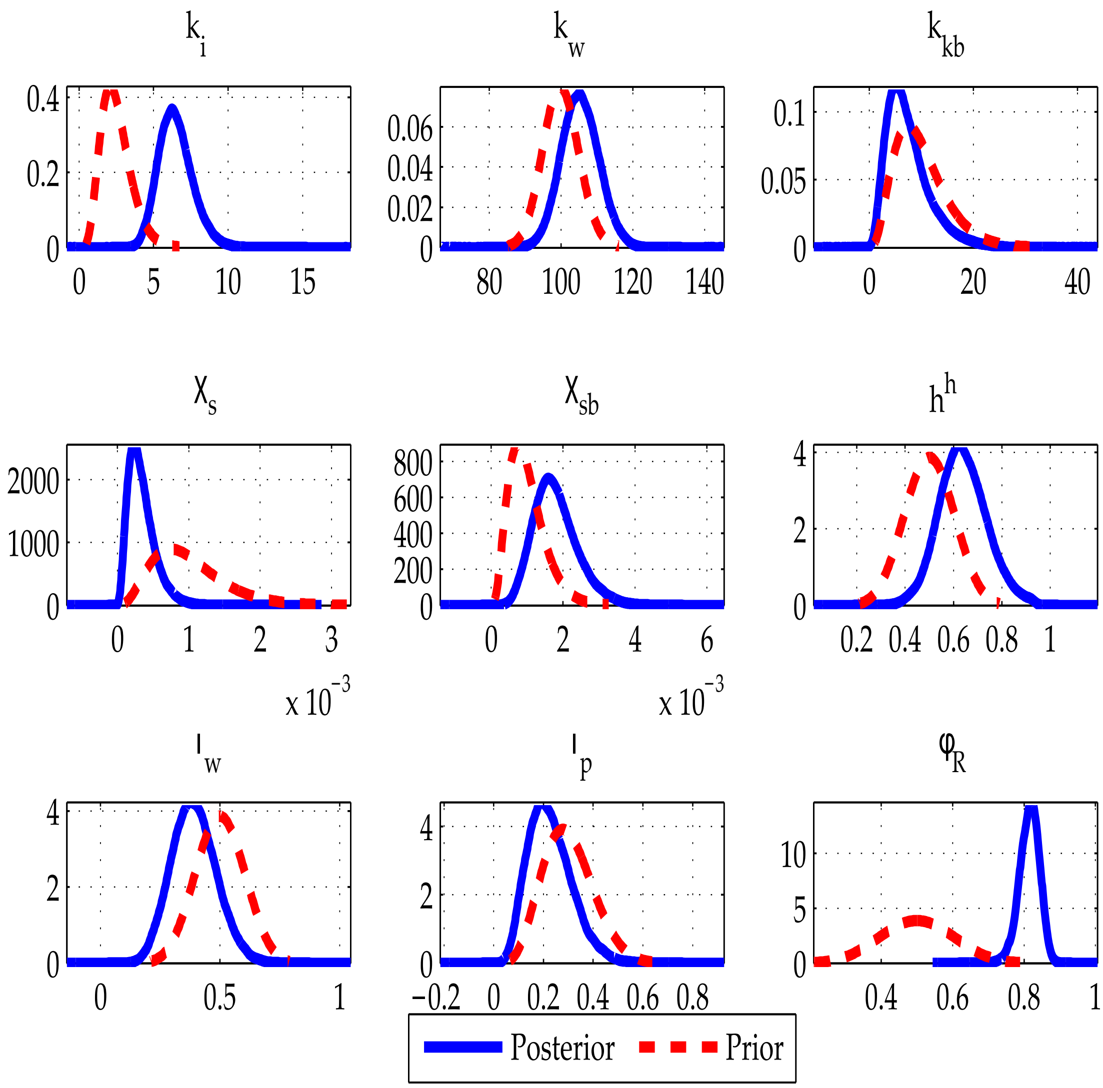
The red dot line represents the prior density, the solid blue line represents the posterior density.

Figure 6: Prior and Posterior density



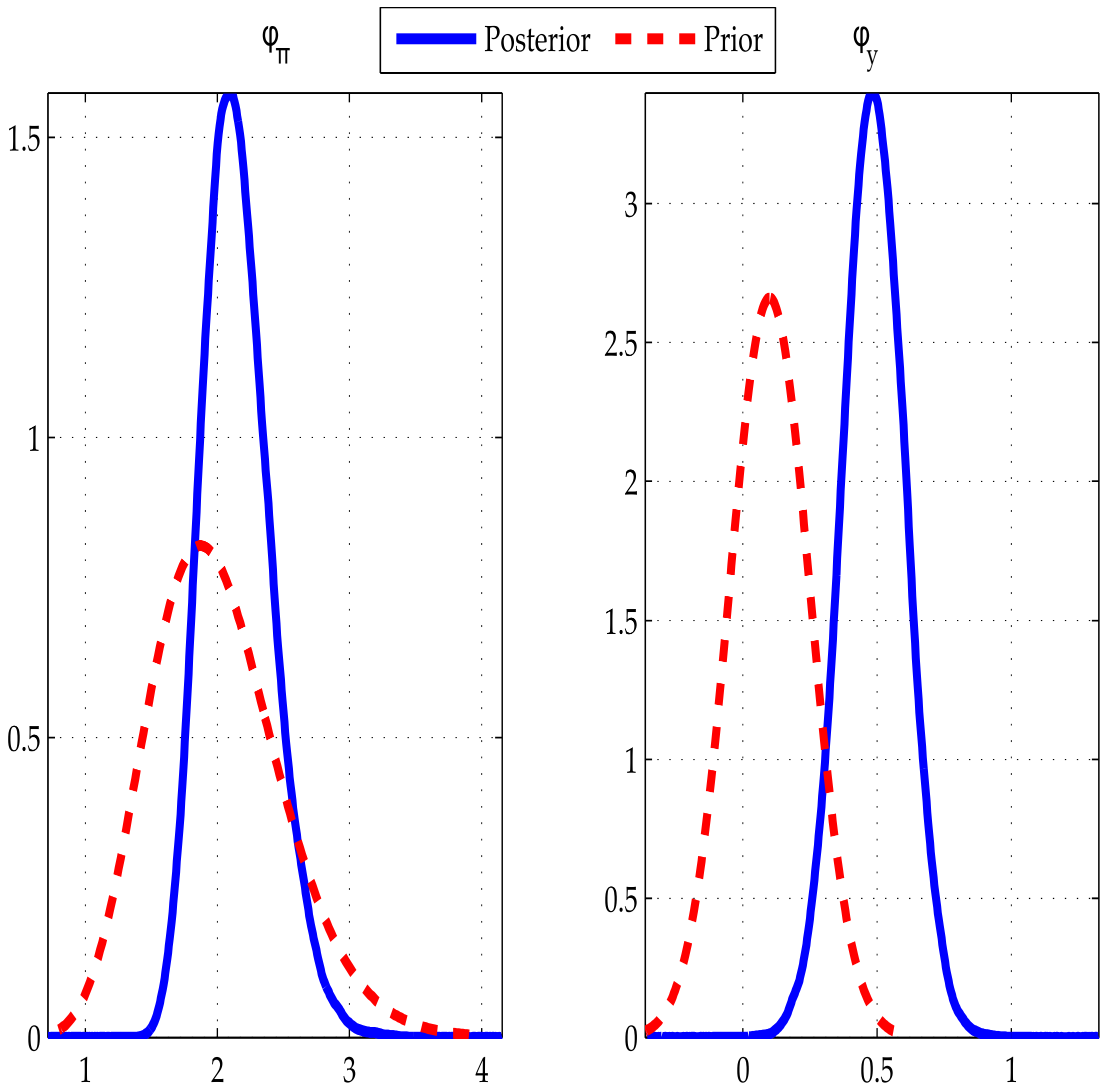
The red dot line represents the prior density, the solid blue line represents the posterior density.

Figure 7: Prior and Posterior density



The red dot line represents the prior density, the solid blue line represents the posterior density.

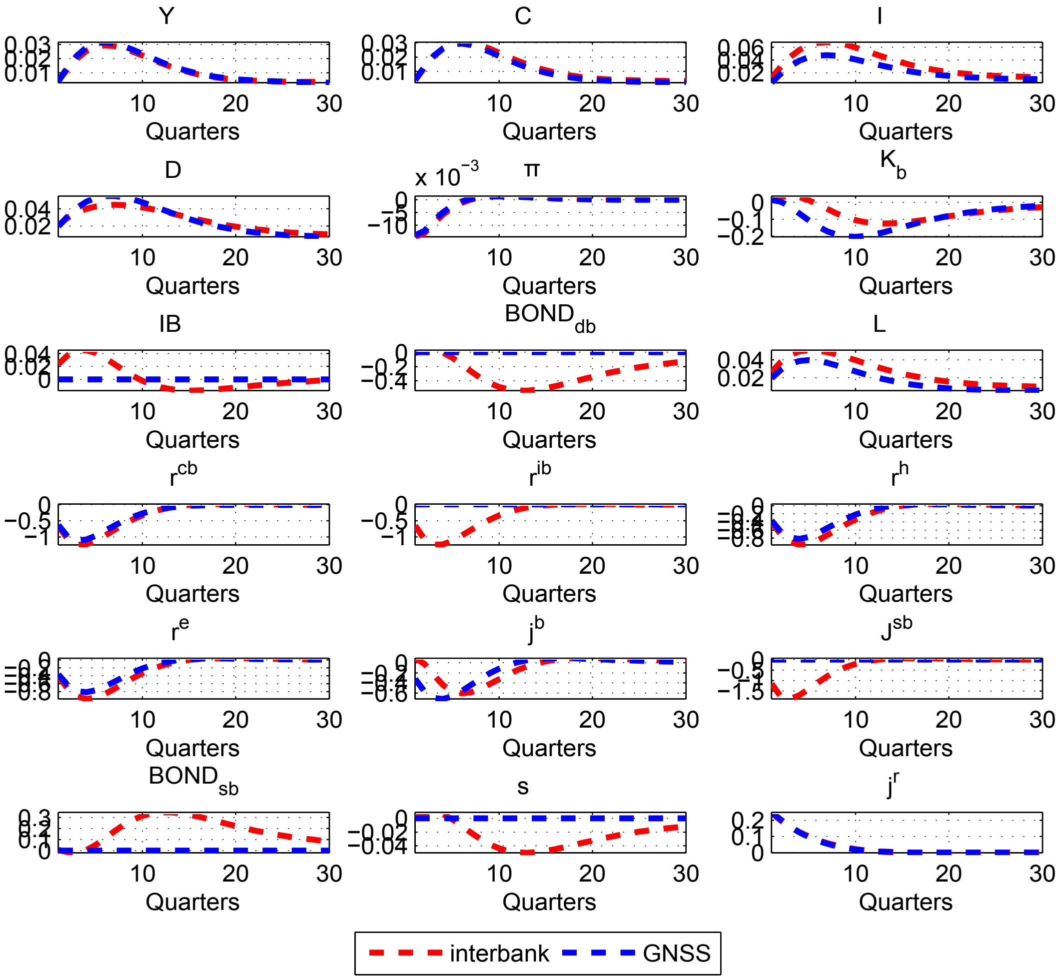
Figure 8: Prior and Posterior density



The red dot line represents the prior density, the solid blue line represents the posterior density.

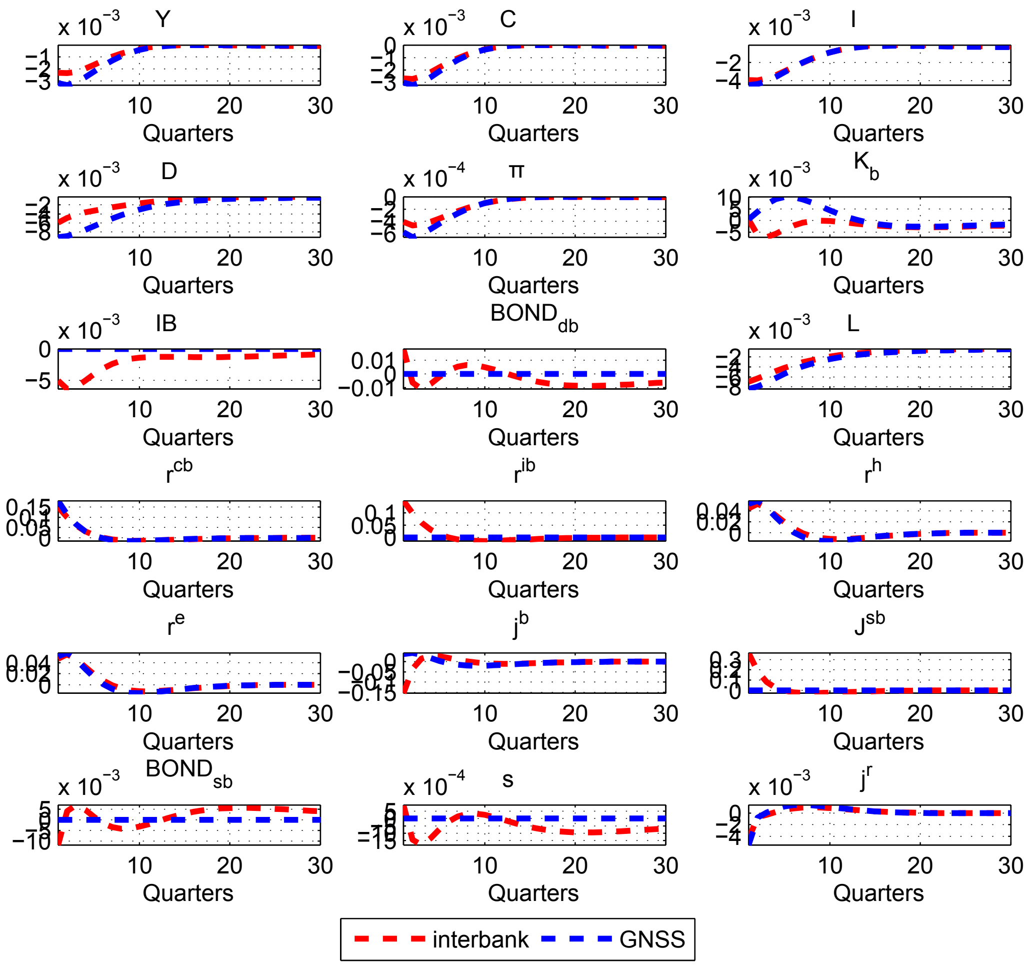
8.7 Appendix A.5

Figure 9: Response to technological shock



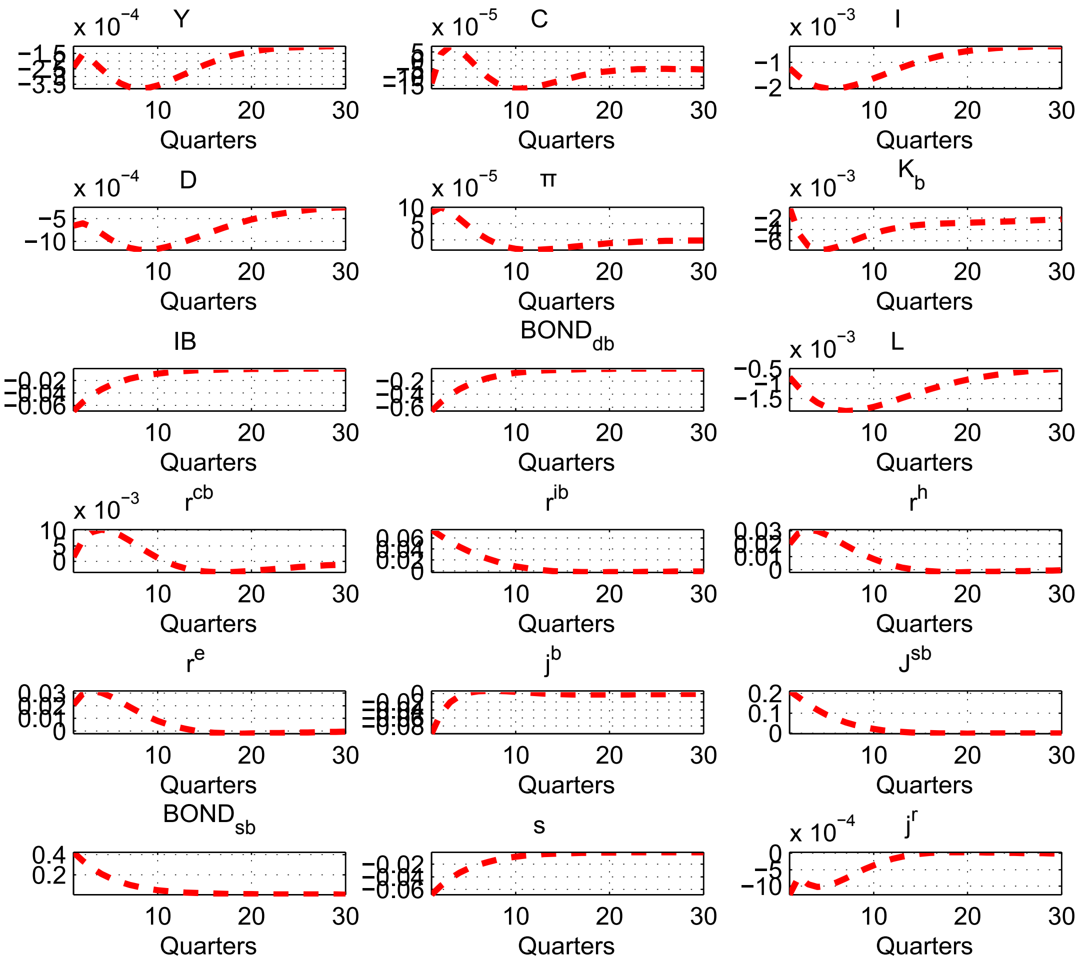
The dot red line represents the response to a monetary policy shock calculated at the posterior mean of the model with the interbank market. The dot blue line is the GNSS model calibrated at our estimated value.

Figure 10: Response to monetary policy shock



The dot red line represents the response to a monetary policy shock calculated at the posterior mean of the model with the interbank market. The dot blue line is the GNSS model calibrated at our estimated value.

Figure 11: Response to an interbank riskiness shock



The dot red line represents the response to an interbank riskiness shock calculated at the posterior mean.

8.8 Appendix A.6

Figure 12: Historical decomposition of loans to firms, loans to households, interest rate on entrepreneurial loans, interest rate on loans to households.

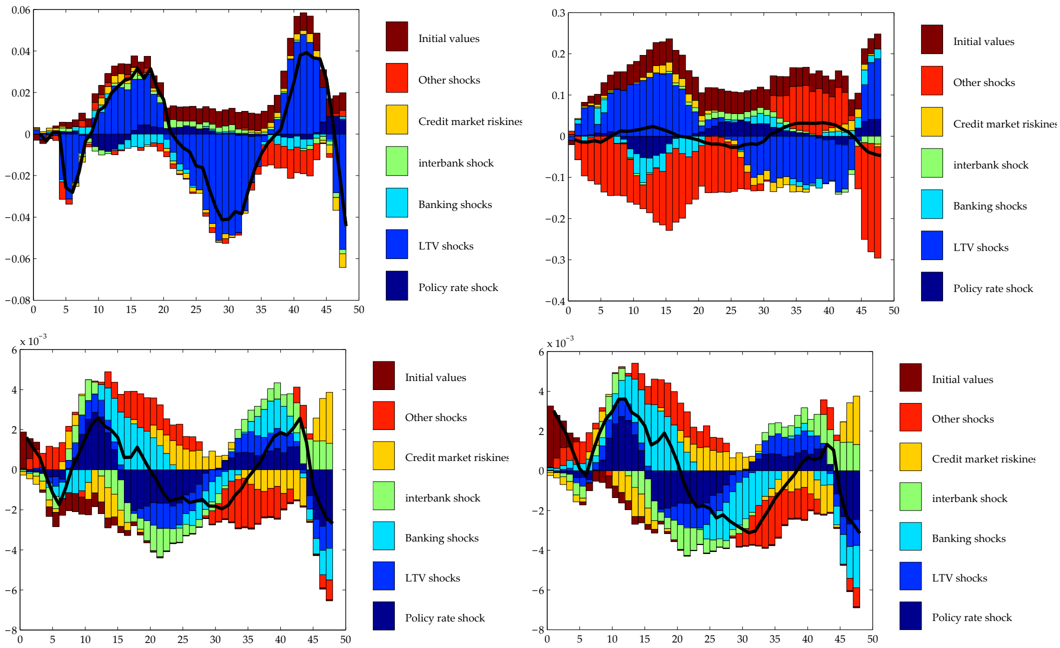
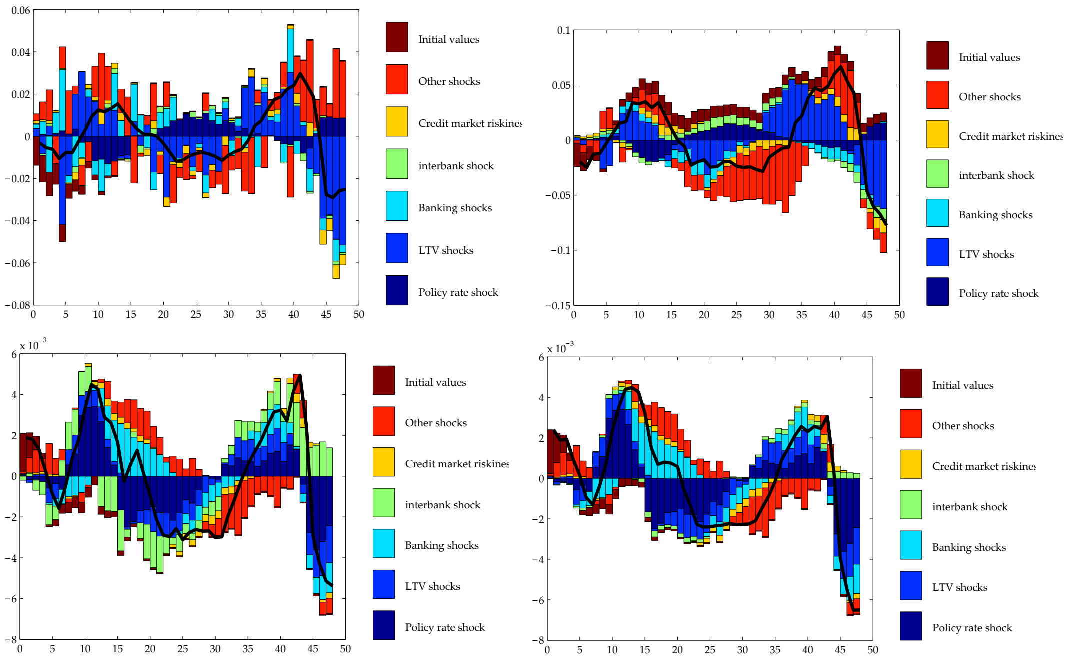


Figure 13: Historical decomposition of Output, Investment, Interbank interest rate, policy rate.



8.9 Appendix A.7

Figure 14: CUSUM convergence test

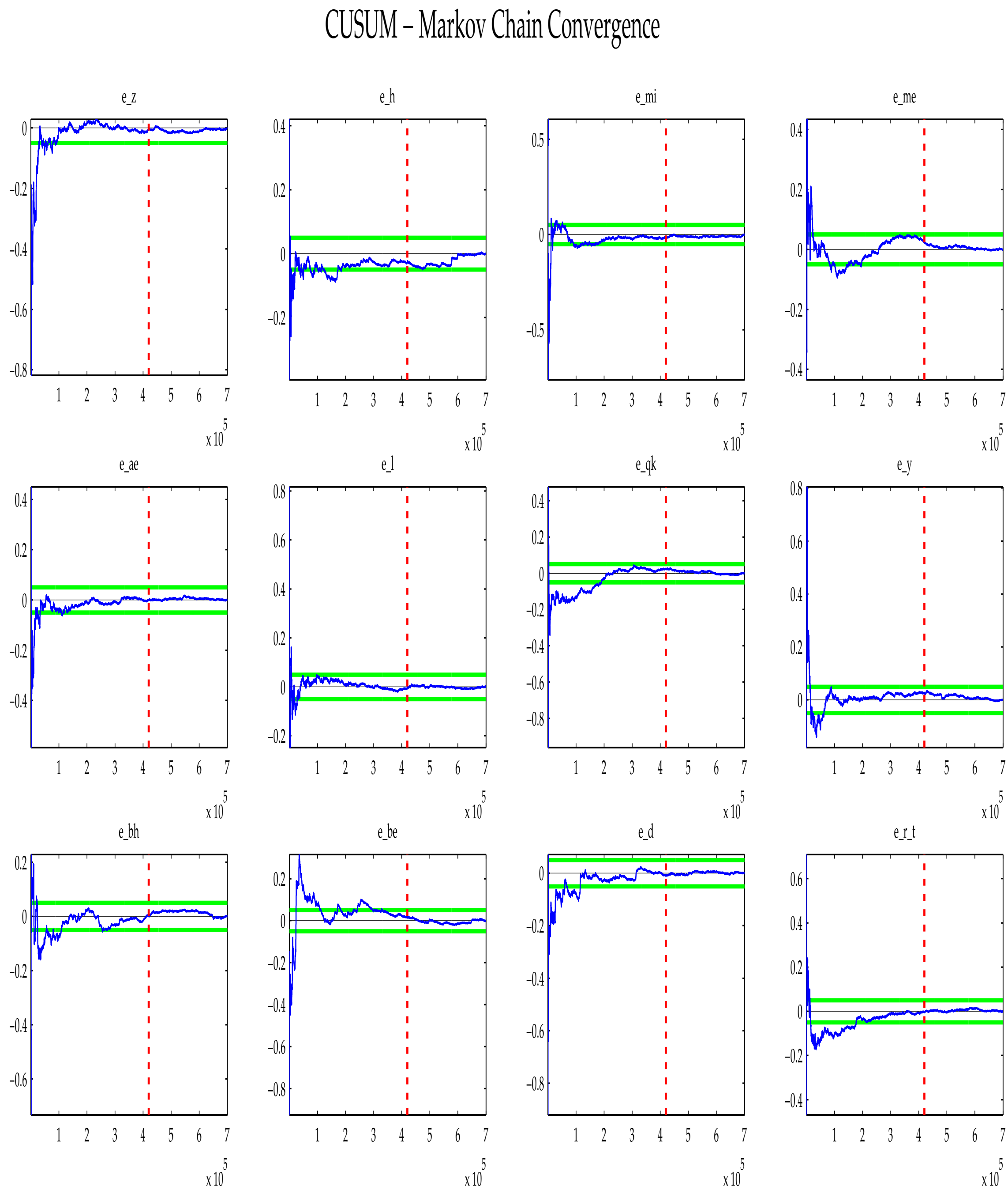


Figure 15: CUSUM convergence test

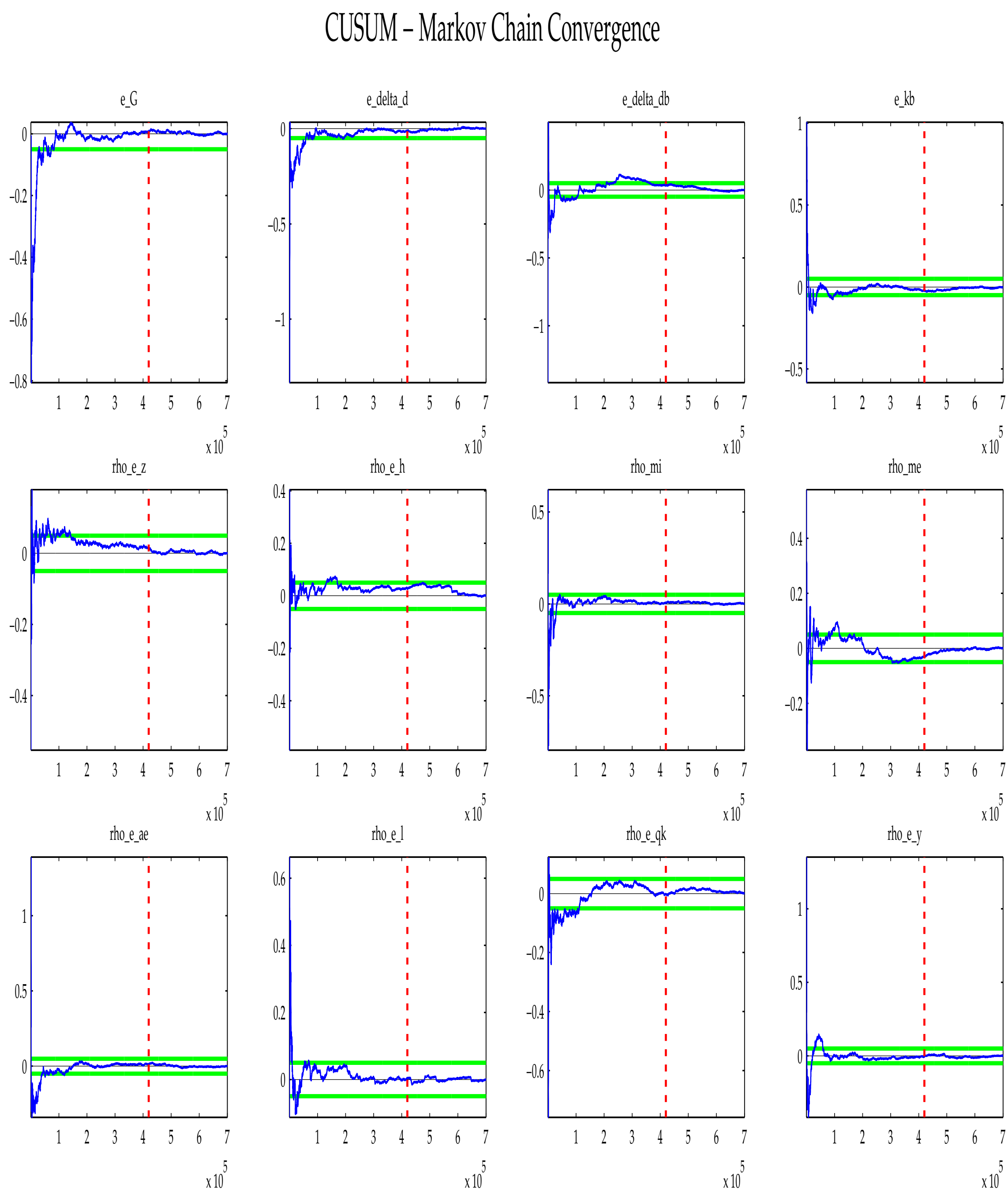


Figure 16: CUSUM convergence test

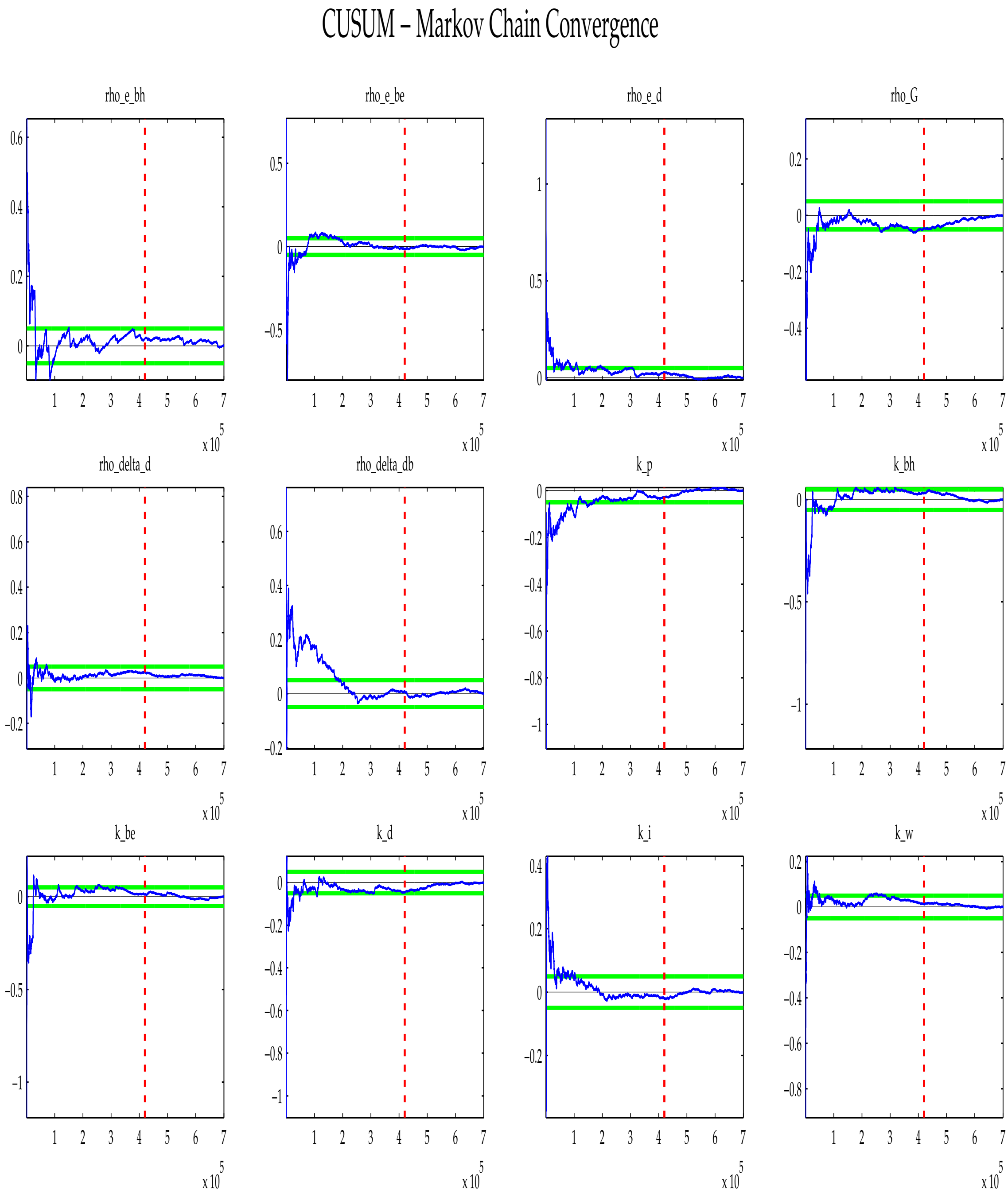
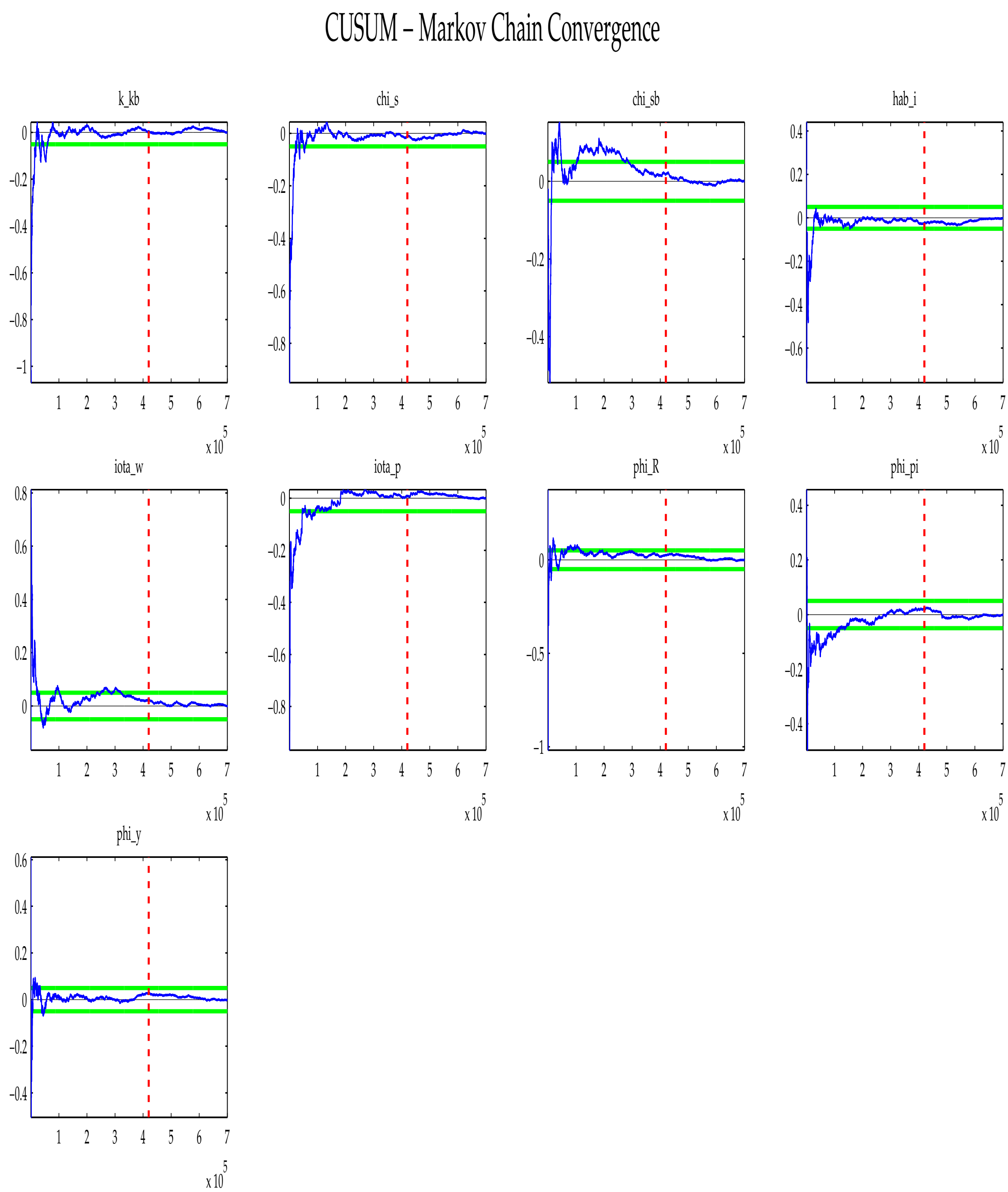
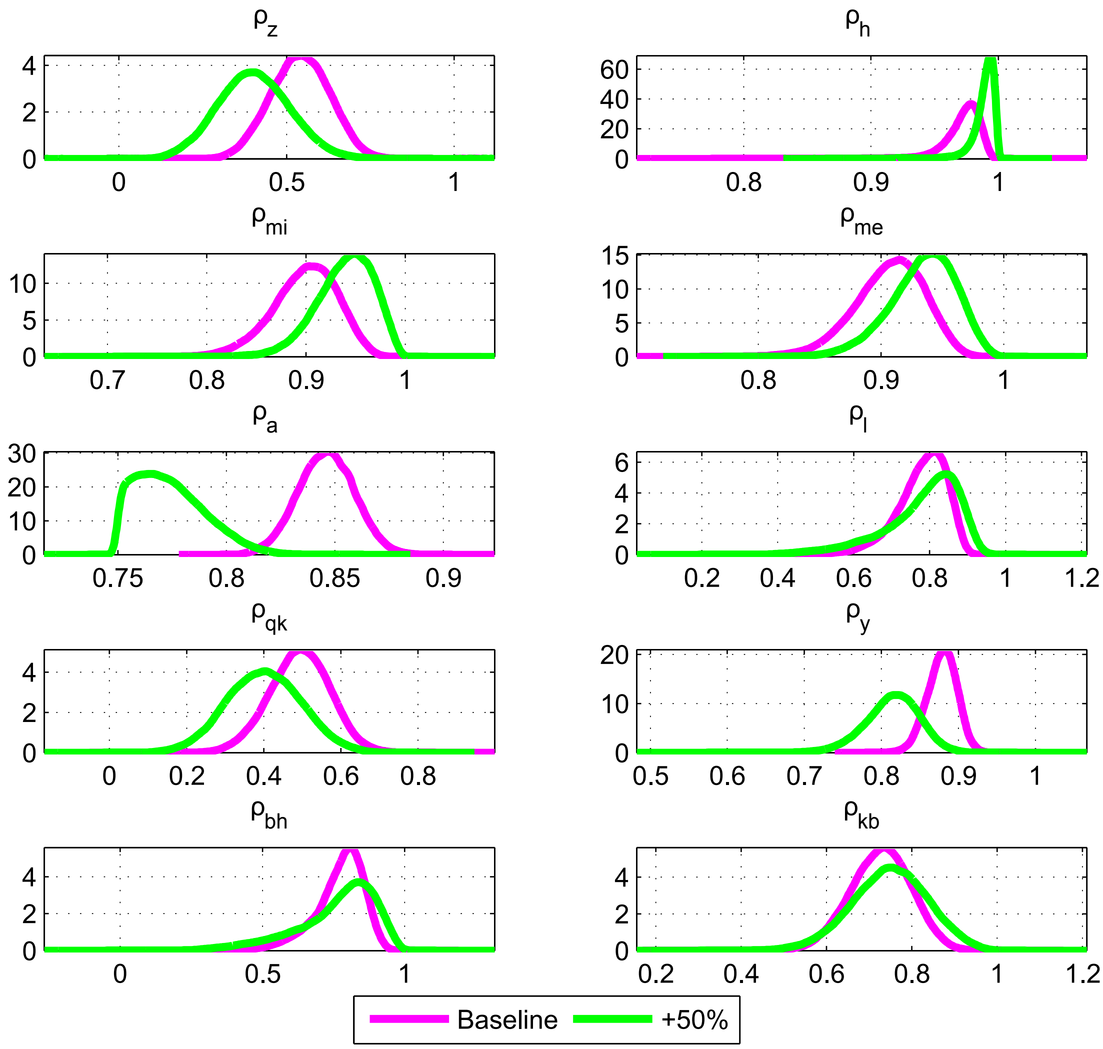


Figure 17: CUSUM convergence test



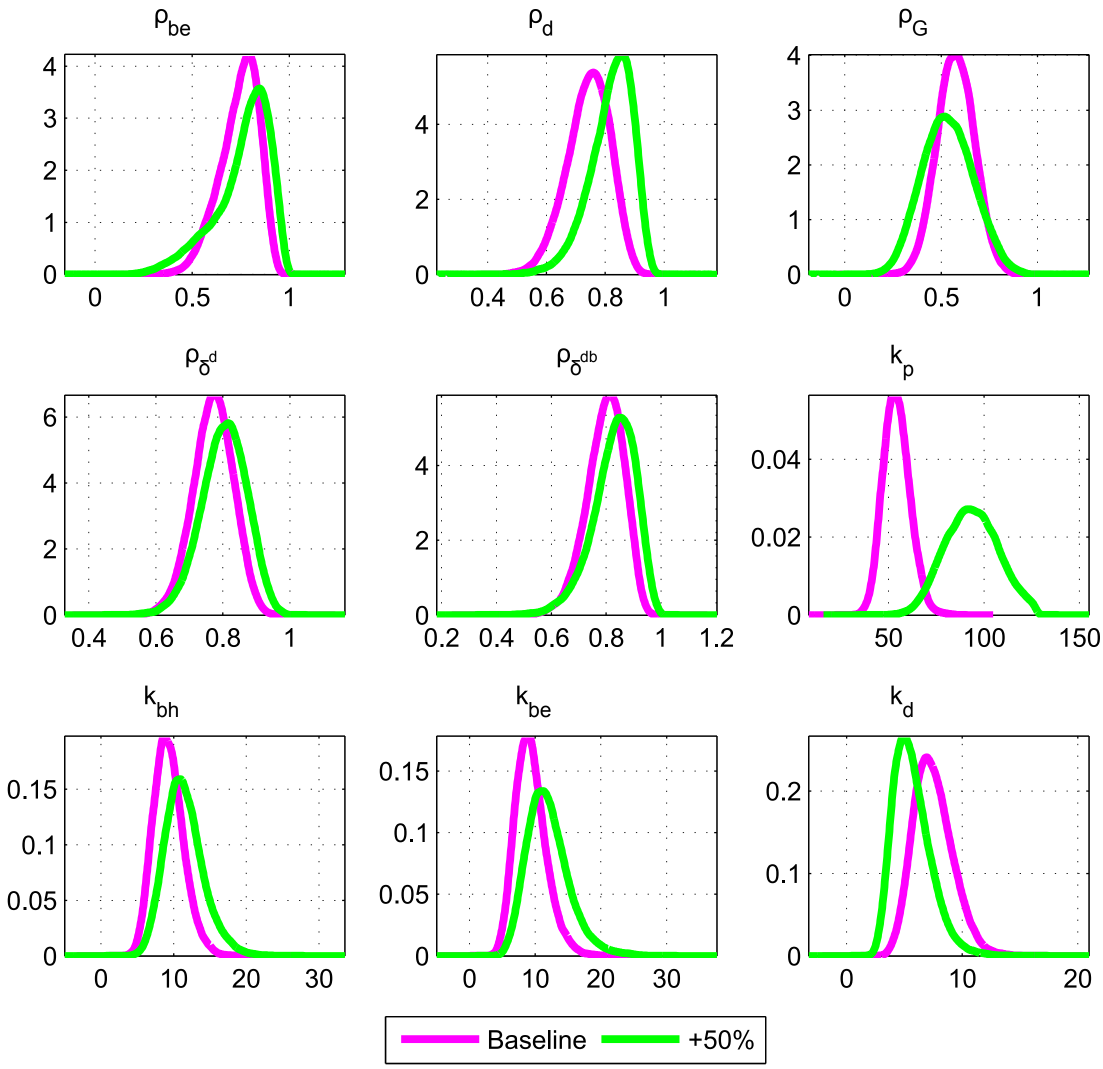
8.10 Appendix A.8

Figure 18: Sensitivity analysis



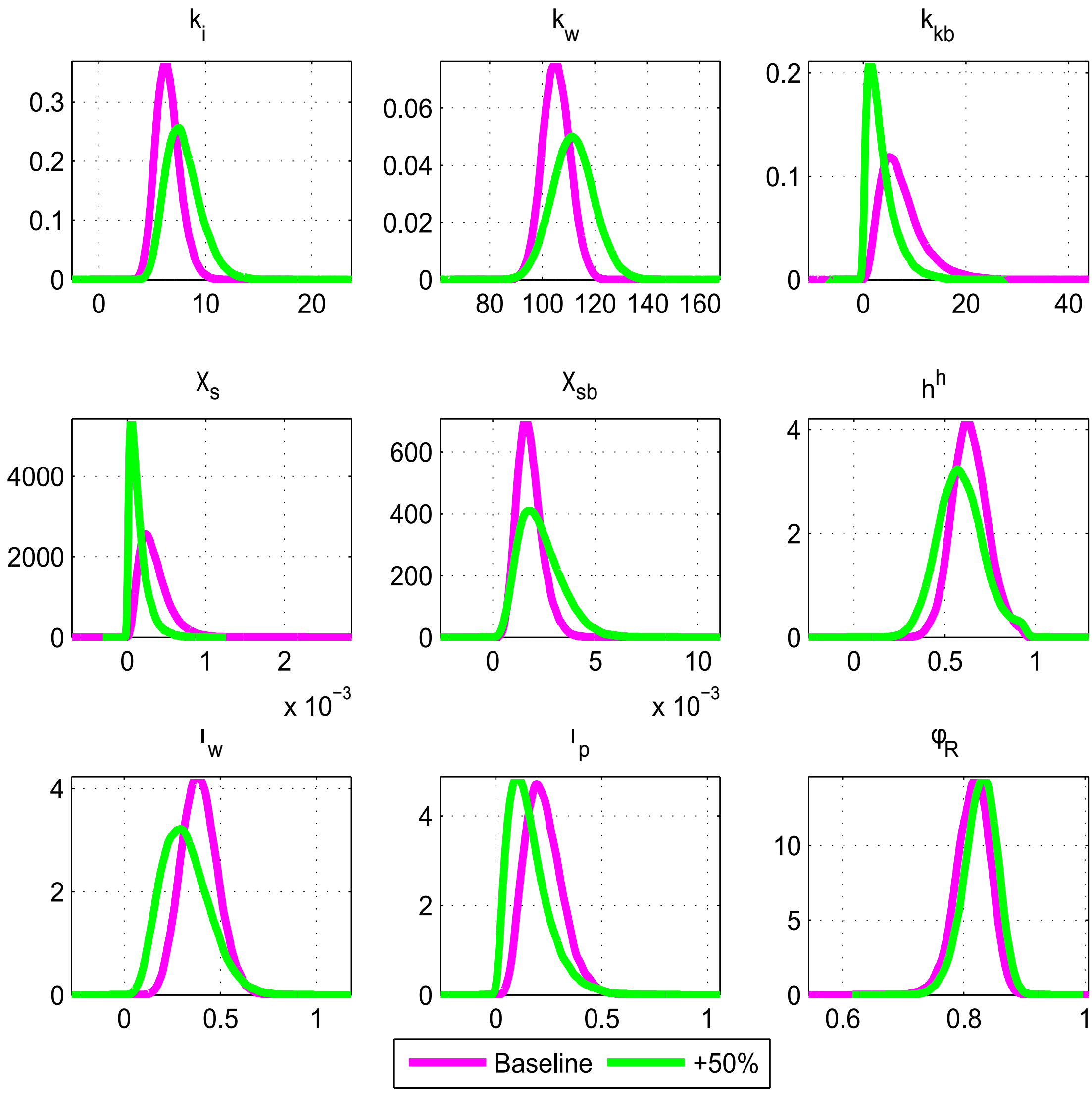
The purple line represents the baseline prior, the green line represents the estimation with 50% larger standard errors.

Figure 19: Sensitivity analysis



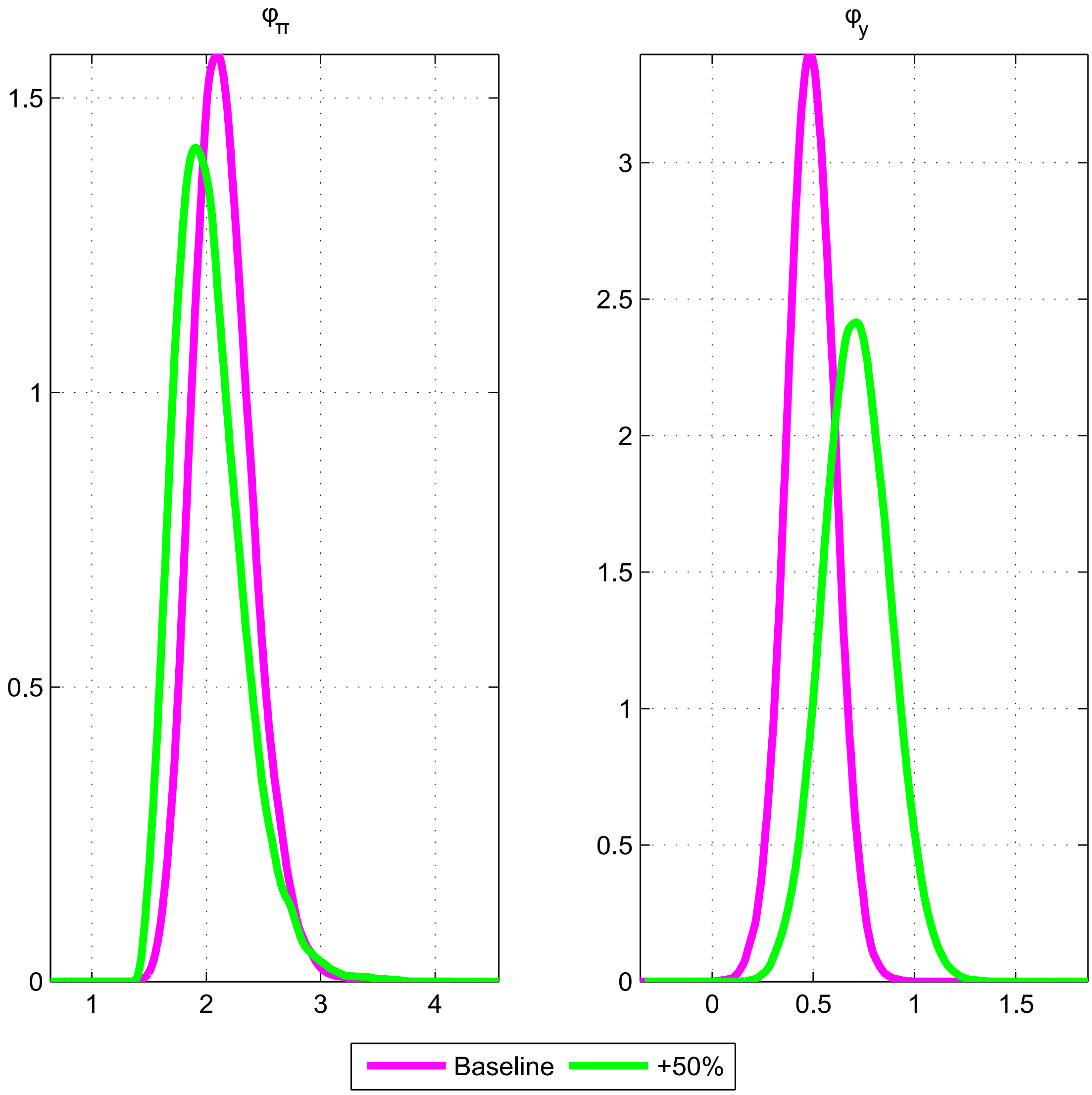
The purple line represents the baseline prior, the green line represents the estimation with 50% larger standard errors.

Figure 20: Sensitivity analysis



The purple line represents the baseline prior, the green line represents the estimation with 50% larger standard errors.

Figure 21: Sensitivity analysis



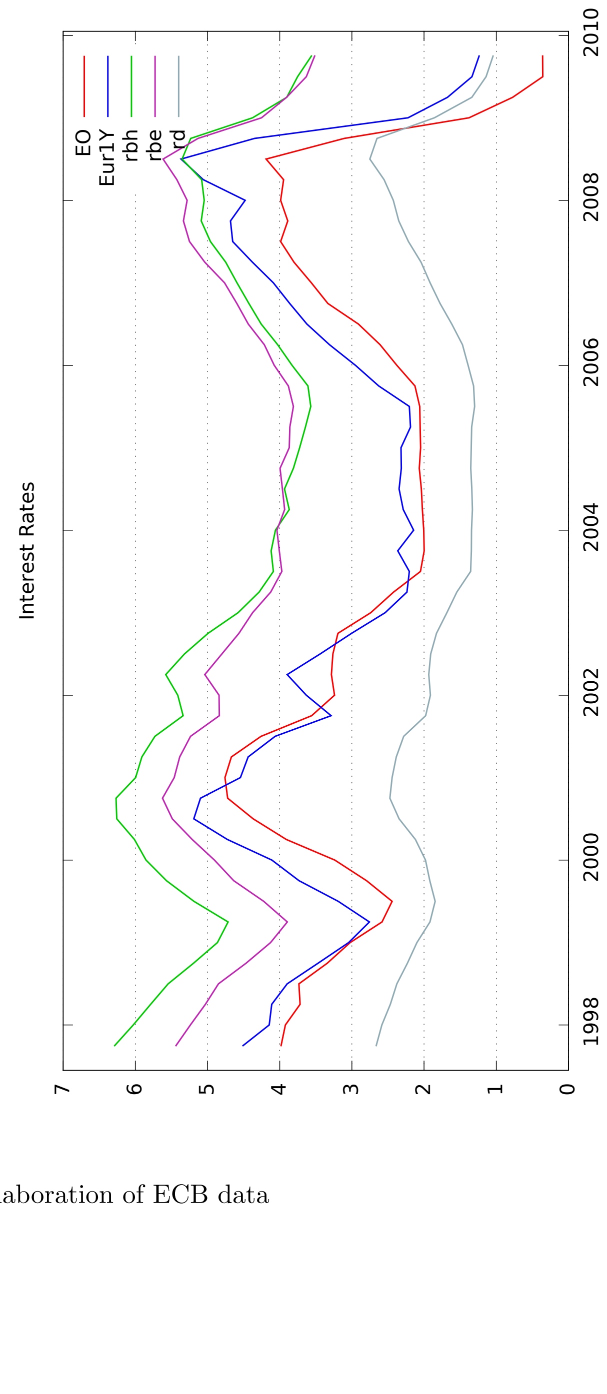
The purple line represents the baseline prior, the green line represents the estimation with 50% larger standard errors.

8.11 Appendix A.9

Table 7: Implied steady state values of the interest rate model and data

Variables	Steady state values	Data
$\bar{r}^{ib}$	3.32	3.42
$\bar{r}$	3.31	3.00
$\bar{r}^{bh}$	4.70	4.84
$\bar{r}^{be}$	4.61	4.60
$\bar{r}^{d}$	2.29	1.90

Figure 22: Interest rate from 1997:4 to 2009:4



Source: Authors elaboration of ECB data