UNIVERSITÀ POLITECNICA DELLE MARCHE

DIPARTIMENTO DI ECONOMIA

A COPULA-GARCH MODEL FOR MACRO ASSET ALLOCATION OF A PORTFOLIO WITH COMMODITIES: AN OUT-OF-SAMPLE ANALYSIS

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QUADERNO DI RICERCA n.355

Gennaio 2011

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Abstract

Many authors have suggested that the mean-variance criterion, conceived by Markowitz (1952), is not optimal for asset allocation, because the investor expected utility function is better proxied by a function that uses higher moments and because returns are distributed in a non-Normal way, being asymmetric and/or leptokurtic, so the mean-variance criterion can not correctly proxy the expected utility with non-Normal returns. In Riccetti (2010) I apply a simple GARCH-copula model and I find that copulas are not useful for choosing among stock indices, but they can be useful in a macro asset allocation model, that is, for choosing the stock and the bond composition of portfolios. In this paper I apply that GARCH-copula model for the macro asset allocation of portfolios containing a commodity component.

I find that the copula model appears useful and better than the mean-variance one for the macro asset allocation also in presence of a commodity index, even if it is not better than GARCH models on independent univariate series, probably because of the low correlation of the commodity index returns to the stock, the bond and the exchange rate returns.

Keywords: Portfolio Choice.
JEL classification codes: C52, C53, C58, G11, G17.

I am very grateful to Dean Fantazzini, Riccardo Lucchetti and Giulio Palomba for helpful comments and suggestions.

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1 Introduction

In Riccetti (2010) I find that the use of a GARCH-copula model can be useful for the macro asset allocation (choosing the stock and the bond composition of portfolios), while it is not useful in the choice of the weights for a portfolio composed by many stock indices. A possible driving factor of the performance of the copula model is the correlation value between the assets: the copula model appears to be useful in the case of a portfolio containing one asset with small absolute values of correlation, while this model is dangerous with highly correlated assets.
In this paper I want to check if that good results of the copula model is confirmed in a portfolio that contains a commodity index, that is uncorrelated with the stock index, the bond index and the exchange rate returns time series. Besides I extend the analysis in two sides:

- analysing also portfolios composed by three and four assets that are all uncorrelated;
- adding the comparison of the copula model to a model that implies independent assets.

Moreover, this is a relevant topic for real asset management, indeed in the last years the investments in commodities are increasing, also thanks to the recent creation of a lot of financial vehicles (exchange traded funds or mutual funds), that make the investment in commodities possible for retail investors too.

The paper proceeds as follows. In this section I briefly review some papers about the use of commodities and of copulas in the asset allocation context, especially with a review of the book of Riccetti (2010), that is the base for the following analysis; section 2 presents the model; section 3 reports the results and section 4 concludes.

1.1 Use of commodities

Authors as Irwin and Landa (1987), Froot (1995), Abanomey and Mathur (1999), Jensen *et al.* (2000), Jensen *et al.* (2002), Gorton and Rouwenhorst (2006), Dempster and Artigas (2010) or Conover *et al.* (2010) found that commodities offer sizeable diversification benefits in the portfolio allocation, especially in equity portfolios, because their returns are uncorrelated to the stock returns[1].
A lot of papers, as Bodie (1983), Halpern and Warsager (1998), Gorton and Rouwenhorst (2006), Amenc *et al.* (2009) or Dempster and Artigas (2010), explain that the low correlation appears to be driven by the returns of the commodities in periods of high inflation (expected

[1]Moreover, there is debate on the utility of commodity during extreme events: Chong and Miffre (2010) finds that the correlations between the S&P500 index and several commodities fell in periods of above-average volatility in equity markets, while Büyüksahin *et al.* (2010) finds that correlations between equity and commodity returns increased sharply in the fall of 2008, so diversification benefits are weaker when they are needed most. On this debate, see also appendix B. Copula models could catch the extreme outcome features.

or not): in these periods, while stock or bond portfolios are negatively affected, commodities provide a hedge against inflation, thus the diversification benefit is coupled by a return benefit. These papers often highlight also that high inflation periods coincide with periods in which the central banks apply a restrictive monetary policy and so, as in Jensen *et al.* (2000) or Conover *et al.* (2010), they show that the allocation can be improved using the information given by the monetary policy (that is to invest more in commodities during restrictive monetary phases and vice versa).
The growing interest in commodities makes the financial engineering to develope investment vehicles to give the opportunity to retail investors to allocate part of their portfolios in commodities without using futures. Indeed, in recent years a lot of exchange traded funds (ETFs) and mutual funds have been introduced, as shown by papers as Mazzilli and Maister (2006), Anderson (2008) or Conover *et al.* (2010). Moreover, authors as Amenc *et al.* (2009) or Dempster and Artigas (2010), point out that today the financial crise makes central banks to lower interest rates and to pump a lot of money into the economy, so there is the risk of a rise in inflation that could be hedged by commodities that are likely to outperform stock and bond returns in these periods. This is the reason for the increasing interest in commodities.

1.2 Use of copulas

The Markowitz model is optimal if investors only care about mean and variance or if returns are distributed as a Normal.
In fact, on one side, financial returns are not Normally distributed, as observed since 1963 with two papers of Mandelbrot and Fama.
On the other side, a lot of authors as Arditti (1967), Kraus and Litzenberger (1976), Simkowitz and Beedles (1978) found, for example, that people prefer high values for skewness. Scott and Horvath (1980) analytically proves that investors prefer returns with higher skewness and lower kurtosis. This proof is also repeated for higher moments and people always prefer higher values for odd moments and lower values for even moments.
Authors as Harvey and Siddique (2000) or Dittmar (2002) prove that higher moments improve the allocation if returns are distributed in a very different way from the Normal.
Copulas are a very useful tool to deal with non standard multivariate distribution. Sklar (1959) shows that all multivariate distribution fuctions contain copulas and that copulas may be used in conjunction with univariate distribution fuctions to construct multivariate distribution functions. Indeed, every joint distribution fuction contains a description of the marginal behaviour of the individual factors, given by the univariate distribution, and a description of their dependence structure, given by the copula. Copulas isolate the description of the dependence and, describing it on a quantile scale, help to understand dependence at a deeper level than correlation. In this way they can describe the extreme outcomes and allow the building of a multivariate model, that combines more developed marginal models with

the variety of dependence models. An overview on copulas is in appendix A, but for a deeper introduction see Embrechts *et al.* (2005) or Joe (1997).
Over recent years, many authors have applied copulas to financial research. Much literature exists on the use of copulas for the computation of VaR in risk management and very advanced papers have been published in this field. However, there are also papers that use copulas to model stock market returns without a direct application to risk management, as in Jondeau and Rockinger (2006a), where the authors investigate the interactions between four major stock indices (S&P500, FTSE100, DAX, CAC) and they find that the Student-t copula is more accurate than the Gaussian one, consistent with the idea that dependence is stronger in the tails.
Patton (2004) is the first work that builds an asset allocation with copulas. Patton tries some models with different copulas and he finds that the best model for the investor's utility uses the Skewed-t marginal distribution and the rotated Gumbel copula and this model gains more utility on the others if the investor is very risk averse.
In Riccetti (2010) I try to develope the analysis published by Patton (2004). I use models with different copulas (Normal, Student-t, Clayton, Gumbel, Frank, mix copulas and Canonical Vine copulas) in an allocation on two, three and four assets, analyzing various combinations of indices, to test whether the use of copulas improves the investor's utility and revenue. From those analyses, I conclude that the best copula model is the one that uses the Student-t copula, but the copula model is better than the naive allocations and than the mean-variance model only for deciding the macro-asset allocation (for choosing the stock and the bond composition of the portfolios) or when the portfolio is composed by one bond index and more stock indices, especially when the investor is very risk averse. Instead, the copula model is not useful neither in the choice between some different stock indices in portfolios composed only by equity, nor in the choice of the weights of portfolios composed by two stock indices and two bond indices. I argue that a possible driving factor of the performance of the copula model is the correlation values between the assets: when there is one index that has small absolute values of correlation to all the other indices, then the copula model appears to be useful. Besides, I find that, when the number of assets increases, the standard copula model shows increasing problems and the use of Vine copulas can reduce these problems, even if they are still present.

2 Data and model

To build the model I make some simplifying assumptions. I assume that the investor does not face any transaction costs, so there are no costs for rebalancing or for short-selling. Moreover, I take savings decisions exogenously specified and ignore intermediate consumption: the investor/asset manager has to invest an amount of money at the beginning of the period (that, for simplicity, I take as equal to 1) and does not receive or disinvest anything until the end[2]. The allocation is done for three years left out-of-sample, with monthly rebalancing, so there are 36 rebalancing every 22 working days (I assume that a month has 22 working days).

The assets used for the analysis are four: a commodity index (the New York CRB), a stock index (the Dow Jones Industrial), a bond index (the Merrill Lynch US Treasury 1-10 years) and the exchange rate between US Dollar and Euro. I use daily log returns for all assets. Different to Patton, who uses monthly returns, daily returns are used in the present analysis, but I aim to obtain monthly allocations. In this way, I cover quite a long time period without rebalancing (and this reduces the problem of not considering rebalancing costs), but I use more information compared to few monthly returns[3]. However, to do this, I need to simulate a path of 22 returns to allocate the portfolio for each out-of-sample month, as I will explain after.

The data are obtained from Datastream for the period that goes from the 1^{st} of January 1996 to the 31^{st} August 2010 (and the descriptive statistics are in table 1), skipping U.S. holidays, but the model is chosen on data from January 1996 to July 2007, because the last three years (22 days for 36 months, thus 792 observations) are left for the out-of-sample analysis. The

Table 1: *Descriptive statistics of the indices used. Full data range: 1996/01/01 - 2010/08/31. NYCRB = New York CRB index, DJ = Dow Jones Industrial index, MLUSTR1-10 = Merrill Lynch US Treasury 1-10 years, US$-EURO = exchange rate US Dollar-Euro.*

Index	Mean (Daily %)	Standard Deviation	Skewness	Excess Kurtosis
NYCRB	0.019	0.824	-0.283	4.345
DJ	0.018	1.245	-0.104	7.223
MLUSTR1-10	0.022	0.199	-0.137	3.897
US$-EURO	-0.001	0.623	0.216	2.880

choice of the model consists in the choice of the copula and it is done according to the best value of the maximum likelihood, together with the analysis of contour and scatter plots. I also check the chosen copula with goodness-of-fit (GOF) measures, performed using the second approach reported in Berg (2008) and suggested by Genest and Remillard (2008). This goodness-of-fit test is one of the best, as reported by power studies such as Berg (2008) or Genest *et al.* (2009). However, an investor or an asset manager that has to allocate the portfolio, has to do it even if none of the proposed models are very appropriate; thus, I allocate these portfolios with the models that show the best log-likelihood, even if they do not appear to be appropriate from the GOF tests and/or other lines of evidence.

I try the following kind of copulas: Clayton, Gumbel, Frank, Normal, Student-t and Canonical Vine copula. These copulas possess different tail dependences (for more details see A.2): the

[2]As remarked by Ang and Bekaert (2002), the use of a CRRA utility does not address market equilibrium, so the investor does not have to be necessarily the representative agent.

[3]Similar to Morana (2009), who compute monthly realized betas from daily data.

Gumbel copula catches the positive tail dependence, the Clayton copula catches the negative one, the Student-t copula implies both tail dependences but symmetrically, while the Normal and the Frank copulas do not have any tail dependence. These copulas are used for the bivariate case and can be generalized to the multivariate case, but in the multivariate case they imply the strong restriction that the dependence is the same across all pairs of variables. To overcome this problem, in Riccetti (2010), in the allocation of a four assets portfolio, I find helpful to use the Canonical Vine copula, a more flexible construction that models multivariate dependence with a cascade of bivariate copulas (see appendix A.3). Thus, also here, I use the Canonical Vine copula in the case of a portfolio with four indices.

The model is structured in the following way:

- means are unconditional. I also try to model the returns using ARMA processes, but these models are not theoretically justifiable and, as often happens in these cases, they are effective in-sample, but not useful for the out-of-sample forecast;
- variances are explained by GARCH(1,1), using error terms extracted from a Skewed-t distribution. The choice of GARCH(1,1) is due to the fact that, in practice, it is the most used GARCH model; moreover, it is often better than other (more complex) GARCH models for the out-of-sample forecast. The error terms is extracted from a Skewed-t distribution, as done by Patton (2004) or by Jondeau and Rockinger (2003), because all GARCH residuals present negative skewness and excess kurtosis. However, differently from the above cited papers, skewness and kurtosis parameters are non conditional;
- the joint behaviour of the residuals of the indices is modelled by the copula that presents the best log-likelihood.

The steps to build the optimal portfolio are:

1. estimate with the Maximum Likelihood the parameters of the above model;
2. simulate the path of assets returns on the investment horizon (22 days) 5000 times;
3. choose the optimal weights in order to maximize the investor's expected utility, using the CRRA utility function as done, for instance, in Patton (2004):

$$U(\gamma) = (1-\gamma)^{-1}(P_0 R_{port})^{1-\gamma} \tag{1}$$

with γ that is the risk aversion parameter, R_{port} that is the portfolio capitalization factor, and P_0=1 that is the initial investment. The values of γ used are: 2, 5, 10 and 15, as suggested by Bucciol and Miniaci (2008) and such as chosen by other authors (for example Jondeau and Rockinger (2006b)).

In the next section I report the result of the copula model compared to the mean-variance model, to a model that supposes all series to be independent and to an allocation equally divided among the assets, that is used as a benchmark. With the use of the CRRA utility function I can not derive a simple closed form for the optimal weights in Markowitz; so I use, as an approximation of the weight for the mean-variance approach, a model that uses GARCH(1,1) for the univariate variances and shocks extracted from a multivariate Normal with correlations equal to the in-sample historical ones.
I use the final amount of the portfolio and the utility obtained by the monthly returns, to compare the allocations. To compare the utility I use the management fee, called also opportunity cost or forecast premium, that is the amount that the investor would pay to switch from the equally divided portfolio (that does not use any information) to the analyzed allocation. In other words, it is the return that needs to be added to the returns obtained by the equally divided portfolio, so that the investor becomes indifferent to the returns obtained by the analyzed model. Formally, denoting r_p^* as the optimal portfolio return obtained using the copula or the Markowitz or the indipendence model, and denoting r_p as the return obtained using the equally divided portfolio, the opportunity cost θ is:

$$U(1 + r_p + \theta) = U(1 + r_p^*) \tag{2}$$

Following an approach similar to Jondeau and Rockinger (2006b) in the context of a fourth-order Taylor approximation with CRRA utility function, the previous equation can be written as follows:

$$\theta = (\mu_p^* - \mu_p) - \frac{\gamma}{2}(m_p^{*2} - m_p^2) + \frac{\gamma(\gamma+1)}{3!}(m_p^{*3} - m_p^3) - \frac{\gamma(\gamma+1)(\gamma+2)}{4!}(m_p^{*4} - m_p^4) \tag{3}$$

where μ represents the mean return and m represents the non central moments: $m_p^i = mean[r_p^i]$.
I use this expression to calculate the forecast premium. It indicates that the investor is willing to pay to use a strategy that decreases variance and kurtosis and increases mean and skewness of his/her portfolio returns.

3 Results

I try the copula model in the macro asset allocation of a financial portfolio on two, three or four uncorrelated assets. The Pearson's correlation, reported in table 2, and above all the Kendall's Tau, reported in table 3, show that the relations are very weak (positive or negative) for all index couples (however, the correlation matrix changes in time, as shown in appendix B).

In the case of two assets the portfolio is composed by the commodity index and the equity index; in the case of three assets I add the bond index and, for the four assets portfolio, the

Table 2: *Pearson's correlations. Full data range: 1996/01/01 - 2010/08/31. NYCRB = New York CRB index, DJ = Dow Jones Industrial index, MLUSTR1-10 = Merrill Lynch US Treasury 1-10 years, US$-EURO = exchange rate US Dollar-Euro.*

	NYCRB	DJ	MLUSTR1-10	US$-EURO
NYCRB	1	0.1504	-0.1265	0.2731
DJ		1	-0.2382	-0.0342
MLUSTR1-10			1	0.1088
US$-EURO				1

Table 3: *Kendall's Tau. Full data range: 1996/01/01 - 2010/08/31. NYCRB = New York CRB index, DJ = Dow Jones Industrial index, MLUSTR1-10 = Merrill Lynch US Treasury 1-10 years, US$-EURO = exchange rate US Dollar-Euro.*

	NYCRB	DJ	MLUSTR1-10	US$-EURO
NYCRB	1	0.0476	-0.0630	0.1469
DJ		1	-0.1164	-0.0457
MLUSTR1-10			1	0.0633
US$-EURO				1

investor has also the possibility to invest in the exchange rate market: in each step there is an increase of the absolute values of the correlations.

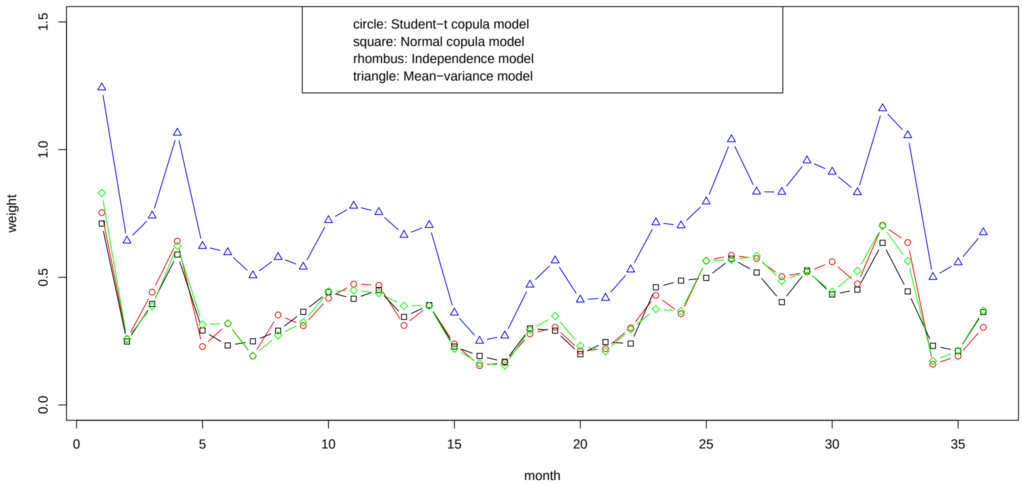
3.1 Two assets: equity and commodity portfolio

I begin with the portfolio composed by the Dow Jones Industrial index and the New York CRB index.

In order to be able to choose between copulas, I analyze the value of the maximum likelihood. The best copula is the Student-t copula, even if it has 75 degrees of freedom, so it is similar to a Normal copula and, indeed, they have similar maximum likelihood values. Moreover, they both pass the GOF test (at 5% of confidence level).

However, all the copulas give similar maximum likelihood values, with parameters that show a weak relation between the two indices: the Clayton copula and the Frank copula present a parameter near 0, the Gumbel copula a parameter near 1 and Normal and Student-t copula a correlation parameter near 0. Thus, I test the independence between the two indices, following Genest and Rémillard (2004), and the test does not reject the hypothesis of independence. So I try both the Student-t and the Normal copulas and a model with no copula (only the GARCH(1,1) with Skewed-t errors, that is the same as the use of an Independence copula). As expected, all these three models give similar weights, as shown in fig.1, where is displayed the Dow Jones weight's evolution during the 36 months for an investor with risk aversion parameter γ=5. In this figure, I also note that the two copula models and the Independence model give less weight to the Dow Jones index compared to the mean-variance model. This feature can be explained by the fact that the marginal distributions in the mean-variance model are Normal, while in the other models they are Skewed-t. So, the mean-variance model is not able to catch the high kurtosis of Dow Jones and then allocates a larger portfolio's slice to the equity index. The conclusion is that the mean-variance portfolio is riskier with a higher exposure to the index with the highest kurtosis, as already found in Riccetti (2010).

Figure 1: *Weights of Dow Jones with different models - γ=5 investor*

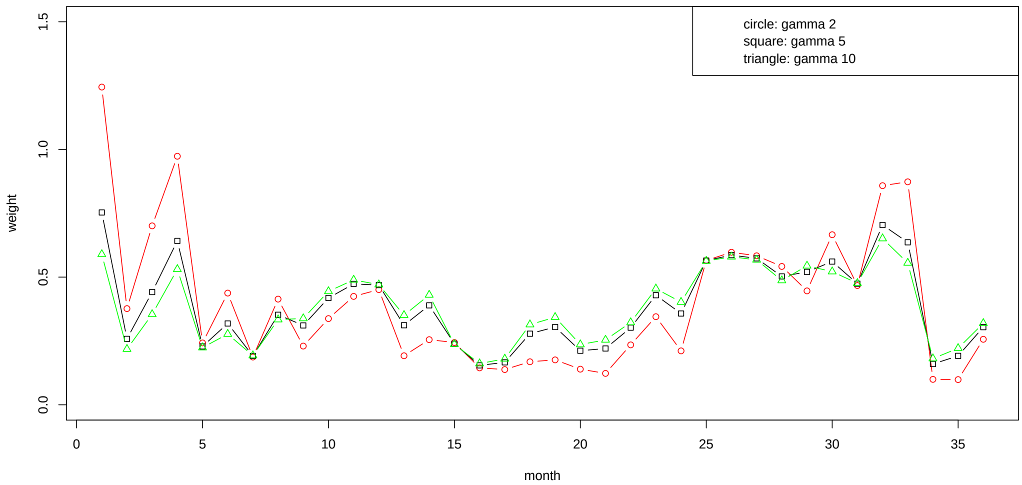


The optimal weights can be studied also in the changes that occur when the risk aversion parameter γ changes. In figure 2 is reported the Dow Jones weight's evolution during the 36 months for the Student-t copula model with three levels of γ (2, 5 and 10). The weights of the two assets are similar (about 50% and 50%) for $\gamma = 10$, while the investor takes some bigger risks for smaller and smaller values of γ, with also a short selling of the commodity index in the first month. In this way, the portfolio for more risk averse investors needs less rebalancing activities (so it is less expensive in the real world). This confirms what I find in Riccetti (2010): investors with little risk aversion especially buy the more remunerative index, while more risk averse investors try to divide their portfolio more equally. The Normal copula, the Independence and the mean-variance portfolios show the same feature.

Now I report the analysis which addresses the results obtained by the models. As already said, I compare these models with the 50%-50% allocation in order to understand whether the use of an analytical model is really useful.

I consider two characteristics: the utility of the allocation and the amount of money obtained at the end of the entire out-of-sample period (all 36 months). The most important value is the utility, indeed it contains all the features of the obtained returns, thus the mean return (and

Figure 2: *Weights of Dow Jones for Student-t copula model with different γ values: 2, 5 and 10*



so the total return) is already included in this value. However, the mean return is the most important feature of the utility, especially for a low risk averse person, so table 4 presents both the utility and the final amount obtained. The utility is calculated using the CRRA function and the monthly out-of-sample obtained returns. I report not only the value of the utility, but also the forecast premium above the 50%-50% portfolio (calculated using formula 3) to give an economic meaning to the utility differences; obviously, the forecast premium is negative if the model gives a utility worse than the 50%-50% allocation.

By observing table 4, it is evident that the copula models are the best at all γ levels for the macro asset allocation of this portfolio composed by equity and commodity. The use of a construction able to divide the modelling of the marginals and of the dependence appears useful when the two parts are uncorrelated. The two copula models give similar results, as expected observing the weights (see fig.1), but the Normal copula is slightly better than the Student-t copula, especially for not very risk averse investors (γ=2 or 5).

Instead, even if the Independence copula seems to fit well the data in-sample, out-of-sample its performance is not as good as the one obtained with a Normal or Student-t copula. However, the result of the Independence model shows that also two simple GARCH(1,1) with Skewed-t errors are enough to obtain a very good performance with these two unrelated indices.

The mean-variance portfolio has the worst performance (terrible in the case of a γ=2 investor), so the naive allocation appears better and this kind of model is not useful. Considering the good performances of the Independence and of the Normal copula models, the trouble of the mean-variance model is caused by the Normal marginals that do not catch the features of the skewness and, especially, of the kurtosis of the indices.

Table 4: *Utilities, monthly forecast premiums (%) and final amounts for Dow Jones - New York CRB portfolios.*

Utility Portfolio	γ 2	γ 5	γ 10	γ 15
T Copula model	-1.0029	-0.2576	-0.1306	-0.1201
Normal Copula model	-1.0016	-0.2571	-0.1304	-0.1200
Independence model	-1.0032	-0.2580	-0.1311	-0.1097
Mean-Variance model	-1.0157	-0.2634	-0.1341	-0.1227
50% - 50% allocation	-1.0034	-0.2595	-0.1330	-0.1193

Utility-Forecast Premium Portfolio	γ 2	γ 5	γ 10	γ 15
T Copula model	0.060%	0.192%	0.248%	0.270%
Normal Copula model	0.193%	0.248%	0.274%	0.275%
Independence model	0.022%	0.156%	0.199%	0.187%
Mean-Variance model	-1.332%	-0.406%	-0.110%	-0.034%

Final Amount Portfolio	γ 2	γ 5	γ 10	γ 15
T Copula model	0.960	0.997	1.010	1.017
Normal Copula model	1.007	1.016	1.019	1.019
Independence model	0.951	0.991	1.004	1.008
Mean-Variance model	0.630	0.840	0.921	0.948
50% - 50% allocation	0.949			

3.2 Three assets: equity, commodity and bond indices portfolio

Now I consider a portfolio allocated in three macro asset classes: commodity, equity and bond. Also in this case the copula used is the Student-t one, because it presents the best log-likelihood. Moreover, the bond index has negative Pearson's correlations (see table 2), thus the use of copulas as Clayton or Gumbel is not feasible.
In this case the Student-t copula has 10 degrees of freedom, so the improvement in the log-likelihood compared to the Normal copula is significant. Besides, the independence test does not pass, so the only reasonable choice is the Student-t copula. However, in the case of two assets, even if the Independence structure seems to be the best in-sample, out-of-sample it is (slightly) beaten by the two copula models. So, also in this case, I check if the out-of-sample results are different than what I could imagine analysing the in-sample data: again I compare the Student-t copula model result with the results of the simplier models that use the Normal

copula and the Independence structure.

The results (forecast premiums and final amounts) are reported in table 5, and they show that the Normal copula model obtains the best utility (also driven by the highest final amount) at the level of risk aversion γ=5, while at higher levels of risk aversion the Independence model appears to be the best and for γ=2 the equally divided portfolio wins.

Table 5: *Monthly forecast premiums (%) and final amounts for Dow Jones - New York CRB - ML US Treasury portfolios.*

Utility-Forecast Premium Portfolio	γ 2	γ 5	γ 10	γ 15
T-Copula model	-0.476%	0.397%	1.024%	1.592%
Normal-Copula model	-0.241%	0.485%	1.067%	1.619%
Independence model	-0.308%	0.475%	1.076%	1.636%
Mean-Variance model	-2.178%	-0.206%	0.751%	1.424%

Final Amount Portfolio	γ 2	γ 5	γ 10	γ 15
T-Copula model	0.879	1.082	1.156	1.181
Normal-Copula model	0.947	1.113	1.172	1.192
Independence model	0.923	1.106	1.173	1.196
Mean-Variance model	0.542	0.905	1.061	1.118
equally divided allocation	1.060			

The Student-t copula, that is the best model in-sample, is never the best in the performance out-of-sample, even if the two copula models and the Independence model present very similar results, especially for very risk averse investors (γ=10 or 15).
There is no clear classification among these allocations, but some results seem clear enough:

- the copula models are always better than the mean-variance one; indeed the mean-variance portfolio has an awful performance in the case of a γ=2 investor, it is the worst also for a γ=5 investor, while it appears useful for a very risk averse person (γ=10 or 15) because it is better than the naive allocation, but it is not better than the copula models and the Independence model. Considering the good performance of the Independence model and of the Normal copula, the improvement is due to the importance of the Skewed-t marginals[4];

[4]This conclusion is different from Patton (2004), indeed Patton finds that the dependence structure is more relevant than the marginal one. However, Patton uses two stock indices and the U.S.Treasury bill interest rate, so two indices have high correlation.

- the use of a construction able to divide the modelling of the marginals and of the dependence, appears useful;
- for the macro asset allocation of a portfolio composed by equity, commodity and bond parts, which are all almost unrelated, the use of a copula model can be useful, but the use of univariate GARCH(1,1) with Skewed-t errors obtains good results also. Thus modelling the dependence structure in a complex way could be a waste of energy and time;
- a good in-sample fit does not assure a good out-of-sample result.

3.3 Four assets: equity, commodity, bond and exchange rate portfolio

In this section the portfolio is allocated on four macro asset classes: commodity, equity, bond and exchange rate. The comparison is done, as usual, with the equidivided portfolio as benchmark (25%-25%-25%-25%). The Student-t copula has the best log-likelihood in this case too. However, this copula doed not pass the GOF test, so I try also a Canonical Vine copula. Indeed, in Riccetti (2010), I find that the Canonical Vine copula improves the performance compared to the Student-t copula, in a four assets portfolio with one uncorrelated index. All the bivariate copulas that compose the Vine copula are Student-t, indeed some assets have negative correlations, so the use of Clayton or Gumbel copulas is not feasible. Moreover, the Student-t copulas give the best fit according to several authors, such as Aas *et al.* (2007).
Now the problem is the choice of the order of the building blocks. The first index is the one that has a bivariate copula with each of the other indices; the second index, conditional to the first index, has two bivariate copulas with the two remained indices; finally there are the two last indices with the copula between them. I try four orders:

1. order 1: bond index, exchange rate (then equity index and commodity index). The first tree is the one that presents the highest values of the tail dependences (calculated using the Student-t copula estimates) reported in table 6, and so on; this is the method suggested by Berg and Aas (2007);
2. order 2: exchange rate, bond index (equity index and commodity index). The first tree is the one that presents the highest absolute values of the Kendall's Tau, reported in table 3, and so on;
3. order 3: commodity index, equity index (exchange rate and bond index). This is the opposite of the first methods, beginnig with the index with the lowest tail dependence;
4. order 4: equity index, commodity index (exchange rate and bond index). The first tree has all negative relations (as Kendall's Tau or Pearson's correlation) with the others,

then, among the remained indices, the commodity one has the lowest Kendall's Tau or Pearson's correlation, and so on.

Table 6: *Tail dependences calculated with the Student-t copula parameters. Both tails have the same value of the tail dependence. NYCRB = New York CRB index, DJ = Dow Jones Industrial index, MLUSTR1-10 = Merrill Lynch US Treasury 1-10 years, US$-EURO = exchange rate US Dollar-Euro.*

	DJ	MLUSTR1-10	US$-EURO
NYCRB	0.0000	0.0000	0.0031
DJ		0.0504	0.0002
MLUSTR1-10			0.0320

The building blocks with order 1 and 2 and the building blocks with order 3 and 4 are very similar, so the results are very similar. Therefore, I present only the first and the fourth ones that give results slightly better than the second and the third respectively.

I report the final amount obtained by the various models in table 7: this is a fast way to show some evidences. From this table it is clear that the standard copulas give awful results:

Table 7: *Final amounts for Dow Jones - New York CRB - ML US Treasury - US$-EURO portfolios.*

Portfolio	γ 2	γ 5	γ 10	γ 15
Vine-Copula order 4	1.600	1.748	1.649	1.585
Vine-Copula order 1	0	0	0.001	0.050
T-Copula model	0	0	0.0002	0.025
Normal-Copula model	0	0	0	0.105
Independence model	1.335	1.543	1.538	1.529
Mean-Variance model	0.915	1.100	1.156	0.458
equally divided allocation	1.026			

- the Normal copula makes the investor fail, unless the investor has a very high risk aversion value (γ=15), when he/she loses 90% of his/her money!
- the Student-t copula makes the investor with γ=2 or 5 fail and, with higher risk aversion, lose almost all the money.

The cause is the use of enormous short-sellings: for example, in the case of a γ=2 investor with the use of a Normal copula, the bond index has weights till 3537%, while the exchange rate has short sellings till 1570%.

In this case, the mean-variance model heavily overperforms the two copula models. Indeed, even if it creates a big loss for investor with γ=15 and a smaller loss with a γ=2 investor, investors with intermediate risk aversion parameter values obtain good returns. This non monotonic result confirms the result found in Riccetti (2010).
In any case, there is a way for the copula model to improve the mean-variance allocation return also with this portfolio: it has to use the Canonical Vine copula. However, with the Canonical Vine copula there is the need to choose the order of the building blocks and this choice is very important, indeed the results obtained range from losing all the money to having a return of 75% in three years. Following the suggestions of Berg and Aas (2007), that is using as the first two factors the indices with the highest tail dependence or Kendall's Tau, the results are disastrous, while the results are very good if the first two factors are the index with the smallest tail dependence and Kendall's Tau. Thus, also in the Vine copula construction, the use of low dependencies seems to improve the out-of-sample results.
The Independence model gives very good returns at all levels of risk aversion and it is beaten only by the Vine copula models with building blocks that follow order 3 or 4. However, this model appears to be a very safe (you can not be wrong in the choice of the building blocks order) and simple way to obtain excellent returns.

However, till now, I have considered only the final amount and not the utility. In table 8 I do not report the forecast premiums of all the above models. Indeed, I skip the forecast premiums of "Vine-copula order 1", Student-t copula and Normal copula models, because when the portfolio fails the forecast premium is less infinite, and also in presence of an heavy loss the absolute value of the forecast premium is enormous and not economically significant.

Table 8: *Monthly forecast premiums (%) for Dow Jones - New York CRB portfolios - ML US Treasury - US$/EURO exchange rate portfolios.*

Portfolio	γ 2	γ 5	γ 10	γ 15
Vine Copula model-inverse	-5.957%	-0.829%	0.060%	0.473%
Indipendence model	-4.354%	-0.509%	0.259%	0.629%
Mean-Variance model	-0.599%	0.405%	1.014%	-61.101%

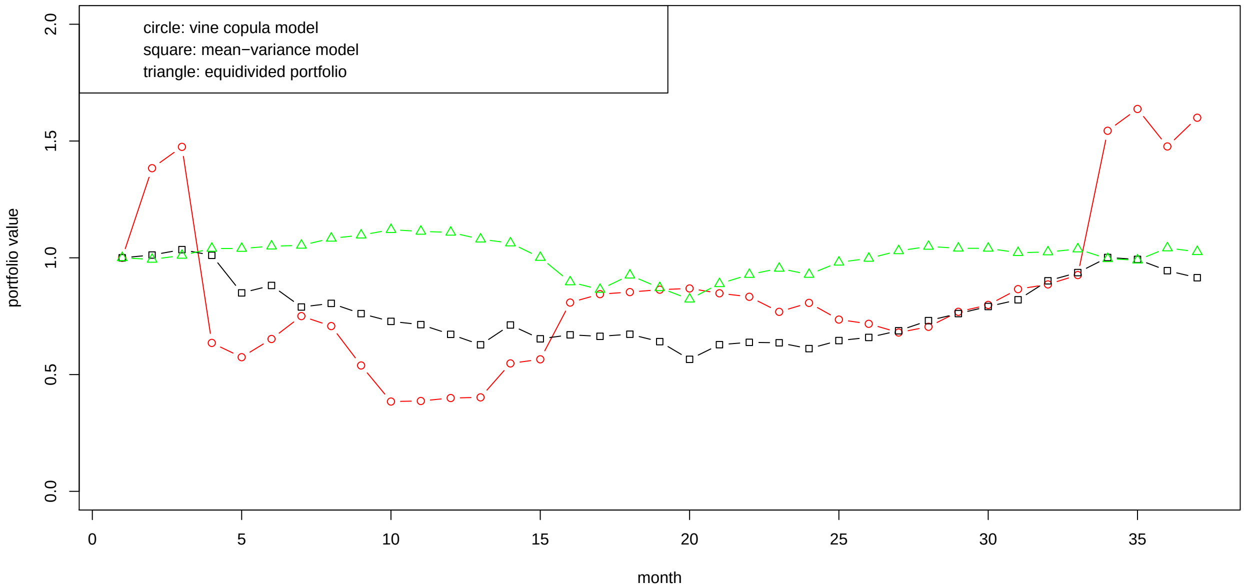
Analysing table 8, I note the enourmous value of the forecast premium when the portfolio has a heavy loss, such as for the mean-variance model when γ=15.
However, the most interesting features are:

- the Independence model beats the Vine copula model;
- both the Independence and the Vine copula models have negative forecast premiums when the investor has low risk aversion (and they are very high in absolute value when γ=2);

- the mean-variance model is the best for investors with intermediate risk aversion parameters (γ=5 or 10).

Both the first two features appears in contrast with the final amount of table 7. Indeed, for example, for a γ=2 or 5 investor, the two considered models obtain a negative forecast premiums, in contrast with the very high returns, especially compared with the low return of the equi-divided allocation (that is the benchmark for computing the forecast premium), that has a return of 2,6% over 3 years. The cause is the high level of short-selling that implies returns distributions with very high volatility and kurtosis. This is evident in fig.3, that shows the path of the value of the portfolio during the 36 months: this value goes up and down in the case of the Vine copula, while it is more stable in the case of the mean-variance model (that obtains the best utility for γ=5 and 10, because it has a distribution of the returns with a good mean, and small variance and kurtosis) and for the equi-divided allocation.

Figure 3: *Total amount of the portfolio during the 36 months for Vine copula model, mean-variance model and equi-divided allocation - γ=2 investor*



4 Conclusions and possible future works

In Riccetti (2010) I find that the use of copulas can be useful in an asset allocation model for the macro asset allocation (choosing the stock and the bond composition of portfolios), while it is not useful in the choice of the weights for a portfolio composed by many stock indices. A possible driving factor of the performance of the copula model is the correlation

value between the assets: the copula model appears to be useful in the case of a portfolio containing one asset with small absolute values of correlation, while this model is dangerous with highly correlated assets.
In this paper I check if that result is confirmed in a portfolio that contains a commodity index, that is uncorrelated with a stock index or a bond index or an exchange rate returns time series. The use of a commodity index is relevant because, in the last years, this kind of investment is increasing, also thanks to the recent creation of a lot of financial vehicles (exchange traded funds or mutual funds) that make the investment in commodities possible for retail investors too. Moreover, commodities are expected to be very profitable after the expansive monetary policy of these years.

The results are contrasting: on the one hand, the good performance of the copula model in the macro asset allocation of uncorrelated asset classes is confirmed. In the case of two macro asset classes the copula model, with Student-t of Normal copulas, is the best for all levels of risk aversion, as in Riccetti (2010). In the case of three assets this model presents again a very good performance, better than the mean-variance portfolio.
On the other hand, the copula model is not always the best one, for sure it is not the simpliest and it implies some arbitrary choices. Indeed, I compare the performance of the copula model with the performance of a model that implies independence among the univariate series of residuals, because it is a reasonable way to model almost uncorrelated asset classes. This kind of model is faster and simplier, because it needs only the univariate GARCH, with a suitable distribution of the error terms, as the Skewed-t one. The comparison shows that the Independence model produces out-of-sample results more or less similar to those obtained by the copula model, but with a simplier method. Moreover, the improvement of the Independence model over the mean-variance one, is high and it means that, with these indices, it is more important to catch the features of the return distribution of the single asset classes, compared to catch the joint behaviour of the multivariate distribution of the returns. Perhaps this feature is tied to the fact that the macro asset allocation is done on almost unrelated classes. However, as shown in Riccetti (2010), the copula model is useless also for portfolios involving very related assets, as all stock indices. Thus, the copula model appears not useful at all, with the exception of portfolios containing one uncorrelated asset. Indeed, the copula model obtains the best (among the tried models) out-of-sample performance, when the portfolio is invested in:

- two uncorrelated assets (or asset classes), such as commodities and stocks, as shown in this paper;
- two or more correlated indices and one less correlated one, as shown in Riccetti (2010) or Patton (2004) with portfolios invested in two or three stock indices and one safer index.

However, there are a lot of possible works in this field to confirm or deny the general conclusions enunciated (if general conclusions, that can be applied in different situations, really exist):

- there could be situations in which the copula model is the best, using other indices and/or using other periods and/or using other allocation horizons and/or using other frequency of returns and/or using a multiperiodal framework and/or using other utility functions and/or considering the uncertainty related to the estimation of the parameters and/or using the short-selling constraint;
- there could be different models of the marginal distributions of returns (for the mean, the variance...) and for the copulas, and/or model extensions for example with dynamic conditional parameters (again for mean forecast or for skewness and kurtosis of the marginal distribution of residuals or for the copula parameters, see also appendix B).

Moreover, in this paper, it seems that the complex models tend to overfit the data in sample, but they are often too specific to obtain a good performance out-of-sample. Further research could analyse the use of even more complex copula models, in order to understand if it improves the out-of-sample forecasting: I can not exclude that the relationship between complexity and the out-of-sample performance could be U shaped, with an initial decrease and a following increase with very complex models (for example using a dynamic copula).
A possible case of a U shaped relation between complexity and performance found in this paper is that, when the number of assets increases, the standard copula model shows increasing problems, but the use of Vine copulas can reduce these problems (even if they are still present). However, very complex structures can imply several other problems. For instance, a Vine copula gives problems in the choice of the single bivariate copulas and problems in the choice of the order of the indices in the building blocks; as shown, these choices can change the results dramatically and the better in-sample choice can be no more good out-of-sample. Indeed, a clear feature that emerges from this paper is that the good in-sample fit does not assure a good result out-of-sample. Instead, methods that do not use strong assumptions to improve the in-sample fit, as the Independence one, often seem more robust than more complex models, as the copula ones.

In conclusion, the use of a simple GARCH(1,1) with Skewed-t errors for each asset class seems to be enough to obtain a good performance in the macro asset allocation of a portfolio that uses commodities. Indeed, even if I find that the copula model appears useful and better than the mean-variance one, it is often worse than a set of independent univariate GARCH(1,1) models, probably because commodities have returns that are almost unrelated to the returns of the other macro asset classes (stock, bond and exchange rate).

A Appendix: an introduction to copulas

A d-dimensional copula C($\mathbf{u}$) is a distribution function defined in $[0,1]^d$ with standard uniform marginal distributions, hence C is a mapping of the form $C:[0,1]^d \rightarrow [0,1]$. To be a copula, the following three **properties** must hold:

1. $C(u_1, ..., u_d)$ is non-decreasing in each component u_i, to be a multivariate distribution function.

2. $C(1, ...1, u_i, 1, ..., 1) = u_i$ for all $i \in [1, ..., d]$ and $u_i \in [0,1]$, to have uniform marginal distributions.

3. For all $(a_1, ..., a_d), (b_1, ..., b_d) \in [0,1]^d$ with $a_i \leq b_i$, I have:

$$\sum_{i_1=1}^{2} ... \sum_{i_d=1}^{2} (-1)^{i_1+...+i_d} C(u_{1i_1}, ..., u_{di_d}) \geq 0$$

   where $u_{j1} = a_j$ and $u_{j2} = b_j$ for all $j \in 1, ..., d$. This property, called rectangle inequality, ensures that if the random vector $(U_1, ..., U_d)'$ has df C, then $P(a_1 \leq U_1 \leq b_1, ..., a_d \leq U_d \leq b_d)$ is non-negative.

These three properties characterize a copula: if a function C possesses all three properties, then it is a copula.
Sklar (1959) shows that all multivariate distribution functions (dfs) contain copulas and that copulas may be used in conjunction with univariate dfs to construct multivariate dfs. This is **Sklar's Theorem**:

let F be a joint distribution function with margins $F_1, ..., F_d$. *Then there exists a copula* $C:[0,1]^d \rightarrow [0,1]$ *such that for all* $x_1, ..., x_d$ *in* $\Re = [-\infty, \infty]$ *then*

$$F(x_1, ..., x_d) = C(F_1(x_1), ..., F_d(x_d)) \quad (4)$$

If the margins are continuous then C is unique; otherwise C is determined on $RanF_1$ *x* $RanF_2$ *x ... x* $RanF_d$, *where* $RanF_i = F_i(\Re)$ *denotes the range of* F_i. *Conversely, if C is a copula and* $F_1, ..., F_d$ *are univariate df, then the function F defined above is a joint df with margins* $F_1, ..., F_d$.

For multivariate **discrete distributions** there is more than one copula that can be used to join the margins to form the joint df.
Sklar's Theorem provides a technique for constructing multivariate distributions with arbitrary margins and copulas. If you use a copula with arbitrary margins, you have a **meta distribution**; for example a meta-Gaussian is a distribution with a Gaussian copula, but arbitrary and non-Gaussian margins.

A.1 Tail dependence

An important measure of pairwise dependence, that depends only on the copula of a couple of random variables with continuous marginal dfs, is the Tail Dependence. It provides a measure of extremal dependence that is the strength of dependence in the tail of a bivariate distribution. The **Upper tail dependence** λ_u is the probability that X_2 exceeds its q-quantile, given that X_1 exceeds its q-quantile with q that goes to its limit:

$$\lambda_u = lim_{u \to 0^+} P(F_2(x_2) > 1 - u \,|F_1(x_1) > 1 - u) \tag{5}$$

$\lambda_u \in [0, 1]$ and if $\lambda_u \in (0, 1]$ then there is upper tail dependence, whereas if $\lambda_u = 0$ then the two random variables are asymptotically independent. Analogously, but reversing the definition, I have also the **Lower tail dependence** $\lambda_l \in [0, 1]$.
The tail dependences of a copula, that is a convex combination of copulas, is the convex combination of the individual coefficients of tail dependences.

A.2 Kinds of copulas

Copulas can be divided into three categories:

1. **Fundamental**: this category represents important special dependence structures;
2. **Implicit**: in this category, copulas are extracted from well-known multivariate distributions, but they do not necessarily possess simple closed-form expressions;
3. **Explicit**: these copulas have simple closed-form expressions and follow general mathematical constructions known to yield copulas.

There are three fundamental copulas:

- the **Independence** copula, defined as $C^{ind}(u_1, ..., u_d) = \prod_{i=1}^{d} u_i$
- the **Comonotonicity** copula, that is the copula for random variables that are perfectly positively dependent.
- the **Countermonotonicity** copula, that is the copula for two random variables that are perfectly negatively dependent.

Example of implicit copulas are the **gaussian** and the **t** copula. Whereas, Archimedean copulas are an example of explicit copulas. I use three distinct bivariate Archimedean copulas, indeed this study uses five copulas, and the Canonical Vine copula, to model the series of residuals: Normal, T, Gumbel, Clayton and Frank. Gumbel, Clayton and Frank are all Archimedean copulas. Archimedean copulas all possess the following expression:

$$C(u_1, u_2) = \phi^{-1}(\phi(u_1) + \phi(u_2)), \tag{6}$$

where ϕ is a decreasing function satisfying $\phi(0) = \infty$ and $\phi(1) = 0$, known as the generator. The generator is different for each archimedean copula. The table below summarizes the functional form of the three generators $\phi(t)$ and the admittable range for the values of the parameter:

Table 9: Generators and admittable values for parameters

Copula	generator	values
Gumbel	$(-\ln t)^\theta$	$\theta \geq 1$
Clayton	$\frac{1}{\theta}(t^{-\theta} - 1)$	$\theta \geq -1$
Frank	$-\ln\left(\frac{e^{-\theta t}-1}{e^{-\theta}-1}\right)$	$\theta \in \Re$

Table 10: Tail Dependence

Copula	λ_u	λ_l
Normal	0	0
t	$2t_{\upsilon+1}(-\sqrt{\frac{(\upsilon+1)(1-\rho)}{1+\rho}})$	$2t_{\upsilon+1}(-\sqrt{\frac{(\upsilon+1)(1-\rho)}{1+\rho}})$
Gumbel	$2 - 2^{1/\theta}$	0
Clayton	0	$\begin{cases} 2^{-1/\theta} & \theta > 0 \\ 0 & \theta \leq 0 \end{cases}$
Frank	0	0

These copulas have five different values of tail dependence, summarized in table 10 where, for the Student-t copula, t_υ is the df of a univariate t with υ degrees-of-freedom and correlation ρ, while, for the archimedean copulas, θ is the parameter of the generator.
The Gumbel copula has very high density in its positive corner (there is high probability that a large positive value of one variable corresponds to a large positive value of the other variable), the Clayton copula has a negative thick tail and the Student-t copula shows more density than the Normal copula in both tails symmetrically.

A.3 Multivariate dependence

Most of the research on copulas is limited to the bivariate case. The set of higher-dimensional copulae is limited and the copulas that can be generalized to the multivariate case, often imply the strong restriction that the dependence is the same across all pairs of variables. In recent years, some authors have tried to construct more flexible multivariate copulas by modelling multivariate dependence with a cascade of bivariate copulas. Some extensions are the nested Archimedean constructions discussed in Joe (1997), Embrechts *et al.* (2003), Whelan (2004) and McNeil (2007). These methods use up to d-1 bivariate Archimedean copulas. However, they have some limits: they only use Archimedean copulas, they do not model all possible mutual dependencies among the d variates and they have to satisfy necessary conditions to lead to valid multivariate copulas.

I therefore use another kind of construction, denoted pair-copula constructions (PCC), first proposed by Joe (1997) and either discussed or applied by Bedford and Cooke (2001), Bedford and Cooke (2002), Kurowicka and Cooke (2006), Aas *et al.* (2007), Berg and Aas (2007), Heinen and Valdesogo (2008) and Haff *et al.* (2009). The PCC is more flexible, indeed it allows for the free specification of $d(d\text{-}1)/2$ copulas and uses all kinds of bivariate copulas; furthermore it does not have parameter constraints.

The most frequently used type of PCC's is the regular vine type and, in particular, two special cases of regular vine: the Canonical Vines and the D-vines. I use the Canonical Vine copula that decomposes the joint probability density function of n variables y_1,...,y_n through iterative conditioning, as follows:

$$f(y_1, ..., y_n) = f(y_1) \cdot f(y_2|y_1) \cdot f(y_3|y_1, y_2)...f(y_n|y_1, ..., y_{n-1}) \tag{7}$$

Each factor in this product can be further decomposed using conditional copulas, for example:

$$f(y_2|y_1) = c_{12}(F(y_1), F(y_2)) \cdot f(y_2) \tag{8}$$

$$f(y_3|y_1, y_2) = c_{23|1}(F(y_2|y_1), F(y_3|y_1)) \cdot f(y_3|y_1) \tag{9}$$

Finally, combining all the expressions I obtain the joint density, that is the product of marginals densities and bivariate conditional copulas densities. For instance, for a three dimensional case, this is the joint density function:

$$f(y_1, y_2, y_3) = c_{23|1}(F(y_2|y_1), F(y_3|y_1))c_{12}(F(y_1), F(y_2))c_{13}(F(y_1), F(y_3))f(y_1)f(y_2)f(y_3) \tag{10}$$

The parameters are estimated by maximum likelihood with a recursive approach, as explained in Aas *et al.* (2007).

The PCC approach has some shortcomings. One is that there are almost unlimited ways to combine each building block, so the problem of not having enough multivariate copulas is reversed. Another problem is that the estimation process has a number of steps that increases rapidly with the dimension and the complexity of the copula, so it becomes time consuming. Moreover, as analysed by Haff *et al.* (2009), the exact decomposition is based on pair-copula of the following form:

$$C_{23|1}(F(y_2|y_1), F(y_3|y_1); y_1) \tag{11}$$

but, for inference to be fast, flexible and robust, I assume that these pair-copulas are independent of the conditioning variables, except through the conditional distribution:

$$C_{23|1}(F(y_2|y_1), F(y_3|y_1)) \tag{12}$$

However, in Haff *et al.* (2009), the authors show that, even if the pair-copula is dependent on the conditioning variable so that the simplyfied decomposition is wrong, this is a good approximation of the general decomposition. Then, in this study, I also use Canonical Vine copulas for multivariate allocations.

B Appendix: why a dynamic copula could be useful

Pearson's correlation matrix and Kendall's Tau matrix change over time. A significant change is present between the in-sample and the out-of-sample data range, as shown in tables 11 and 12: the bond index decreases its Kendall's Tau values with the other indices, while the other three indices increase the ones among them. Moreover, the absolute values of the coefficients are often higher in the out-of-sample data.

Table 11: *Kendall's Tau. In-sample data range. NYCRB = New York CRB index, DJ = Dow Jones Industrial index, MLUSTR1-10 = Merrill Lynch US Treasury 1-10 years, US$-EURO = exchange rate US Dollar-Euro.*

	NYCRB	DJ	MLUSTR1-10	US$-EURO
NYCRB	1	-0.0008	-0.0265	0.0872
DJ		1	-0.0475	-0.0911
MLUSTR1-10			1	0.0917
US$-EURO				1

The increase of Kendall's Tau and of Pearson's correlation (not reported) between stock index and commmodity index confirms the findings of Büyüksahin *et al.* (2010) (see footnote 1) about the last financial crises. Thus, for example, a dynamic copula model could improve the forecast. Instead, in my model, copula parameters are non conditional, even if the estimates are calculated every month out-of-sample; in this way the copula observes the new features of increased dependence during the last financial crisis, but it is a slow "learning process".

However, the dependences are low also during the out-of-sample period, as shown by table 12, thus it is preserved the characteristic of the portfolio to be composed by assets with low

Table 12: *Kendall's Tau. Out-of-sample data range. NYCRB = New York CRB index, DJ = Dow Jones Industrial index, MLUSTR1-10 = Merrill Lynch US Treasury 1-10 years, US$-EURO = exchange rate US Dollar-Euro.*

	NYCRB	DJ	MLUSTR1-10	US$-EURO
NYCRB	1	0.1833	-0.1559	0.3148
DJ		1	-0.3075	0.0929
MLUSTR1-10			1	-0.0163
US$-EURO				1

correlations.
Moreover, the changes in the correlation coefficients are present in all the data range, apart from the crises. For instance, Pearson's correlation between stock and commodity indices begins slightly negative in 1996-1997 and then it slowly increases, till the faster growth during the last financial market crisis. Also the crises are different from each other, indeed during the first crisis present in the sample (in 2001-2002), the correlation increase is not significant. The in-sample correlations among the other indices change even more. On the one hand, these features implies that the copula model could have difficulties in forecasting the correlations out-of-sample, but this is a common problem in the econometrics of time series: all models could make mistakes in forecasting happenings that appears for the first time. On the other hand, the particular out-of-sample period analysed increases the interest in the performance of the models, indeed an investor or an asset manager has to face new events in the real world, and the possibility to use a model that is efficient in facing also extreme event is a very important characteristic.

In conclusion, as already said in section 4, it could be interesting to develop further research to enrich the model, in order to understand if a more complex model can improve the out-of-sample forecasting.

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