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# FROM MOMENTS, CO-MOMENTS AND MEAN-VARIANCE WEIGHTS TO COPULA PORTFOLIO ALLOCATION

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# From Moments, Co-Moments and Mean-Variance weights to Copula Portfolio Allocation

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**Abstract**

In Riccetti (2010) I find that the use of copulas can be useful in an asset allocation model for choosing the stock and the bond composition of portfolios (the macro asset allocation) or if the portfolio is composed by one bond index and some stock indices. Thus, in these cases, easy methods to reconstruct the copula allocation without estimating the copula, could be important for an asset manager/investor.

In this paper I build a model that considers moments and co-moments of the returns till the fourth power (respectively the mean of the returns and the mean of the crossed products of the returns raised up to fourth power) in order to understand whether they can approximate the use of copulas to obtain optimal weights. I analyse two models: the first reconstructs the copula model's weights using only moments and co-moments, while the second models the weights using moments, co-moments and the mean-variance weights.

I also use the moments and co-moments of the excess returns of the stock indices over the bond index return as independent variables.

The in-sample and the out-of-sample analyses show that it is possible to have an approximation of the weights obtained by a copula model using moments and co-moments of returns. Even if these models are different for each asset, changeable in time, with explanatory variables and signs that are not predictable and with accuracy that is uncertain, both models appear useful: the first appears to be easier (because the weights of the Markowitz model are not needed), while the second is more accurate in-sample and out-of-sample. Moreover the regression with the excess returns of the stock indices over the less risky index seems to be useful: it is a bit less accurate, but it needs to calculate less combinations of moments and co-moments.

**Keywords:** Asset Allocation, Portfolio Choice, Copulas, Mean-Variance, Moments and Co-moments, OLS, Excess Returns, Out-of-sample.

**JEL classification codes:** C20, C52, C58, G11, G17.

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# 1 Introduction

In Riccetti (2010) I find that the use of copulas can be useful in an asset allocation model for choosing the stock and the bond composition of portfolios (the macro asset allocation) or if the portfolio is composed by one bond index and some stock indices. Thus, in these cases, easy methods to reconstruct the copula allocation without estimating the copula, could be important for an asset manager/investor.
High moments seem also useful in portfolio choice, so I analyse whether the use of high moments can produce allocations similar to those produced by the copula model.
In this paper I build a model that considers moments and co-moments of the returns till the fourth power (respectively the mean of the returns and the mean of the crossed products of the returns raised up to fourth power) in order to understand whether they can approximate the use of copulas to obtain optimal weights. I analyse two models: the first reconstructs the copula model's weights using only moments and co-moments, while the second models the weights using moments, co-moments and the mean-variance weights. In the first case I run an OLS regression on the weights obtained with the copula models, using as independent variables the non central moments and co-moments of the returns. In the second case, I repeat the OLS adding the mean-variance weights as independent variable.

The paper proceeds as follows. Section 2 reports the literature review and the analysis done in Riccetti (2010) that is the base for the present analysis; section 3 contains the two models used to reconstruct the copula weights; section 4, 5 and 6 present the in-sample results and section 7 reports the out-of-sample results; section 8 concludes.

# 2 Literature review and data

In this section I briefly review some papers about the use, in the asset allocation context, of return moments of higher order than the variance and of copulas. Then I conclude with a review of the book of Riccetti (2010), that is the base for the following analysis.

## 2.1 Asset allocation with high order moments

The Markowitz model is optimal if investors only care about mean and variance or if returns are distributed as a Normal.
In fact financial returns are not Normally distributed as observed since 1963 with two papers of Mandelbrot and Fama, and it is especially true in cases like the Emerging Markets (see, for example, Bekaert *et al.* (1998)).
On the other side, a lot of authors as Arditti (1967), Kraus and Litzenberger (1976), Simkowitz and Beedles (1978) found, for example, that people prefer high values for skewness.
Scott and Horvath (1980) analytically prove, in their study "On the direction of preference for

moments of higher order than the variance", that people prefer returns with higher skewness and lower kurtosis. This proof can also be repeated for higher moments, and investors always prefer higher values for odd moments and lower values for even moments.
Authors as Bekaert *et al.* (1998), Harvey and Siddique (2000), Dittmar (2002) or Cvitanic *et al.* (2008) prove that higher moments improve the allocation if returns are distributed in a very different way from the Normal. Indeed, they find that "ignoring higher moments can lead to significant overinvestment in risky securities, especially when volatility is high" (Cvitanic *et al.* (2008)).
Jondeau and Rockinger (2006b) try to improve the mean-variance allocation using a skewed-t distribution for "innovations", allowing third and fourth moments to catch skewness and kurtosis features. They find that the opportunity cost of using a mean-variance allocation, compared to a four-moment optimisation, can be high under large departures from Normality.
Ioannidis and Williams (Williams and Ioannidis (2002) and Ioannidis and Williams (2007)) highlight that also the multivariate co-dependency between assets is often assumed to be conditional multivariate Normal, while there is non-linearity between risky assets returns. Therefore, they use conditional higher co-moments[1] in the asset allocation models, in order to capture the tail dependency and improve the diversification of the portfolio.

## 2.2 Use of copulas

Every joint distribution function contains a description of the marginal behaviour of the individual factors and a description of their dependence structure. Copulas isolate the description of the dependence and, describing it on a quantile scale, help to understand dependence at a deeper level than correlation. In this way they can describe the extreme outcomes and allow the building of a multivariate model that combines more developed marginal models with the variety of dependence models. Sklar (1959) shows that all multivariate distribution fuctions contain copulas and that copulas may be used in conjunction with univariate distribution functions to construct multivariate distribution functions. For an introduction on copulas see Embrechts *et al.* (2005) or Joe (1997).
Over recent years, many authors have applied the use of copulas to finance research. Much literature exists on the use of copulas for the computation of VaR in risk management and very advanced papers have been published in this field. However, there are also papers that use copulas to model stock market returns without a direct application to risk management, one of which is Jondeau and Rockinger (2006a). In this paper the authors first estimate the multivariate distributions of returns using copulas with time-varying parameters. This model is used by Jondeau and Rockinger to investigate the interactions between four major stock

[1]There are some papers on asset pricing, as Chung *et al.* (2006), that use co-moments till a very high order and not only co-skewness or co-kurtosis. A book that analyses moments and co-moments both in the field of asset pricing and in the field of asset allocation is edited by Jurczenko and Maillet (2006).

indices (S&P500, FTSE100, DAX, CAC) and they find that the Student-t copula is more accurate than the Gaussian one, consistent with the idea that dependence is stronger in the tails (high joint gains and high joint losses for indices).
In section 2.3 an asset allocation done with a copula model is reviewed in detail.

## 2.3 Patton (2004)

Patton (2004) does not only analyse multivariate returns series with copulas, but also builds an asset allocation on a portfolio composed by two stock indices and a risk free asset. He tries some models with different copulas and he finds that the best model for the investor's utility uses the skewed-t marginal distributions and the rotated Gumbel copula and this model (that I will call "Gumbel") gains more utility on the others if the investor is very risk adverse. After the asset allocation, Patton makes an analysis similar to the one that I am going to implement, trying to determine the causes of the differences in portfolio weights between the mean-variance model and the copula model. He uses nine parameters for the regression on the difference between the two weights: the two expected excess returns (of the stock indices over the risk-free return), the two volatilities, the two skewness parameters, the two kurtosis parameters and the copula parameter. This regression helps to highlight the causes of the differences, even if it is probably misspecified (because optimal weights are nonlinear function of the parameters of the joint density, so this is a first-order approximation of the true function relating parameters to weights) and suffer of error-in-variables bias (because all variables are estimated).
Patton finds that "Normal" portfolio weights react more strongly to changes in forecasted return and volatility. On the other side, the "Gumbel" portfolio reacts to the degrees-of-freedom parameter. Moreover, the dependence parameter is also important: greater dependence leads to more conservative weights in "Gumbel" portfolio, while the "Normal" portfolio responds in the opposite way, being more aggressive.

## 2.4 Riccetti (2010)

In Riccetti (2010) I try to develope the analysis published by Patton (2004). I use models with copulas (Normal, Student-t, Clayton, Gumbel, Frank, mix copulas and Canonical Vine copulas) in an allocation on two, three and four assets, analyzing various combinations of indices to test whether the use of copulas improves the investor's utility and revenue.
The copula model is so structured:

- means are unconditional;
- variances are explained by a GARCH(1,1), using error terms extracted from a Student-t distribution;

- the joint behaviour of the residuals of the indices is modelled by a copula.

I assume that the investor does not face any transaction costs, so there are no costs for rebalancing or for short-selling. Moreover, I take savings decisions exogenously specified and I ignore intermediate consumption: the investor/asset manager has to invest an amount of money at the beginning of the period (that, for simplicity, I take as equal to 1) and does not receive or disinvest anything until the end. As remarked by Ang and Bekaert (2002), the use of a CRRA utility does not address market equilibrium, so the investor can not necessarily be the representative agent.
Differently to Patton, who uses monthly returns, in my analysis I use daily returns, but I aim to obtain weekly, monthly and yearly allocations. In this way, I cover quite a long time period without rebalancing (and this reduces the problem of not considering rebalancing costs), but I can use more information compared to few monthly returns[2]. However, to do this for a weekly allocation, for example, I need to simulate a path of 5 returns (I assume that a week has 5 working days).
The daily time series are obtained from Datastream, for the period that goes from January 1999 to September 2008, but the model is chosen on data from January 1999 to September 2005, because the last three years are left for the out-of-sample analysis.
The steps to build the optimal portfolio are:

1. estimate with the Maximum Likelihood the parameters of the above model;

2. simulate 5000 times the path of assets returns on the investment horizon (for example 5 days for the weekly allocation);

3. choose the optimal weights in order to maximize the investor's expected utility, using the CRRA utility function:

$$U(\gamma) = (1-\gamma)^{-1}(P_0 R_{port})^{1-\gamma} \tag{1}$$

   with $R_{port}$ that is the portfolio capitalization factor and the initial investment $P_0$=1. The values of $\gamma$ used are: 2, 5, 10 and 15.

From those analyses I conclude that the best copula model for a portfolio composed by 2 or 3 assets, is the one that uses the Student-t copula, but it is only useful for deciding the macro-asset allocation (for choosing the stock and the bond composition of the portfolios) or if the portfolio is composed by one bond index and some stock indices, especially when the investor is very risk averse. In the other cases the copula model does not beat mean-variance of naive allocations. Here I report the result of the copula model compared to the mean-variance model and to an allocation equally divided among the assets, in the case of a portfolio composed by three indices: Italian Mibtel, U.S. Dow Jones and an Italian Long

[2]Similar to Morana (2009) who compute monthly realized betas from daily data

Term Bond index. The allocation is rebalanced weekly for three years (156 rebalancing every 5 working days). The results are in table 1 and they show that the copula model obtains the best utility (also driven by the highest final amount) for all levels of risk aversion except $\gamma$=2 (when the equally divided portfolio is the best).

Table 1: *Utility and final amount with weekly allocation for a portfolio composed by Mibtel, Dow Jones and an Italian Long Term Bond index.*

| | | | | | |
|---|---|---|---|---|---|
| Utility | Portfolio | $\gamma$ 2 | $\gamma$ 5 | $\gamma$ 10 | $\gamma$ 15 |
| | Copula | -1,0023 | -0,2511 | -0,1119 | -0,0721 |
| | Mean-Variance | -1,0038 | -0,2517 | -0,1122 | -0,0723 |
| | Equal Division | -1,0009 | -0,2512 | -0,1126 | -0,0734 |
| Final Amount | Portfolio | $\gamma$ 2 | $\gamma$ 5 | $\gamma$ 10 | $\gamma$ 15 |
| | Copula | 0,8425 | 0,9564 | 0,9674 | 0,9677 |
| | Mean-Variance | 0,6750 | 0,8781 | 0,9274 | 0,9411 |
| | Equal Division | 0,8747 | | | |

For the analyses of this paper I will use the weights obtained for this portfolio allocation, so I have 156 weights for the three years of out-of-sample allocation. With this portfolio composed by 3 assets, I can study in the same simulation the way to reconstruct the series of the weights of two assets (the third series could be calculated as 1 less the other two weights): Mibtel and Dow Jones. I will analyse the case of an investor with $\gamma$=10, in which the copula model has a very good performance, but I will also try the case of a $\gamma$=5 investor as a robustness check.

# 3 Models

I build two models that consider the non central moments and co-moments of the returns till the fourth order (respectively the mean of the returns and the mean of the crossed products of the returns raised up to fourth power) in order to understand if they can approximate the use of copulas to obtain optimal weights. In practice, I run OLS regressions using as independent variables the non central moments and co-moments of the returns.
The first model reconstructs the weights using only moments and co-moments, thus I run the following OLS regression on the weights obtained with the copula models ($W_t^C$):

$$W_t^C = \alpha + mom_t\beta' + \epsilon_t \tag{2}$$

where $mom_t$ is the row vector of all moments and co-moments.
The second model reconstructs the copula model weights using moments, co-moments and

the mean-variance weights:

$$W_t^C = \alpha + W_t^{MV}\phi + mom_t\beta' + \epsilon_t \tag{3}$$

Then I repeat the two OLS regressions using as independent variable not moments and co-moments of the returns, but moments and co-moments of the excess returns of Mibtel and Dow Jones over the return of the Italian Long Term Bond index; in this way I need to calculate less combinations of moments and co-moments.

# 4 In-sample results

## 4.1 Reconstruct copula model weights using only moments and co-moments - Model 1

I run an OLS on the weights obtained by the copula model using as independent variables the mean of the returns and of the (cross) products of the returns raised up to fourth power. The labels in the regressions are so composed: the first part represents how high is the moment and the second part means which assets are present in that combination (1 = Mibtel, 2 = Dow Jones, 3 = Italian Long Term Bond Index). For example:

- $m1\_1$ = mean ($r_{mibtel}$);
- $m1\_2$ = mean ($r_{dowjones}$);
- $m2\_1$ = mean ($r_{mibtel} * r_{mibtel}$), like a variance, but it is not a centered moment;
- $m2\_12$ = mean ($r_{mibtel} * r_{dowjones}$), like a covariance, but again it is not a centered moment;
- $m3\_1$ = mean ($r_{mibtel} * r_{mibtel} * r_{mibtel}$), similar to a skewness measure, but it is not a centered moment;
- $m3\_112$ = mean ($r_{mibtel} * r_{mibtel} * r_{dowjones}$);
- $m3\_123$ = mean ($r_{mibtel} * r_{dowjones} * r_{longtermbond}$);
- $m4\_1$ = mean ($r_{mibtel} * r_{mibtel} * r_{mibtel} * r_{mibtel}$), similar to a kurtosis measure.

The vector $mom_t$ contains all these moments and co-moments, thus the model is the one reported in eq.2:

$$W_t^C = \alpha + mom_t\beta + \epsilon_t$$

As already noted by Patton (2004), this regression is probably misspecified (because optimal weights are nonlinear function of the parameters of the joint density, so this is a first-order

approximation of the true function relating parameters to weights) and suffer from error-in-variables bias (because all variables are estimated). However, I want to find a simple and fast method to reproduce the copula weights without estimating a copula.
Differently from Patton, I do not use the copula parameter as a regressor (it is obvious: I do not want to understand the cause of the differences in weights between the two mean-variance and the copula models, but I want to have the copula model weights without the copula), but I insert the co-moments up to the fourth order, to catch the interactions between assets[3].

I begin with the analysis of table 4 that contains the results of the regression on the weights of the copula model for Mibtel index, for an investor with risk aversion $\gamma$=10. The Ramsey test (RESET test) shows a misspecification trouble, as supposed by Patton, and the test for omission of variables points out that some variables are useless.
Moreover the signs of the coefficients are sometimes different from what I expect. For example the fourth moment of Mibtel (m4_1) is significantly positive while I expect it to be negative: if a measure similar to the kurtosis of the Mibtel increases, the copula model should give less weight to Mibtel. This problem could be caused by the very high multicollinearity among all the independent variables (shown by the Variance Inflation Factor, not reported here): to really understand what happens when the kurtosis of the return of Mibtel increases, all moments and co-moments have to be considered together.
However the regression is very powerful, indeed the $R^2$ is 0.89 and it shows that, in this case, it is possible to reproduce quite well the copula weights using moments and co-moments of returns.
To reduce the multi-collinearity problem without losing large explanatory power, I exploit the result of the test for omission of variables and I progressively eliminate the variable with the highest p-value till all variables have a p-value under 10%[4]; the final regression, that I call "reduced", is reported in table 5. The results obtained in the "reduced" regression are very close to those obtained in the OLS with all the moments and co-moments:

- the $R^2$ value is very high (88%);
- all remained variables have the previous sign;
- the multicollinearity problem is reduced, but it is still strongly present.

[3]The use of co-moments is often adopted also in the asset pricing, as an extension of the CAPM. In these models the use of co-moments is also extended to higher moments and co-moments, till an order of 10 or 15, as done by Chung *et al.* (2006) or Nguyen and Puri (2006).

[4]This procedure is often called "backward iteration". The model is estimated with all variables. Then the variable with the highest p-value is eliminated. The model is estimated again without the deleted variable. Among the remained variables, the one with the highest p-value is eliminated. The model is estimated again without the two eliminated variables. The process goes on till there is an estimate in which all variables have a p-value smaller than 10%.

In other words, with the model with less explanatory variables the multi-collinearity problem is reduced without losing explanatory power. Moreover, the reduced models improve the out-of-sample forecast (see section 7).
These features are common for both models and at all levels of risk aversion, and also with the use of the excess returns instead of the returns, so I will report only the reduced regressions in the rest of the paper.

I study the series of weights of Dow Jones index for the same portfolio of the same investor (that has $\gamma$=10); estimates of the reduced model are in table 6. Again I can observe that:

- the $R^2$ is high, 83%, so the model reconstructs the copula weights in a good way;
- the Ramsey test shows a misspecification trouble;
- the variables that are statistically significant for this regression are different from the variables used for the OLS in the case of Mibtel. This means that each index has a different model for its weight, so there is not a standard way to reproduce these series, but each index has a peculiar model;
- the multicollinearity problem is strongly present (as shown by the Variance Inflation Factors, not reported) and probably it is the cause of the unexpected signs for some coefficients; for example, a higher second moment of Mibtel index (m2_1) should increase the weight of Dow Jones, instead it has a negative coefficient. However, with this kind of model, as already explained, I can not expect to forecast the signs of moments and co-moments.

The previous results are confirmed by all regressions at all $\gamma$ levels (see section 5).
Similar results are obtained without using the three series of returns, but using the two series of excess returns of Mibtel and Dow Jones over the Italian Bond index, as done by Patton (2004). These results are in section 6.

I can conclude that I am able to obtain an approximation of the weights of the copula model with these simple models, that are using moments and co-moments of returns.

## 4.2 Reconstruct the copula weights using the mean-variance weights - Model 2

I reproduce the weights of the copula model using also the mean-variance weights, as expressed in equation 3:

$$W_t^C = \alpha + W_t^{MV}\phi + mom_t\beta + \epsilon_t$$

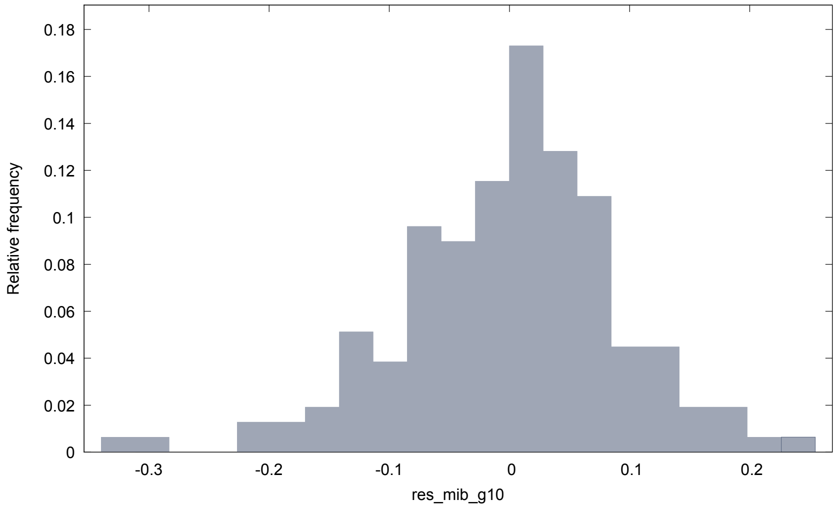
In the tables the mean-variance weights are denoted as "norm" plus the $\gamma$ coefficient of the investor and the name of the index, for example "$norm10\_mib$" In the case of an investor with risk aversion $\gamma$=10, the reduced model for the weights of Mibtel is reported in table 7, while the reduced model for the weights of Dow Jones is in table 8. Observing both tables, I note that:

- these regressions have very high $R^2$ (96% for Mibtel and 87% for Dow Jones weights), thus these models reconstruct well the copula weights;
- the coefficient of the mean-variance weights ($\phi$ in equation 3) is always positive and significant, thus this variable is surely useful; moreover the coefficient is smaller than 1 (indeed copula model's weights are usually less large than mean-variance weights);
- moments and co-moments help in explaining the copula weights (as already found with model 1). Moreover some first and second order moments and co-moments are significant, even if the mean-variance weights already consider these moments; however, compared with the first model, I note, as expected, that signs are often (but not always) the same for moments and co-moments of the first and second order; instead moments and co-moments of higher order always have the same sign of the model without mean-variance weights (even if the significant moments and co-moments are not always the same);
- in this case the signs of the coefficients are more consistent with what I expect, but I can not forecast all the signs of the coefficients of the moments and of the co-moments, and which variables are significant;
- the problems with the signs are probably due to the multicollinearity problem; however the models seem correctly specified, indeed Ramsey's RESET and White test do not signal problems;
- the model on Mibtel weights and the model on Dow Jones weights are very different about the statistically significant variables (except than the mean-variance weights that always have a coefficient significant and positive).

I analyse better the goodness-of-fit: as already noted, these regressions have very high $R^2$ (between 87% and 96%), higher than those of the first models. The improvement in the value of $R^2$ is small, but it could have a relevant impact on the improvement in the sum of squared residuals: in the case of Mibtel, this value falls significantly from 4.28 in the first model to 1.38 in the second one, while in the case of Dow Jones it falls only from 1.51 to 1.18. This difference is also more evident observing the standard errors of the residuals: in the case of Mibtel they pass from 17.8% to 9.8%, while in the case of Dow Jones they pass only from 10.4% to 8.9%.
However, with these small values for the standard errors of residuals (even if for $\gamma$=5 they are higher and they appear economically significant, see section 5) I can confirm that these models

reconstruct well the copula weights. Moreover the errors are often very small; for example, for Mibtel weights, over 90% of errors are between -14% and +14%, with few relevant errors, as show in figure 1.

Figure 1: Frequency plot of the residuals of the regression on Mibtel weights for an investor with $\gamma$=10




I can conclude that both models analyzed are useful to model the copula's weights: the first appears easier (because the weights of the Markowitz model are not needed), but the use of the mean-variance weights renders the estimate more accurate. In situations like Mibtel (or in presence of small $\gamma$ as we will see in section 5), perhaps, it is more useful to use also mean-variance weights because there is a good improvement in performance, while the use of regressions without this variable is faster in cases like Dow Jones (or with very risk averse people), when these regressions are already good and the use of the mean-variance model weights does not improve significantly the estimation.

# 5 Robustness checks - $\gamma$=5 investor

The previous results are confirmed by all regressions at all $\gamma$ levels and both models present the same features, so I report the results only for model 1 in the case of an investor with risk aversion $\gamma$=5. Table 9 presents the results of the regression (after the backward iteration on the independent variables) on Mibtel weights, while table 10 reports the results for Dow Jones weights. I can observe that the regressions have the same features of the regression on weights obtained for an investor with $\gamma$=10. For example observing the regression on weights for Mibtel index, it is again very powerful with the same $R^2$ of 0.89, it has the same problems of multicollinearity and of specification (shown by the Ramsey test), it has almost the same significant variable (with only small differences) and the same sign of the coefficient.
A difference is that the signs of the coefficients have higher absolute values (and higher

standard error) in the case of an investor with $\gamma$=5 and it is normal because the absolute values of weights are more extreme with a less risk averse investor.
Another difference is that the sum of the squared residuals is much higher in the case of $\gamma$=5, indeed it passes from 4.28 to 15.93, due to the fact that weights are more extreme. In this case the standard errors of the residuals are economically significant, indeed the value is about 34% for the Mibtel index. However, as shown by the high value of $R^2$, these values are small if I consider that, for $\gamma$=5, the weights of the indices (without a short-selling constraint) are very large: in the case of Mibtel the weights vary from a minimum of -86% to a maximum of +356%, with a standard deviation of 95%. Moreover, also in this case, the errors are often very small, with only few relevant errors.
All the previous considerations can be replied in the case of Dow Jones index's weights, even if with this index the weights are less extreme, so the standard error of residuals is smaller than in the case of Mibtel: the weights of the portfolio vary from a minimum of -177% to a maximum of +40%, with a standard deviation of 49%, so the standard error of the residuals of the regression is about 21%.
As already said, the previous features are present also for model 2 and also the comparison between model 1 and 2 has the same characteristic at every level of risk aversion. So I can conclude that the results found in section 4 are confirmed by all regressions at all $\gamma$ levels: both models analyzed are useful to model the copula's weights, but the first appears easier (because the weights of the Markowitz model are not needed), while the second renders the estimate more accurate; so in situations like Mibtel and/or in presence of small $\gamma$, it appears more useful to use also mean-variance weights because there is a good reduction of the standard error of the residuals, while the use of regressions without this variable is faster in cases like Dow Jones and/or with very risk averse people, when these regressions have already a small value of standard error of the residuals and the use of the mean-variance model weights does not improve significantly the estimation.

# 6 Regressions with excess returns

## 6.1 Model 1

I run the same regressions on the copula model weights, using as independent variables the moments and co-moments calculated on the series of excess returns of Mibtel and Dow Jones over the Italian Long Term Bond index:

- excess return 1 = $r_{mibtel} - r_{longtermbond}$ ;
- excess return 2 = $r_{dowjones} - r_{longtermbond}$ .

Thus the independent variables are written, for example, in this way:

- $exm1\_1$ = mean (excess return 1);
- $exm2\_1$ = mean (excess return 1*excess return 1);
- $exm2\_12$ = mean (excess return 1*excess return 2);
- $exm3\_2$ = mean (excess return 2*excess return 2*excess return 2);

Now I am not using the three series of returns but the two series of excess returns, as done by Patton (2004). In table 11 there are the results (after the backward iteration) for Mibtel weights for an investor with risk aversion $\gamma$=10. I can note that these results present differences compared to the ones obtained with the three returns used separately:

- the $R^2$ is a bit lower (0.76 versus 0.88), thus the sum of squared residuals is higher;
- with excess returns, signs of the coefficients are more coherent with our expectations (observe the significant variables: $exm1\_1$ shows that if the return of the Mibtel increases, the weight of the Mibtel increases; $exm2\_2$ shows that if the "variance" of the Dow Jones increases, the weight of the Mibtel increases; $exm3\_2$ shows that if the "skewness" of Dow Jones improves, the weight of the Mibtel decreases).

However, there are also some similarities compared to the model obtained with the three returns used separately:

- the model residuals show a misspecification trouble (see the Ramsey test);
- the multicollinearity problem is strongly present (as shown by the Variance Inflation Factors, not reported), even if it is smaller than in the case of total returns (and perhaps it is the reason for the coefficients more coherent with our expectations);
- some similar variables (for example mean return of Mibtel in the regression with separated returns and mean excess return of Mibtel in this OLS) have coefficients with the same sign, even if the significant variables are not the same, so it means that there is not a standard way to reproduce these weights and each case needs a peculiar model.

I also repeat the OLS on the weights of Dow Jones index. The result of the reduced model, for a $\gamma$=10 investor, are in table 12. I can confirm the considerations already done for Mibtel:

- the $R^2$ is a bit lower and the sum of squared residuals is higher;
- the model residuals show a misspecification trouble;
- the multicollinearity problem is strongly present, even if smaller than in the case of total returns and, with excess returns, signs of the coefficients are consistent with my expectations;

- the similar variables that were significant in the other regression with returns (and not excess returns) are not the same of this case;

These results are confirmed by all regressions at all $\gamma$ levels. Moreover the coefficients are almost the same at different $\gamma$, with the only difference that for higher $\gamma$ the absolute values of the coefficients are smaller.

## 6.2 Model 2

For Model 2 I run the same regressions on the copula model weights using as independent variable the moments and co-moments calculated on the series of excess returns of Mibtel and Dow Jones over the Italian Long Term Bond index and the mean-variance weights. As an example, I report in table 13 the estimate of the reduced model for Mibtel weights for an investor with $\gamma$=10. This model is very similar to model 2 done with the total return (see table 7):

- the performance is almost the same: the $R^2$ decreases from 96.2% to 96.1%, the sum of squared residuals increases from 1.38 to 1.45 and the standard error of the residuals is the same around 9.8%;
- the most important variable is the correspondent weight of the mean-variance model, that has a positive coefficient with a value above 0.8, but, with the reduced model, there are also moments and co-moments statistically significant;
- there are not misspecification problems.

I repeat the analysis on the weights for the Dow Jones index and I find the same results found in the case of Mibtel, as shown in table 14. So, observing these tables, I note that these regressions have a very good performance (high $R^2$, small standard errors of residuals), thus these models reconstruct well the copula weights, and this performance is almost as good as the performance of model 2 that uses the three returns and not the two excess returns.

In conclusion I can use the returns or the excess returns of the stock indices over the less risky index. The first way is more accurate (has a higher $R^2$ in the regressions and smaller standard errors of residuals), while the model with excess returns needs to calculate less combinations of moments and co-moments.
It is better to use the total returns in situations in which the mean-variance weights does not improve significantly the estimates (in cases like Dow Jones weights and/or with very risk averse people) and I prefer to use the faster model 1 (moments and co-moments only), therefore the use of excess return creates a significant loss in the accuracy of the regressions. Meanwhile, in situations in which the use of the mean-variance weights is relevant (in cases like Mibtel weights and/or in presence of small $\gamma$), so I prefer model 2, the use of the excess returns can be a valid alternative to the use of total returns.

# 7 Out-of-sample results

Till now I have analysed the in-sample goodness-of-fit of the regressions. However in the real world an investor/asset manager that wants to use one of the two models to have an allocation similar to the one obtained by a copula model, has to forecast the weights out-of-sample.
I leave the observations of the last year out-of-sample (so the last 52 observations).
The out-of-sample analysis is done only in the cases that use the three separated returns and not with the use of the two excess returns.
To measure the goodness-of-fit I will use the mean absolute error (M.A.E.):

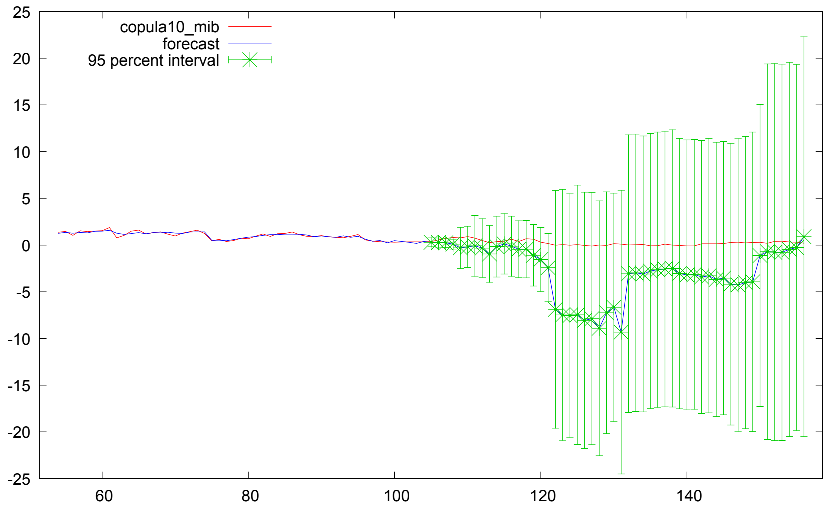
$$M.A.E. = \sum |errors| /n \tag{4}$$

where $n$ is the number of the out-of-sample observations.

## 7.1 Model 1

Here I report the results for the Mibtel weights in the case of an investor with $\gamma$=10. The first analysis hypothesizes that I want to forecast all the copula weights using the model estimated at the beginning of the year (so with the first 104 observations): I call this the "static" forecast. This model obtains a very bad M.A.E. equal to 310%. Observing the plot of the forecast and of the real values (fig.2) I note that the model begins its very bad performance from week 122, with a very unrealistic forecast of -689% against a real weight of -2%.

Figure 2: Forecast and real weights for Mibtel - static forecast



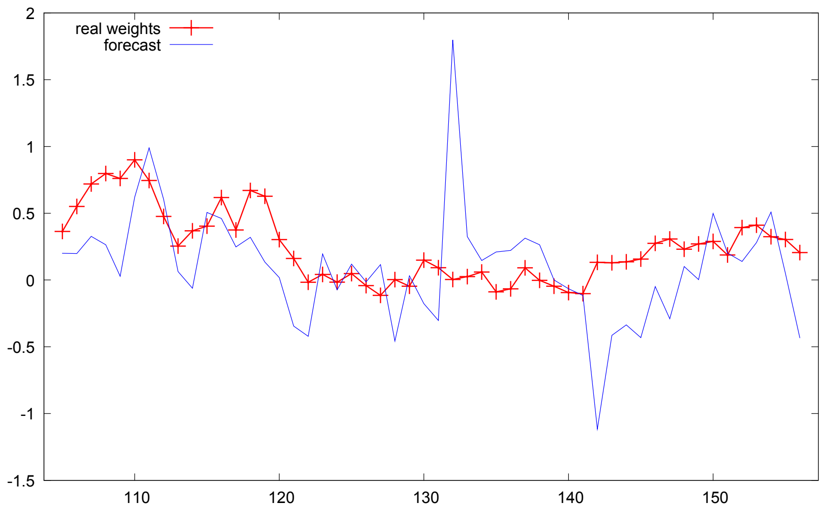

However, if I use the reduced model, which estimates are shown in table 16, the M.A.E. is again very bad, but much better: 156%. This is not a general rule, indeed the reduced model does not improve the M.A.E. in the case of Dow Jones weight for the static model (that forecasts all the 52 weeks). However, the reduced model is often better than the model that contains all the independent variables: for example it happens with the dynamic forecast (see after for the meaning of "dynamic" forecast) and with the static forecast done on shorter periods (for example 9 or 17 weeks, such as I will show in table 3). For this reason, I will only analyse the out-of-sample performance of the reduced models.

Going back to the reduced model 1, I have already observed that it obtains a very bad M.A.E. equal to 156%. So, the model estimated at week 104 is not good in the forecast for all the year. Perhaps the use of a one-step ahead forecast, done with the up-to-date model, should be better: for every week of the last year I estimate the model and I forecast the weights for the next week, then I compare the forecast weights with the real copula model weights. I call this the "dynamic" forecast.

In this case the M.A.E. is much smaller (32.4%).

Again, I report the plot with the forecast and the real values of the copula model's weights (fig.3). I note that the model presents a very bad forecast for some weeks, such as week

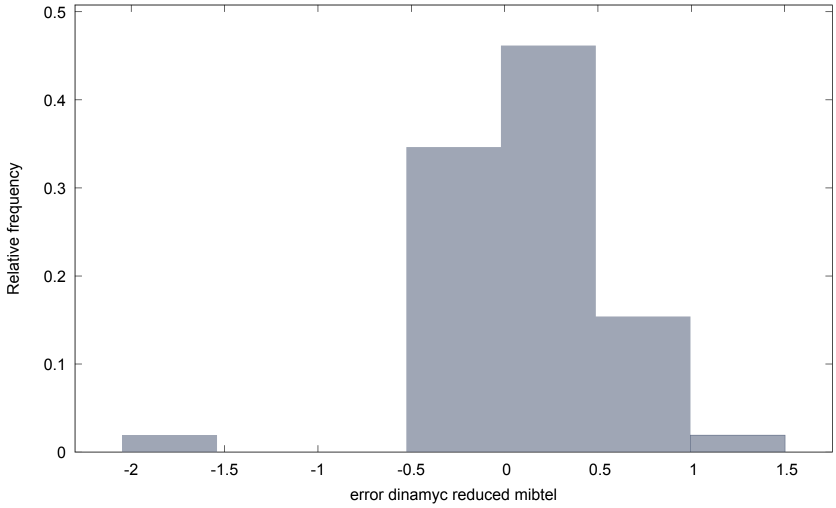
Figure 3: Forecast and real weights for Mibtel - dynamic forecast




132 and 142, but in this case the model does not repeat the error for the following weeks. Observing the output of the regression done on 104 observations and the one done with the full sample, it is evident that there are some important changes during the 52 weeks. Estimating the model every week, I observe that there are only small changes in the estimates and the biggest changes are in weeks after a bad forecast, such as week 142. Thus, a good strategy could be to use the first estimate of the model till the error is not too big, then re-estimate (the re-estimation is done with all moments and co-moments as independent variables and not only the ones used in the previous reduced model) the model and keep it till a new unrealistic forecast.

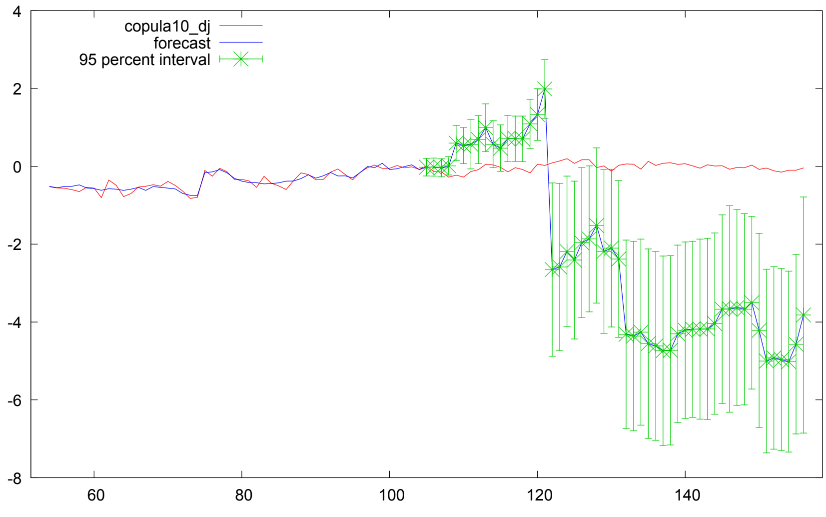
As already said the M.A.E. is 32.4%, which is not a huge value (compared to the high values of the weights, caused by the fact that in this portfolio there are only three assets and there are not short-selling constraints), but it is not an optimal prediction. However, this value is due to few very bad forecasts, indeed the forecasts are usually enough accurate, as shown in fig.4.

Figure 4: Frequency of the forecast error - Mibtel weights




Now I briefly show the results for Dow Jones weights, that are qualitatively the same as Mibtel.I report in table 17 the reduced model estimated at week 104 and in figure 5 the graph of the forecast done with this static model for all the 52 observations out-of-sample.

Figure 5: Forecast and real weights for Dow Jones - static forecast

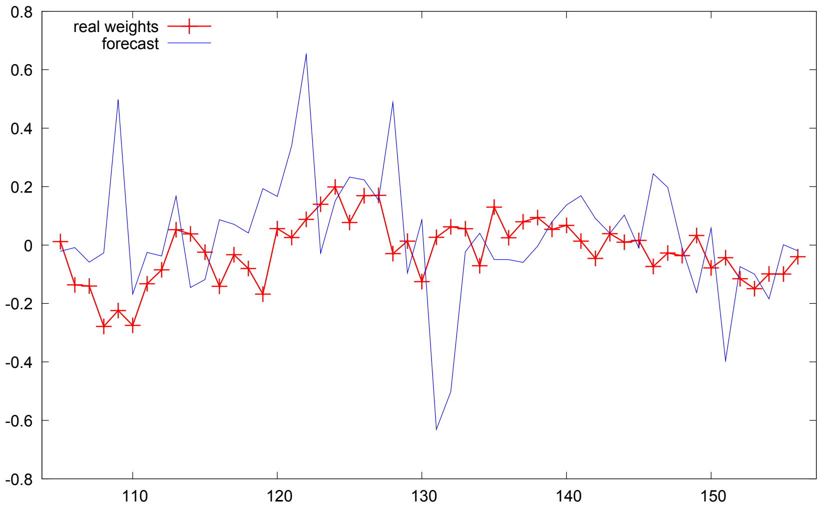



Again the model presents a very bad M.A.E. of 273%; however, I can note from the plot of the forecast and of the real values (fig.5) that the model has not a very bad performance for some weeks, then the error becomes larger and larger. So the model estimated at week 104 is not good in the forecast for all the year, but only for the following weeks.

The use of a one-step ahead forecast, done with the up-to-date model, has a much smaller M.A.E.= 17.1%.

As in the case of Mibtel, I report the plot with the forecast and the real values of the copula model's weights (fig.6), to show that there are only a few heavy errors. Observing that the

Figure 6: Forecast and real weights for Dow Jones - dynamic forecast




estimates of the model at time 104 are quite different from those at time 156, I can confirm that the strategy proposed in the case of Mibtel can be a good solution to apply to this model overcoming the temporal instability of the estimates, but avoiding the computation of the copula model every week (indeed, in that case the weights of the copula model could be used directly!) or too often: an investor/asset manager could use the model estimates till a bad forecast, than he/she has to re-estimate the model till a new unreal or bad forecast.

## 7.2 Model 2

I repeat the same analysis done in the previous section, using also the mean-variance weight as independent variable. I begin estimating the reduced model 2 for Mibtel weights at week 104 (see table 17). The mean-variance weight is again statistically significant and positive (near 0.8)

This model obtains a bad M.A.E. of 134.4%, even if it is better than the correspondent M.A.E. obtained by model 1. Observing the plot of the forecast and of the real values (fig.7), I see that the prediction is almost perfect for the first 9 weeks and not very bad till week 122. So, also with model 2, the estimates at week 104 are not good in the forecast for all the year. I compare this forecast with the dynamic forecast (one-step ahead), done with the up-to-date model. Now the M.A.E. is much smaller at 10.7%. The plot with the forecast and the real values of the copula model's weights (fig.8) shows that there are not huge errors (except at week 114). Comparing the two forecasts, I can understand that at week 122 there is a change in the estimates that makes the static model to be permanently no more useful, so after that week there is the need to estimate again the model. This is confirmed by the observation of

Figure 7: Forecast and real weights for Mibtel - static forecast

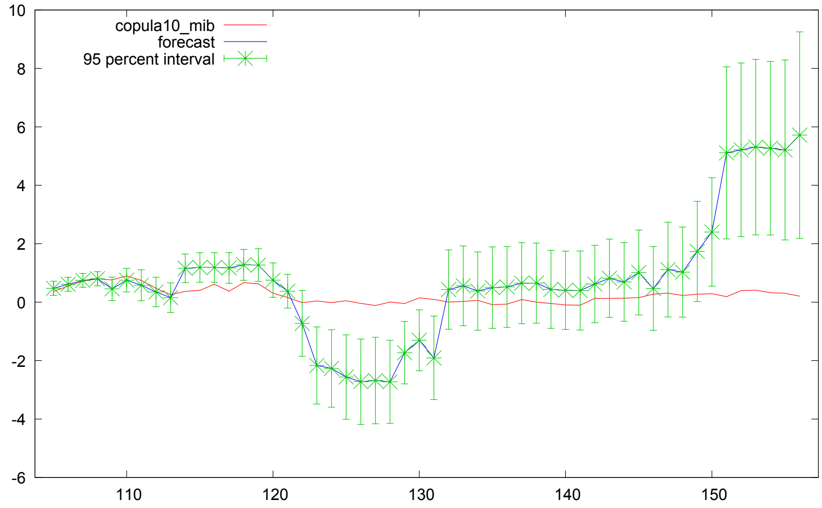
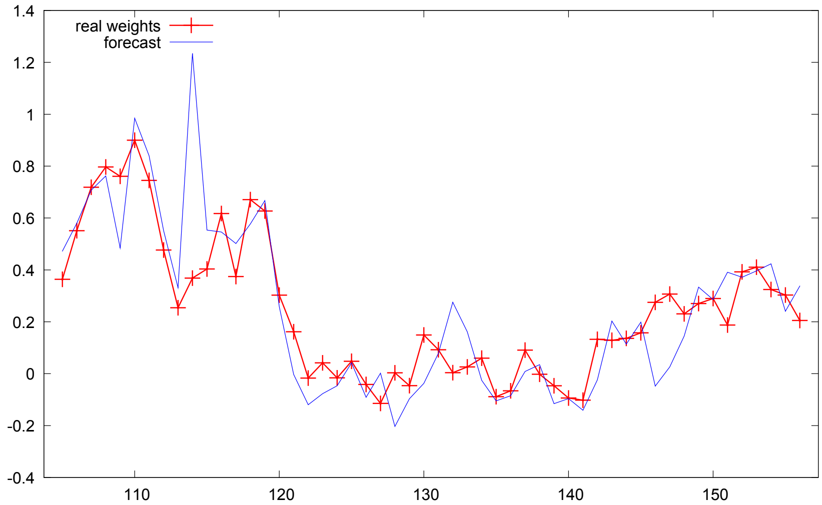



Figure 8: Forecast and real weights for Mibtel - dynamic forecast




the outputs of the regressions done on 104 and 156 observations, that show some important changes during the 52 weeks. Thus, again, a good strategy could be to use the model without re-estimating it till an unrealistic (the changes in weights are usually not so big as it appears in the forecast) or bad forecast, then be careful in the weight of that week and then re-estimate the model the following week and keep it till a new unrealistic or bad forecast.

I do not report the results for Dow Jones weights, because they are qualitatively the same as Mibtel:

- the estimates of the model at time 104 are quite different from the estimates at time 156 (with different moments and co-moments statistically significant and with different sign);
- the static forecast (done with the model estimated only at time 104) has a bad M.A.E.

Table 2: *M.A.E. of the static and dynamic forecast of reduced model 1 and 2 on Mibtel and Dow Jones weights.*

| M.A.E. | Model 1 static | Model 2 static | Model 1 dynamic | Model 2 dynamic |
|---|---|---|---|---|
| Mibtel | 156.4% | 134.4% | 32.4% | 10.7% |
| Dow Jones | 273.4% | 158.8% | 17.1% | 10.1% |

Table 3: *M.A.E. of the static forecast of model 1 and 2, full and reduced (red), on the first 9 and 17 weeks of Mibtel and Dow Jones weights.*

| Mibtel | Model 1 | Model 1 red | Model 2 | Model 2 red |
|---|---|---|---|---|
| 9 weeks | 73% | 75% | 43% | 12% |
| 17 weeks | 95% | 115% | 36% | 35% |
| Dow Jones | Model 1 | Model 1 red | Model 2 | Model 2 red |
| 9 weeks | 55% | 9% | 28% | 19% |
| 17 weeks | 69% | 24% | 38% | 24% |

of 158.8%, but with very good estimates till week 121;

- the dynamic forecast (done with the model estimated every week) presents a M.A.E. of 10.1% (with a not small error at week 122).

I summarize all the results in table 2.

In conclusion model 2 presents better M.A.E. than model 1, especially for static forecast which does not estimate the model often, while in the dynamic forecast (the use of a one-step ahead forecast, done with the up-to-date model) the improvement of M.A.E. is smaller (for example in the Dow Jones case it passes from 17.1% to 10.1%), even if economically significant (especially in cases as Mibtel when it passes from 32.4% to 10.7%).
As already said and as evident from the difference in the M.A.E of the static and dynamic forecast, the static forecast has a good performance only for some weeks, so a strategy could be to use the model without re-estimating it till an unrealistic or bad forecast. In table 3, I report the M.A.E. of the static forecast done for the first 9 weeks and for the first 17 weeks. These values are chosen because they are the periods in which the static forecasts have a good performance and also because they represent roughly 2 and 4 months respectively.
The table shows some features:

- model 2 is better (except reduced model for Dow Jones weights on 9 weeks horizon) than model 1 and the improvement is stronger in the case of Mibtel, the index with the highest variance, as already found in the in-sample analysis;
- the reduced model is better (except model 1 for Mibtel weights) than the model with all moments and co-moments;

- the static model is more unsatisfactory when the forecast week is further away from the estimate. Indeed the M.A.E. on 9 weeks is better (except non reduced model 2 for Mibtel weights) than the M.A.E. on 17 weeks.

So, if I use the strategy to estimate the copula model weights with reduced model 2 (using also the mean-variance weights), for the first 9 weeks I have a mean absolute error of 12% for Mibtel and of 19% for Dow Jones. That is a very good result considering the high values of the weights of these indices.

# 8 Conclusions

In conclusion, I am able to have an approximation of the weights obtained by a copula model using moments and co-moments of returns. I analyse two models: the first one models the copula model weights using only moments and co-moments and the second one models the weights using moments, co-moments and the mean-variance weights. Both models are useful: the first appears to be easier (because the mean-variance weights are not needed), while the second is more accurate in-sample and, above all, out-of-sample. In situations when the time series of the weights have a high variance (like Mibtel or in presence of small $\gamma$), perhaps, it is more useful to use the second model because there is a good improvement in performance, while the use of a regression without the mean-variance weights is faster in cases when the series of weights have small variance (like Dow Jones and with very risk averse people), when this regression is already good and the use of the mean-variance model does not improve significantly the estimation.
Moreover, I can use both the returns or the excess returns of the stock indices over the less risky index. The first way is more accurate, while the model with excess returns needs to calculate less combinations of moments and co-moments. In situations in which the use of the mean-variance weights does not improve significantly the estimation (in cases like Dow Jones weights with very risk averse people) and I prefer the faster use of the moments and co-moments only (model 1), the use of excess return creates a significant loss in the accuracy of the regressions, so it is better to use the total returns. In situations in which the use of the mean-variance weights is relevant (in cases like Mibtel weights or in presence of small $\gamma$) and I prefer regressions that use also this variable (model 2), the use of the excess returns can be a valid alternative to the use of total returns.
The estimated models are often similar at various levels of risk aversion, but they are different for each asset, with explanatory variables and signs that are not predictable and with accuracy that is uncertain (uncertain $R^2$ and sum of squared residuals).
Moreover, observing the out-of-sample analysis, I understand that these models are also changeable in time, so, to put into practice, a good strategy could be to use the model estimating it once till an unrealistic forecast, then estimate again the model and keep it till a new un-

realistic forecast. In this way there is no need to estimate both copula model weights and the OLS too often and, at the same time, the prediction is quite accurate.

## References

Ang, A. and G. Bekaert (2002), "International Asset Allocation with Regime Shifts", *Review of Financial Studies*, 15(4), pp. 1137–1187.

Arditti, F. (1967), "Risk and the Required Return on Equity", *Journal of Finance*, 22, pp. 19–36.

Bekaert, G., C. Erb, C. Harvey and T. Viskanta (1998), "Distributional Characteristics of Emerging Market Returns and Asset Allocation", *The Journal of Portfolio Management*, 24(2), pp. 102–116.

Chung, Y., H. Johnson and M. Schill (2006), "Asset Pricing when Returns are Nonnormal: Fama-French factors vs higher-order systematic co-moments", *Journal of Business*, 79, pp. 923–940.

Cvitanic, J., V. Polimenis and F. Zapatero (2008), "Optimal portfolio allocation with higher moments", *Annals of Finance*, 4(1), pp. 1–28.

Dittmar, R. (2002), "Nonlinear Pricing Kernels, Kurtosis Preferences, and Evidence from the Cross Section of Equity Returns", *Journal of Finance*, 57, pp. 369–403.

Embrechts, P., R. Frey and A. McNeil (2005), *Quantitative Risk Management: Concepts, Techniques and Tools*, Princeton University Press.

Fama, E. (1963), "Mandelbrot and the stable Paretian hypothesis", *Journal of Business*, 36, pp. 420–429.

Fang, H. and T. Lai (1997), "Co-Kurtosis and Capital Asset Pricing", *The Financial Review*, 3(2), pp. 293–307.

Harvey, C. and A. Siddique (2000), "Conditional skewness in asset pricing tests", *Journal of Finance*, 55(3), pp. 1263–1295.

Ioannidis, C. and J. Williams (2007), "Static and Dynamc Asset Allocation with Higher Moments", Working paper, University of Aberdeen Business School and University of Bath School of Management.

Joe, H. (1997), *Multivariate Models and Dependence Concepts*, Chapman & Hall.

Jondeau, E. and M. Rockinger (2006a), "The Copula-GARCH model of conditional dependencies: An international stock market application", *Journal of International Money and Finance, Elsevier*, 25, pp. 827–853.

Jondeau, E. and M. Rockinger (2006b), "Optimal Portfolio Allocation under Higher Moments", *European Financial Management*, 12(1), pp. 29–55.

Jurczenko, E. and B. Maillet (2006), *Multi-moment Asset Allocation and Pricing Models*, John Wiley & Sons, Ltd.

Kraus, A. and R. Litzenberger (1976), "Skewness Preference and the Valuation of Risk Assets", *Journal of Finance*, 31(4), pp. 1085–1100.

Mandelbrot, B. (1963), "The Variation of Certain Speculative Prices", *Journal of Business*, 36, pp. 394–419.

Markowitz, H. (1952), "Portfolio Selection", *The Journal of Finance*, 7, pp. 77–91.

Morana, C. (2009), "Realized betas and the cross-section of expected returns", *Applied Financial Economics*, 19, pp. 1371–1381.

Nguyen and Puri (2006), "Higher order systematic co-moments and asset-pricing: new evidence", Working paper.

Patton (2004), "On the Out-of-Sample Importance of Skewness and Asymmetric Dependence for Asset Allocation", *Journal of Financial Econometrics*, 2(1), pp. 130–168.

Riccetti, L. (2010), *The Use of Copulas in Asset Allocation: When and How a Copula Model can be Useful?*, LAP Lambert.

Scott, R. and P. Horvath (1980), "On the direction of preference for moments of higher order than the variance", *The Journal of finance*, XXXV(4), pp. 915–919.

Simkowitz, M. and W. Beedles (1978), "Diversification in a Three Moment World", *Journal of Financial and Quantitative Analysis*, December, pp. 927–941.

Williams, J. and C. Ioannidis (2002), "Higher Moments, Utility Functions and Asset Allocation", Working paper, University of Bath - School of Management.

# A Appendix: Tables with Estimated Models

## List of Tables

## Table 4: Model 1 - Mibtel weights

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | −16.9945 | 12.3356 | −1.3777 | 0.1708 |
| m1_1 | 126.689 | 28.3894 | 4.4626 | 0.0000 |
| m1_2 | 63.6412 | 23.2406 | 2.7384 | 0.0071 |
| m1_3 | 193.535 | 87.3440 | 2.2158 | 0.0286 |
| m2_1 | 12.7422 | 23.0882 | 0.5519 | 0.5820 |
| m2_2 | 40.0890 | 20.3435 | 1.9706 | 0.0511 |
| m2_3 | 194.439 | 330.120 | 0.5890 | 0.5570 |
| m2_12 | −63.3158 | 29.5129 | −2.1454 | 0.0339 |
| m2_13 | −136.903 | 150.960 | −0.9069 | 0.3663 |
| m2_23 | 246.766 | 136.507 | 1.8077 | 0.0731 |
| m3_1 | 1.85847 | 13.0211 | 0.1427 | 0.8867 |
| m3_2 | −8.49073 | 5.46248 | −1.5544 | 0.1227 |
| m3_3 | −9.86500 | 381.927 | −0.0258 | 0.9794 |
| m3_123 | −118.137 | 93.3024 | −1.2662 | 0.2079 |
| m3_112 | −32.8922 | 13.5257 | −2.4318 | 0.0165 |
| m3_122 | 22.5713 | 12.6572 | 1.7833 | 0.0770 |
| m3_113 | −45.1638 | 77.4488 | −0.5831 | 0.5609 |
| m3_133 | −301.594 | 198.724 | −1.5177 | 0.1317 |
| m3_223 | −31.5079 | 49.0310 | −0.6426 | 0.5217 |
| m3_233 | 19.6199 | 174.726 | 0.1123 | 0.9108 |
| m4_1 | 9.39097 | 3.42000 | 2.7459 | 0.0070 |
| m4_2 | −0.757155 | 1.99700 | −0.3791 | 0.7052 |
| m4_3 | −767.731 | 697.452 | −1.1008 | 0.2732 |
| m4_1112 | −4.72961 | 4.84150 | −0.9769 | 0.3306 |
| m4_1122 | −21.7814 | 6.74711 | −3.2283 | 0.0016 |
| m4_1222 | 15.2295 | 5.78505 | 2.6326 | 0.0096 |
| m4_2223 | 18.3088 | 25.2783 | 0.7243 | 0.4703 |
| m4_2233 | −46.3872 | 127.641 | −0.3634 | 0.7169 |
| m4_2333 | −789.313 | 404.285 | −1.9524 | 0.0532 |
| m4_1113 | 112.672 | 38.5954 | 2.9193 | 0.0042 |
| m4_1133 | −30.6776 | 216.548 | −0.1417 | 0.8876 |
| m4_1333 | −895.556 | 490.500 | −1.8258 | 0.0703 |
| m4_1123 | −135.863 | 60.1853 | −2.2574 | 0.0258 |
| m4_1223 | 2.19340 | 47.3694 | 0.0463 | 0.9631 |
| m4_1233 | −413.988 | 198.224 | −2.0885 | 0.0389 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.576623 | S.D. dependent var | 0.487096 |
| Sum squared resid | 4.000780 | S.E. of regression | 0.181836 |
| $R^2$ | 0.891211 | Adjusted $R^2$ | 0.860643 |
| $F(34, 121)$ | 29.15438 | P-value($F$) | 1.03e–43 |
| Log-likelihood | 64.38819 | Akaike criterion | −58.77637 |
| Schwarz criterion | 47.96859 | Hannan–Quinn | −15.42119 |
| $\hat{\rho}$ | 0.084562 | Durbin–Watson | 1.826556 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 119) = 3.80304$
with p-value $= P(F(2, 119) > 3.80304) = 0.0250611$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 48.0825
with p-value $= P(\chi^2(68) > 48.0825) = 0.96798$

Table 5: Model 1 reduced - Mibtel weights

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | −13.7563 | 4.10504 | −3.3511 | 0.0010 |
| m1_1 | 125.331 | 17.6437 | 7.1035 | 0.0000 |
| m1_2 | 60.7860 | 18.2246 | 3.3354 | 0.0011 |
| m1_3 | 151.147 | 36.0343 | 4.1945 | 0.0000 |
| m2_1 | 26.6158 | 11.3705 | 2.3408 | 0.0207 |
| m2_2 | 40.8253 | 4.66708 | 8.7475 | 0.0000 |
| m2_12 | −76.1407 | 16.2284 | −4.6918 | 0.0000 |
| m2_23 | 184.589 | 77.0512 | 2.3957 | 0.0180 |
| m3_2 | −7.53856 | 3.21790 | −2.3427 | 0.0206 |
| m3_112 | −18.1228 | 8.43388 | −2.1488 | 0.0334 |
| m3_122 | 14.2183 | 6.38388 | 2.2272 | 0.0276 |
| m3_223 | −58.6037 | 24.9149 | −2.3522 | 0.0201 |
| m4_1 | 6.17405 | 1.42432 | 4.3347 | 0.0000 |
| m4_3 | −722.593 | 173.518 | −4.1644 | 0.0001 |
| m4_1122 | −20.0774 | 4.35849 | −4.6065 | 0.0000 |
| m4_1222 | 11.9277 | 3.19261 | 3.7360 | 0.0003 |
| m4_2333 | −664.326 | 206.164 | −3.2223 | 0.0016 |
| m4_1113 | 85.3283 | 17.5391 | 4.8650 | 0.0000 |
| m4_1333 | −1072.26 | 202.249 | −5.3017 | 0.0000 |
| m4_1123 | −65.1868 | 24.4122 | −2.6703 | 0.0085 |
| m4_1233 | −396.280 | 122.894 | −3.2246 | 0.0016 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.576623 | S.D. dependent var | 0.487096 |
| Sum squared resid | 4.280299 | S.E. of regression | 0.178062 |
| $R^2$ | 0.883611 | Adjusted $R^2$ | 0.866368 |
| $F(20, 135)$ | 51.24506 | P-value($F$) | 4.51e–53 |
| Log-likelihood | 59.12058 | Akaike criterion | −76.24117 |
| Schwarz criterion | −12.19419 | Hannan–Quinn | −50.22806 |
| $\hat{\rho}$ | 0.193397 | Durbin–Watson | 1.608530 |

Table 6: Model 1 reduced - Dow Jones weights

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_dj

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 6.71128 | 2.11074 | 3.1796 | 0.0018 |
| m1_1 | −59.1814 | 7.24674 | −8.1666 | 0.0000 |
| m1_3 | −105.650 | 15.8540 | −6.6639 | 0.0000 |
| m2_1 | −18.4188 | 6.10024 | −3.0194 | 0.0030 |
| m2_2 | −7.81119 | 3.26167 | −2.3948 | 0.0180 |
| m2_12 | 18.8610 | 8.64823 | 2.1809 | 0.0309 |
| m3_223 | 64.3433 | 9.60711 | 6.6975 | 0.0000 |
| m3_233 | 145.879 | 38.7787 | 3.7618 | 0.0002 |
| m4_1 | −1.74963 | 0.631899 | −2.7688 | 0.0064 |
| m4_1122 | 6.69345 | 2.29181 | 2.9206 | 0.0041 |
| m4_1222 | −4.08848 | 1.53950 | −2.6557 | 0.0088 |
| m4_2223 | −15.9512 | 3.91651 | −4.0728 | 0.0001 |
| m4_1113 | −42.9057 | 10.2050 | −4.2044 | 0.0000 |
| m4_1333 | 404.948 | 115.446 | 3.5077 | 0.0006 |
| m4_1123 | 43.4508 | 12.2380 | 3.5505 | 0.0005 |
| m4_1233 | 159.452 | 48.7699 | 3.2695 | 0.0014 |

| | | | |
|---|---|---|---|
| Mean dependent var | −0.191118 | S.D. dependent var | 0.239912 |
| Sum squared resid | 1.513287 | S.E. of regression | 0.103967 |
| $R^2$ | 0.830376 | Adjusted $R^2$ | 0.812202 |
| $F(15, 140)$ | 45.69040 | P-value($F$) | 2.60e–46 |
| Log-likelihood | 140.2202 | Akaike criterion | −248.4404 |
| Schwarz criterion | −199.6427 | Hannan–Quinn | −228.6209 |
| $\hat{\rho}$ | 0.148044 | Durbin–Watson | 1.687030 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 138) = 4.31155$
with p-value $= P(F(2, 138) > 4.31155) = 0.0152648$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 131.272
with p-value $= P(\chi^2(135) > 131.272) = 0.57468$

Table 7: Model 2 reduced - Mibtel weights

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 0.970087 | 0.559287 | 1.7345 | 0.0850 |
| m1_3 | −19.8850 | 11.6646 | −1.7047 | 0.0904 |
| m2_12 | −5.83966 | 2.73288 | −2.1368 | 0.0343 |
| m3_1 | 4.52977 | 1.46644 | 3.0889 | 0.0024 |
| m3_112 | −4.23525 | 2.33950 | −1.8103 | 0.0723 |
| m3_113 | 30.6474 | 14.0242 | 2.1853 | 0.0305 |
| m4_1 | 1.26914 | 0.477434 | 2.6583 | 0.0087 |
| m4_1113 | 11.1571 | 5.91880 | 1.8850 | 0.0614 |
| m4_1333 | −164.331 | 76.3589 | −2.1521 | 0.0330 |
| m4_1233 | −67.8216 | 26.1954 | −2.5891 | 0.0106 |
| norm10_mib | 0.856943 | 0.0218606 | 39.2003 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.576623 | S.D. dependent var | 0.487096 |
| Sum squared resid | 1.383112 | S.E. of regression | 0.097666 |
| $R^2$ | 0.962391 | Adjusted $R^2$ | 0.959797 |
| $F(10, 145)$ | 371.0423 | P-value($F$) | 5.80e–98 |
| Log-likelihood | 147.2362 | Akaike criterion | −272.4723 |
| Schwarz criterion | −238.9239 | Hannan–Quinn | −258.8464 |
| $\hat{\rho}$ | −0.078664 | Durbin–Watson | 2.148210 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 118) = 0.466584$
with p-value $= P(F(2, 118) > 0.466584) = 0.628293$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 70.6677
with p-value $= P(\chi^2(70) > 70.6677) = 0.455176$

Table 8: Model 2 reduced - Dow Jones weights

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_dj

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 2.40184 | 0.559532 | 4.2926 | 0.0000 |
| m1_1 | −25.7238 | 4.64009 | −5.5438 | 0.0000 |
| m2_2 | −7.04751 | 1.31984 | −5.3397 | 0.0000 |
| m2_12 | 9.72590 | 2.24566 | 4.3310 | 0.0000 |
| m3_3 | −153.061 | 46.8990 | −3.2636 | 0.0014 |
| m3_223 | 20.0485 | 6.98808 | 2.8690 | 0.0047 |
| m4_1113 | −3.04979 | 1.40846 | −2.1653 | 0.0320 |
| norm10_dj | 0.660231 | 0.0603359 | 10.9426 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | −0.191118 | S.D. dependent var | 0.239912 |
| Sum squared resid | 1.184599 | S.E. of regression | 0.089465 |
| $R^2$ | 0.867219 | Adjusted $R^2$ | 0.860939 |
| $F(7, 148)$ | 138.0879 | P-value($F$) | 1.37e–61 |
| Log-likelihood | 159.3208 | Akaike criterion | −302.6417 |
| Schwarz criterion | −278.2428 | Hannan–Quinn | −292.7319 |
| $\hat{\rho}$ | 0.019429 | Durbin–Watson | 1.957599 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 118) = 0.298423$
with p-value $= P(F(2, 118) > 0.298423) = 0.742546$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM $= 61.934$
with p-value $= P(\chi^2(70) > 61.934) = 0.743077$

Table 9: Model 1 reduced - Mibtel weights - $\gamma$=5

OLS estimates using the 156 observations 1–156
Dependent variable: copula5_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | −10.6555 | 7.55794 | −1.4098 | 0.1609 |
| m1_1 | 214.160 | 33.2677 | 6.4375 | 0.0000 |
| m1_2 | 131.893 | 35.5260 | 3.7126 | 0.0003 |
| m1_3 | 302.606 | 70.2962 | 4.3047 | 0.0000 |
| m2_2 | 71.4378 | 13.0156 | 5.4886 | 0.0000 |
| m2_12 | −108.267 | 33.5272 | −3.2292 | 0.0016 |
| m2_13 | −274.988 | 118.114 | −2.3282 | 0.0214 |
| m2_23 | 305.904 | 160.060 | 1.9112 | 0.0581 |
| m3_2 | −22.2533 | 6.29402 | −3.5356 | 0.0006 |
| m3_112 | −50.3248 | 16.7968 | −2.9961 | 0.0033 |
| m3_122 | 42.7615 | 12.8493 | 3.3279 | 0.0011 |
| m3_223 | −128.280 | 48.9170 | −2.6224 | 0.0097 |
| m4_1 | 16.2107 | 2.96367 | 5.4698 | 0.0000 |
| m4_3 | −1316.21 | 423.434 | −3.1084 | 0.0023 |
| m4_1122 | −35.6315 | 9.24770 | −3.8530 | 0.0002 |
| m4_1222 | 18.9968 | 6.64850 | 2.8573 | 0.0050 |
| m4_2223 | 33.7535 | 19.7695 | 1.7074 | 0.0901 |
| m4_2333 | −1155.49 | 413.664 | −2.7933 | 0.0060 |
| m4_1113 | 188.791 | 36.5319 | 5.1678 | 0.0000 |
| m4_1333 | −1902.22 | 385.644 | −4.9326 | 0.0000 |
| m4_1123 | −125.053 | 46.0064 | −2.7182 | 0.0074 |
| m4_1233 | −660.445 | 236.129 | −2.7970 | 0.0059 |

| | | | |
|---|---|---|---|
| Mean dependent var | 1.065582 | S.D. dependent var | 0.954051 |
| Sum squared resid | 15.92739 | S.E. of regression | 0.344762 |
| $R^2$ | 0.887106 | Adjusted $R^2$ | 0.869414 |
| $F(21, 134)$ | 50.14089 | P-value($F$) | 4.41e–53 |
| Log-likelihood | −43.37276 | Akaike criterion | 130.7455 |
| Schwarz criterion | 197.8424 | Hannan–Quinn | 157.9973 |
| $\hat{\rho}$ | 0.179765 | Durbin–Watson | 1.639416 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 132) = 6.89733$
with p-value = $P(F(2, 132) > 6.89733) = 0.00141557$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 38.2791
with p-value = $P(\chi^2(42) > 38.2791) = 0.635053$

Table 10: Model 1 reduced - Dow Jones weights - $\gamma$=5

OLS estimates using the 156 observations 1–156
Dependent variable: copula5_dj

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 11.4361 | 4.24337 | 2.6951 | 0.0079 |
| m1_1 | −116.940 | 14.5686 | −8.0268 | 0.0000 |
| m1_3 | −206.128 | 31.8724 | −6.4673 | 0.0000 |
| m2_1 | −33.2321 | 12.2638 | −2.7098 | 0.0076 |
| m2_2 | −15.4710 | 6.55717 | −2.3594 | 0.0197 |
| m2_12 | 37.9615 | 17.3862 | 2.1834 | 0.0307 |
| m3_223 | 128.856 | 19.3139 | 6.6717 | 0.0000 |
| m3_233 | 287.183 | 77.9597 | 3.6837 | 0.0003 |
| m4_1 | −3.94423 | 1.27035 | −3.1048 | 0.0023 |
| m4_1122 | 14.1340 | 4.60740 | 3.0677 | 0.0026 |
| m4_1222 | −8.71456 | 3.09497 | −2.8157 | 0.0056 |
| m4_2223 | −33.4417 | 7.87364 | −4.2473 | 0.0000 |
| m4_1113 | −86.4525 | 20.5159 | −4.2139 | 0.0000 |
| m4_1333 | 829.928 | 232.090 | 3.5759 | 0.0005 |
| m4_1123 | 89.8678 | 24.6029 | 3.6527 | 0.0004 |
| m4_1233 | 328.978 | 98.0456 | 3.3554 | 0.0010 |

| | | | |
|---|---|---|---|
| Mean dependent var | −0.445834 | S.D. dependent var | 0.492921 |
| Sum squared resid | 6.116094 | S.E. of regression | 0.209013 |
| $R^2$ | 0.837599 | Adjusted $R^2$ | 0.820199 |
| $F(15, 140)$ | 48.13758 | P-value($F$) | 1.31e–47 |
| Log-likelihood | 31.28231 | Akaike criterion | −30.56462 |
| Schwarz criterion | 18.23308 | Hannan–Quinn | −10.74510 |
| $\hat{\rho}$ | 0.151412 | Durbin–Watson | 1.678889 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 138) = 4.80292$
with p-value $= P(F(2, 138) > 4.80292) = 0.00962751$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 132.024
with p-value $= P(\chi^2(135) > 132.024) = 0.556381$

Table 11: Model 1 reduced - Mibtel weights - excess returns

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | −2.97127 | 3.47062 | −0.8561 | 0.3933 |
| exm1_1 | 61.2431 | 16.4103 | 3.7320 | 0.0003 |
| exm2_1 | −7.27118 | 3.39292 | −2.1430 | 0.0337 |
| exm2_12 | −42.1267 | 7.12076 | −5.9160 | 0.0000 |
| exm2_2 | 38.1680 | 3.39557 | 11.2405 | 0.0000 |
| exm3_122 | 20.8892 | 2.79692 | 7.4687 | 0.0000 |
| exm3_2 | −10.7828 | 1.90780 | −5.6520 | 0.0000 |
| exm4_1122 | −2.55456 | 0.554933 | −4.6034 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.576623 | S.D. dependent var | 0.487096 |
| Sum squared resid | 8.698919 | S.E. of regression | 0.242439 |
| $R^2$ | 0.763460 | Adjusted $R^2$ | 0.752273 |
| $F(7, 148)$ | 68.24113 | P-value($F$) | 3.61e–43 |
| Log-likelihood | 3.804856 | Akaike criterion | 8.390287 |
| Schwarz criterion | 32.78914 | Hannan–Quinn | 18.30004 |
| $\hat{\rho}$ | 0.578494 | Durbin–Watson | 0.830555 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 146) = 10.3245$
with p-value $= P(F(2, 146) > 10.3245) = 6.39967\text{e-}005$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM $= 51.5148$
with p-value $= P(\chi^2(35) > 51.5148) = 0.0355204$

Table 12: Model 1 reduced - Dow Jones weights - excess returns

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_dj

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 2.87362 | 1.99779 | 1.4384 | 0.1525 |
| exm1_1 | −39.6703 | 9.76722 | −4.0616 | 0.0001 |
| exm1_2 | 20.2123 | 9.38499 | 2.1537 | 0.0329 |
| exm2_1 | 6.79435 | 1.82658 | 3.7197 | 0.0003 |
| exm2_12 | 14.2591 | 4.87434 | 2.9253 | 0.0040 |
| exm2_2 | −19.8374 | 1.95848 | −10.1290 | 0.0000 |
| exm3_122 | −7.37855 | 2.04480 | −3.6084 | 0.0004 |
| exm3_2 | 3.23898 | 1.31157 | 2.4695 | 0.0147 |
| exm4_1112 | −0.681372 | 0.409822 | −1.6626 | 0.0985 |
| exm4_1222 | 1.81322 | 0.438231 | 4.1376 | 0.0001 |

| | | | |
|---|---|---|---|
| Mean dependent var | −0.191118 | S.D. dependent var | 0.239912 |
| Sum squared resid | 2.396625 | S.E. of regression | 0.128122 |
| $R^2$ | 0.731363 | Adjusted $R^2$ | 0.714803 |
| $F(9, 146)$ | 44.16496 | P-value($F$) | 2.31e–37 |
| Log-likelihood | 104.3576 | Akaike criterion | −188.7151 |
| Schwarz criterion | −158.2165 | Hannan–Quinn | −176.3279 |
| $\hat{\rho}$ | 0.437032 | Durbin–Watson | 1.104548 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 144) = 8.80715$
with p-value $= P(F(2, 144) > 8.80715) = 0.00024635$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 79.8724
with p-value $= P(\chi^2(54) > 79.8724) = 0.0126167$

Table 13: Model 2 reduced - Mibtel weights - excess returns

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 0.752401 | 0.266827 | 2.8198 | 0.0055 |
| exm2_12 | −3.44862 | 1.18061 | −2.9210 | 0.0040 |
| exm3_112 | −2.92516 | 1.33776 | −2.1866 | 0.0303 |
| exm3_122 | 2.23812 | 0.916297 | 2.4426 | 0.0158 |
| exm4_1112 | −0.350041 | 0.193958 | −1.8047 | 0.0731 |
| exm4_1122 | 0.620813 | 0.257838 | 2.4078 | 0.0173 |
| norm10_mib | 0.855878 | 0.0156823 | 54.5760 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.576623 | S.D. dependent var | 0.487096 |
| Sum squared resid | 1.448043 | S.E. of regression | 0.098582 |
| $R^2$ | 0.960625 | Adjusted $R^2$ | 0.959039 |
| $F(6, 149)$ | 1343.935 | P-value($F$) | 5.2e–127 |
| Log-likelihood | 143.6577 | Akaike criterion | −273.3154 |
| Schwarz criterion | −251.9665 | Hannan–Quinn | −264.6444 |
| $\hat{\rho}$ | −0.042752 | Durbin–Watson | 2.076987 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 147) = 1.48657$
with p-value = $P(F(2, 147) > 1.48657) = 0.229528$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 30.0248
with p-value = $P(\chi^2(27) > 30.0248) = 0.31304$

Table 14: Model 2 reduced - Dow Jones weights - excess returns

OLS estimates using the 156 observations 1–156
Dependent variable: copula10_dj

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 1.66964 | 1.27028 | 1.3144 | 0.1907 |
| exm1_1 | −14.0113 | 6.10952 | −2.2934 | 0.0232 |
| exm2_12 | 7.85182 | 3.44042 | 2.2822 | 0.0239 |
| exm2_2 | −6.58501 | 1.54207 | −4.2702 | 0.0000 |
| exm3_122 | −3.20473 | 1.14673 | −2.7947 | 0.0059 |
| exm3_2 | 1.89533 | 0.765610 | 2.4756 | 0.0144 |
| exm4_1222 | 0.408061 | 0.223536 | 1.8255 | 0.0699 |
| norm10_dj | 0.719254 | 0.0552248 | 13.0241 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | −0.191118 | S.D. dependent var | 0.239912 |
| Sum squared resid | 1.229133 | S.E. of regression | 0.091132 |
| $R^2$ | 0.862227 | Adjusted $R^2$ | 0.855711 |
| $F(7, 148)$ | 132.3187 | P-value($F$) | 2.07e–60 |
| Log-likelihood | 156.4423 | Akaike criterion | −296.8845 |
| Schwarz criterion | −272.4857 | Hannan–Quinn | −286.9748 |
| $\hat{\rho}$ | 0.051076 | Durbin–Watson | 1.881407 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 146) = 0.0474202$
with p-value $= P(F(2, 146) > 0.0474202) = 0.953701$

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 33.0158
with p-value $= P(\chi^2(35) > 33.0158) = 0.564198$

Table 15: Model 1 reduced - Mibtel weights - 104 observations

OLS estimates using the 104 observations 1–104
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | −3.41233 | 4.87400 | −0.7001 | 0.4856 |
| m1_1 | 132.467 | 18.4806 | 7.1679 | 0.0000 |
| m2_2 | 54.9885 | 12.8507 | 4.2790 | 0.0000 |
| m2_12 | −173.270 | 29.8799 | −5.7989 | 0.0000 |
| m2_13 | −215.848 | 77.5129 | −2.7847 | 0.0065 |
| m3_122 | 16.4099 | 6.06883 | 2.7040 | 0.0082 |
| m3_223 | 140.740 | 24.7413 | 5.6884 | 0.0000 |
| m3_233 | 904.975 | 191.177 | 4.7337 | 0.0000 |
| m4_1222 | 19.5894 | 4.60351 | 4.2553 | 0.0001 |
| m4_2223 | −80.4272 | 46.9938 | −1.7114 | 0.0904 |
| m4_2233 | −478.430 | 119.763 | −3.9948 | 0.0001 |
| m4_1223 | 335.489 | 79.3205 | 4.2295 | 0.0001 |
| m4_1233 | 758.897 | 171.499 | 4.4251 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.742604 | S.D. dependent var | 0.488075 |
| Sum squared resid | 2.463132 | S.E. of regression | 0.164522 |
| $R^2$ | 0.899613 | Adjusted $R^2$ | 0.886375 |
| $F(12,91)$ | 67.95768 | P-value($F$) | 5.02e–40 |
| Log-likelihood | 47.06417 | Akaike criterion | −68.12833 |
| Schwarz criterion | −33.75125 | Hannan–Quinn | −54.20117 |
| $\hat{\rho}$ | 0.025715 | Durbin–Watson | 1.943373 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2,141)$ = -31.768
with p-value = $P(F(2,141) >$ -31.768) = 1.79769e+308

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 78.2774
with p-value = $P(\chi^2(90) > 78.2774) = 0.806367$

Table 16: Model 1 reduced - Dow Jones weights - 104 observations

OLS estimates using the 104 observations 1–104
Dependent variable: copula10_dj

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | −6.70381 | 2.87362 | −2.3329 | 0.0220 |
| m1_1 | −99.0090 | 18.9395 | −5.2277 | 0.0000 |
| m1_2 | 65.8535 | 19.0844 | 3.4506 | 0.0009 |
| m1_3 | −146.134 | 58.9723 | −2.4780 | 0.0152 |
| m2_12 | 31.3717 | 14.4145 | 2.1764 | 0.0323 |
| m3_1 | 11.6378 | 4.75300 | 2.4485 | 0.0164 |
| m3_2 | −22.7497 | 6.63525 | −3.4286 | 0.0009 |
| m3_3 | 522.630 | 223.186 | 2.3417 | 0.0215 |
| m3_113 | 147.046 | 46.7181 | 3.1475 | 0.0023 |
| m3_133 | 556.016 | 173.060 | 3.2129 | 0.0019 |
| m3_223 | −77.0080 | 43.1596 | −1.7843 | 0.0779 |
| m3_233 | −615.546 | 123.776 | −4.9731 | 0.0000 |
| m4_2 | −3.86941 | 1.85605 | −2.0848 | 0.0401 |
| m4_1112 | 21.7657 | 6.26306 | 3.4753 | 0.0008 |
| m4_1122 | −17.2387 | 7.53938 | −2.2865 | 0.0247 |
| m4_2223 | 83.4617 | 37.6597 | 2.2162 | 0.0293 |
| m4_2233 | 525.937 | 113.562 | 4.6313 | 0.0000 |
| m4_1223 | −138.880 | 63.6228 | −2.1829 | 0.0318 |
| m4_1233 | −783.461 | 226.303 | −3.4620 | 0.0008 |

| | | | |
|---|---|---|---|
| Mean dependent var | −0.276961 | S.D. dependent var | 0.242036 |
| Sum squared resid | 0.778757 | S.E. of regression | 0.095718 |
| $R^2$ | 0.870937 | Adjusted $R^2$ | 0.843605 |
| $F(18, 85)$ | 31.86615 | P-value($F$) | 3.22e–30 |
| Log-likelihood | 106.9416 | Akaike criterion | −175.8832 |
| Schwarz criterion | −125.6398 | Hannan–Quinn | −155.5282 |
| $\hat{\rho}$ | −0.091027 | Durbin–Watson | 2.176045 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 135)$ = -30.5547
with p-value = $P(F(2, 135) >$ -30.5547) = 1.79769e+308

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 36.6757
with p-value = $P(\chi^2(36) > 36.6757) = 0.43735$

Table 17: Model 2 reduced - Mibtel weights - 104 observations

OLS estimates using the 104 observations 1–104
Dependent variable: copula10_mib

| | Coefficient | Std. Error | $t$-ratio | p-value |
|---|---|---|---|---|
| const | 0.917532 | 4.57907 | 0.2004 | 0.8417 |
| m1_1 | 44.6758 | 22.8587 | 1.9544 | 0.0539 |
| m1_2 | −41.8620 | 17.2452 | −2.4275 | 0.0173 |
| m2_1 | 49.4419 | 17.7865 | 2.7797 | 0.0067 |
| m2_2 | −28.8912 | 11.3289 | −2.5502 | 0.0126 |
| m2_13 | 164.295 | 73.2912 | 2.2417 | 0.0276 |
| m3_1 | −28.9866 | 9.49052 | −3.0543 | 0.0030 |
| m3_2 | 21.3942 | 7.32295 | 2.9215 | 0.0045 |
| m3_112 | 20.1131 | 9.95035 | 2.0213 | 0.0464 |
| m3_133 | −335.096 | 192.798 | −1.7381 | 0.0858 |
| m3_233 | 362.656 | 114.882 | 3.1568 | 0.0022 |
| m4_1 | −13.7555 | 3.75945 | −3.6589 | 0.0004 |
| m4_2 | 8.69683 | 2.18298 | 3.9839 | 0.0001 |
| m4_3 | −704.105 | 233.546 | −3.0148 | 0.0034 |
| m4_1113 | −47.8414 | 25.1805 | −1.8999 | 0.0608 |
| m4_1133 | −332.301 | 134.941 | −2.4626 | 0.0158 |
| m4_1333 | −919.039 | 322.616 | −2.8487 | 0.0055 |
| m4_1233 | 358.830 | 176.160 | 2.0370 | 0.0448 |
| norm10_mib | 0.813892 | 0.0522572 | 15.5747 | 0.0000 |

| | | | |
|---|---|---|---|
| Mean dependent var | 0.742604 | S.D. dependent var | 0.488075 |
| Sum squared resid | 0.861601 | S.E. of regression | 0.100680 |
| $R^2$ | 0.964885 | Adjusted $R^2$ | 0.957449 |
| $F(18, 85)$ | 129.7556 | P-value($F$) | 6.75e–54 |
| Log-likelihood | 101.6848 | Akaike criterion | −165.3696 |
| Schwarz criterion | −115.1262 | Hannan–Quinn | −145.0145 |
| $\hat{\rho}$ | −0.180582 | Durbin–Watson | 2.353138 |

RESET test for specification –
Null hypothesis: specification is adequate
Test statistic: $F(2, 135)$ = -23.8348
with p-value = $P(F(2, 135) >$ -23.8348) = 1.79769e+308

White's test for heteroskedasticity –
Null hypothesis: heteroskedasticity not present
Test statistic: LM = 32.5556
with p-value = $P(\chi^2(36) > 32.5556) = 0.633196$