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Abstract

Assuming random matching productivity, we present a search equilibrium model where each match ends in a vacancy, in a temporary job or in a permanent job. Centralized bargaining sets the wage rate of permanent workers whereas firms decide unilaterally the wage rate of temporary workers. In this *segmented* labour market: a) the wage setting function can be downward sloping; b) higher union bargaining power leads to higher wage and higher unemployment; c) average worker productivity shows a maximum with respect to union bargaining power.

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1 Introduction

The aim of the paper is to analyze the implications of the presence of labour unions for macroeconomic variables in a segmented labour market where permanent workers, covered by contractual arrangements, coexist with temporary workers, paid at their reservation wage.

Regulation of temporary contracts has been deeply extensively explored in the economic literature that previously focused on the impact of temporary contracts on unemployment and job turnover. The behaviour of employment over the business cycle was studied by means of a traditional partial equilibrium framework of labour demand under uncertainty. In these models, firms have a stable permanent workforce, and adjust temporary workers to fluctuations in economic activity, so that temporary contracts serve as *buffer stocks* (see among others Bentolila and Saint-Paul (1992), Garibaldi (1998), Boeri (1999), Pissarides and Mortensen (1994)). These studies show that the introduction of temporary contracts has an ambiguous impact on overall employment but increases employment volatility over the business cycle.

Using the Pissarides and Mortensen (1994) matching model with endogenous job destruction, Cahuc and Postel-Vinay (2002) widen its scope by analysing the consequences of the specific combination of temporary and permanent jobs on unemployment. Assuming that long-term contracts, which can be terminated in any period with a fixed *firing cost*, coexist with temporary contracts that can be either terminated at *no cost* or converted into a long-term contract, the paper shows that looser restrictions on the use of temporary contracts have a beneficial impact on employment that can be offset by the increase in job turnover when there are positive firing costs.

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In their matching model where firms create *entry level* jobs which can be converted or destroyed after a given period of time Blanchard and Landier (2001) conclude that lowering firing costs for entry-level jobs while keeping them high for regular jobs can have two effects: firms are more willing to hire new workers and see how they perform, but they are also more reluctant to keep them in regular jobs. As a result, transition rates to permanent jobs are low and the excess turnover induced by the coexistence of temporary and permanent contracts can be high enough to offset the efficiency gains of higher flexibility. In other words, the effects of partial reform may be perverse, leading to higher unemployment and lower workers' welfare.

Dolado *et al.* (2007) build on the same "search and matching" literature that assesses the effects of EPL reforms in dual labour markets, but focus on the spillover effects of targeted EPL. The authors develop a model in which two groups of workers with different productivity levels interact through the matching process in a labour market subject to search frictions. In this way, they are able to compare the effects of a reduction of firing costs concerning only one group of workers with the effects of a more general reform affecting all workers. Calibrating the model on Spanish data, the authors find that targeting firing cost reductions in low productivity workers and in jobs subject to frequent productivity shocks is the most effective way to reduce aggregate unemployment. However, as the authors themselves acknowledge, changes in firing costs could affect labour productivity since higher turnover improves the reallocation of production factors and the adoption of new technologies.

In a similar framework, Casquel and Cunyat (2008) design a model with heterogeneous workers according to which temporary contracts can serve different functions depending on workers' skills. Firms hire workers on temporary positions that can be later converted into permanent jobs or end in a vacancy. Three equilibria emerge in which (1) the temporary job could be a stepping stone, so that all temporary jobs are converted, (2) all temporary jobs are dead-end jobs or (3) only skilled workers can access permanent jobs (a segmentation equilibrium). The kind of equilibrium prevailing depends crucially on the institutional labour market framework, with lower firing costs or unemployment benefits making it easier for skilled and unskilled workers to gain access to permanent jobs.

The papers presented above rarely consider the presence, the behaviour and the effects of trade unions in labour markets characterized by the coexistence of temporary and permanent workers.

Therefore, in order to investigate the role played by trade unions in a segmented labour market, where firms can offer workers both a temporary and a permanent contract, we present a search model that takes segmentation explicitly into account. In particular, we assume that temporary workers, being paid at their reservation wage, continue to search for a better job whereas permanent workers do not. We concentrate our analysis on union bargaining power: we show that stronger unions raise permanent workers' wages, unemployment and the *shakiness* index (defined by the ratio between short-term and total contracts) but, by inducing firms to retain permanently only those workers who show very high productivity, they increase average worker productivity.

The paper is organized as follows. In the next Section we present the model, Section 3 presents a simplified version of the model that allows us to obtain analytical solutions and Section 4 discusses the main results.

2 The model

We define by v the number of vacancies, by u the number of unemployed and by $\theta = \frac{v}{u}$ the tightness in the labour market. We assume that the number of matching, m, is given by a Leontief matching function[1]:

$$m = \eta \quad min(u, v)$$

We also assume that in the labour market there are more workers than jobs, so that $v < u$ holds. Therefore, η is the given probability of a firm filling a vacancy. Hence, with these hypotheses, we obtain that the probability of finding a job for an unemployed person is $\frac{m}{u} = \eta\theta$ and the probability of filling a vacancy is $\frac{m}{v} = \eta$. Therefore, labour market tightness does not influence firm's probability of finding workers and influences positively workers' probability of finding jobs.

A match between a worker and a job gives a random productivity x. $F(x)$ is the cumulative distribution function and $\mu = E(x)$ is the average matching productivity. Given the match arrives, the firm immediately evaluates its productivity and chooses whether or not to hire the worker[2].

When a worker is hired, the firm chooses to offer her either a *temporary* or *permanent* contract.

Assumption 1. *The wage rate of temporary contracts is set by firms at a level such that the utility of temporary workers is equal to that of the unemployed (participation constraint). The wage rate of permanent workers is*

[1] As in Lagos (2000) and Shimer (2007), which assume workers and jobs in fixed proportions. In a previous version of the model we used a Cobb-Douglas matching function, obtaining very similar results but with a more complicated algebra.

[2] In a previous version of the model, we assumed that screening required one period of time and we obtained very similar results.

negotiated at a centralized level and depends on the average productivity of permanent workers.

At least theoretically, one would expect temporary workers to earn higher wages as compensation for instability. But segmentation on the labour market and the hypothesis that unions do not cover temporary workers justify the above assumption. The empirical evidence, at least fot Italy, supports this fact[3].

Assumption 2. *Temporary workers continue to search and they find another job with the same probability as the unemployed; permanent workers do not search any more.*

The contractual arrangement is decided by the firm and depends on the productivity x_i of the match and on the characteristics of the two available contracts. Let us use $\underline{x}$ to denote the endogenous bottom level of productivity which makes the match profitable for the firm and call it *hiring productivity* and let us assume that there exists a higher endogenous value of x, $\bar{x}$, the *keeping productivity*, that makes the firm willing to employ the worker on a permanent basis. The latter threshold must exist once we consider that the firm, in each period, earns more on temporary than permanent workers but this surplus lasts for a lower number of periods because temporary workers continue to search and therefore are more likely to leave the firm.

Given assumptions 1 and 2, a match therefore gives rise to:

- a vacancy with endogenous probability $F(\underline{x})$,
- a temporary contract with endogenous probability $F(\bar{x}) - F(\underline{x})$,
- a permanent contract with endogenous probability $1 - F(\bar{x})$.

Permanent contracts can be hit by a negative shock with exogenous probability λ. The probability that the shock hits temporary workers is assumed to be higher, namely $\lambda + \phi$.[4].

[3]For the Italian labour market, both Picchio (2006) and Berton *et al.* (2009) conclude for an estimated wage penalty for temporary workers (*parasubordinati*) between 20% and 30% with respect to permanent ones. Nevertheless, some of the temporary positions are covered by collective contractual arrangement and the wage rate should be, at least theoretically, the same as the one of permanent workers apart, obviously, for tenure premia.

[4]ϕ may also represent a way to model firing costs. Adding pure firing costs would produce a less tractable model (whereas severance payment does not change the results). Therefore, we simply assume that the expected length of temporary works, given by $\frac{1}{\lambda+\phi}$ is lower than that of permanent ones, given by $\frac{1}{\lambda}$, i.e. temporary workers have higher probability of being fired.

2.1 Wage setting: temporary workers

Given the above hypotheses and following standard job search models, the asset value of being in the state of unemployment is given by:

$$rU = B + \eta\theta \left[[F(\bar{x}) - F(\underline{x})](W^T - U) + [1 - F(\bar{x})](W^P - U) \right] \quad (1)$$

where B is the per period utility of being unemployed (that we call unemployment benefits hereafter), $\eta\theta$ is the probability of finding a match, U, W^T and W^P are the average asset values of being in the states of unemployment, temporary and permanent work, respectively.

Given that the temporary worker continues to search (hypothesis 2), the asset value of being employed on a temporary basis is[5]:

$$rW^T = w^T + (\lambda + \phi)(U - W^T) + \eta\theta[1 - F(\bar{x})](W^P - W^T) \quad (2)$$

where w^T is the wage rate of temporary workers, and $\lambda + \phi$ the exogenous probability of negative shocks. Finally, the asset value of being in a permanent job is:

$$rW^P = w + \lambda(U - W^P) \quad (3)$$

where w is the wage rate of permanent workers. Assume now that $U = W^T$ (hypothesis 1). Given the symmetric equilibrium, it can be readily seen from equations 1 and 2 that $B = w^T$ must hold, i.e. the wage rate of temporary workers equals the per period utility of being unemployed.

2.2 Wage setting: permanent workers

According to Nash bargaining, we assume that the firm and the union maximize their payoffs with respect to w:

$$(W^P - U)^\alpha (J^P - V)^{1-\alpha} \quad (4)$$

where α is union bargaining power, J^P is the average asset value of permanent workers and V is that of vacant jobs. $(W^P - U)$ can be derived using equations 3 and 1 with $W^T = U$. It gives:

$$W^P - U = \frac{w - B}{R + \eta\theta[1 - F(\bar{x})]} \quad (5)$$

where $R \equiv r + \lambda$.

[5]Unless necessary, we do not write in the following equation the index i referring to the productivity of the matching. For instance, the wage rate of temporary workers is equal for all of them and therefore does not depend on matching productivity.

The average asset value of permanent workers is given by:

$$rJ^P = \gamma^P(\bar{x}) - w + \lambda(V - J^P) \tag{6}$$

where $\gamma^P(\bar{x})$ is the endogenous average productivity of permanent workers defined as:

$$\gamma^P(\bar{x}) = \frac{\int_{\bar{x}}^{\infty} xf(x)dx}{1 - F(\bar{x})} \tag{7}$$

Equation 6, for $V = 0$ because of firms' free entry, gives $J^P = \frac{\gamma^P(\bar{x}) - w}{R}$.

By maximizing equation 4 with respect to w we obtain:

$$w(\bar{x}) = \alpha\gamma^P(\bar{x}) + (1 - \alpha)B \tag{8}$$

As expected, the wage rate of permanent workers is a weighted sum of the average endogenous productivity of permanent workers and unemployment benefits.

2.3 Hiring and keeping thresholds

Given a match of productivity x_i, the firm decides: a) whether or not to hire the worker b) in the former case, whether to offer her a permanent or a temporary contract.

Hiring the worker on a temporary basis gives the following asset value:

$$rJ_i^T = x_i - B + (\lambda + \phi + \eta\theta)(V - J_i^T) \tag{9}$$

because temporary workers leave the firm with probability $\eta\theta$ (and they find a permanent position with probability $\eta\theta(1 - \bar{x})$). Given $V = 0$ in equilibrium, the firm will hire the worker if $J_i^T \geq 0$, which gives $x_i > B$ as the *hiring* condition. This leads to the definition of the *hiring productivity*:

$$\underline{x} = B$$

Consider now the asset value of the same matching when the firm offers a permanent contract:

$$rJ_i^P = x_i - w + \lambda(V - J_i^P) \tag{10}$$

The *keeping productivity* threshold $\bar{x}$ is defined by comparing the asset value of a permanent contract with that of a temporary contract. Solving in J_i^T and J_i^P equations 9 and 10, using $\underline{x} = B$ and $V = 0$, solving $J_i^T = J_i^P$ in x_i, we obtain the bottom level of productivity which yields the permanent job as more profitable than the transitory one:

$$\bar{x}(w, \theta) = w\kappa(\theta) + [1 - \kappa(\theta)]B \tag{11}$$

where $\kappa(\theta) \equiv 1 + \frac{R}{\phi + \eta\theta}$, so that $\frac{d\kappa}{d\theta} < 0$.

Remark 1. *The* keeping productivity *is increasing in the wage rate of permanent workers and decreasing in labour market tightness.*

Proof. The results emerge immediately given that $\kappa(\theta)$ decreases with θ and that $w > B$ because the wage rate of permanent workers must be higher than unemployment benefits. □

Higher labour market tightness gives temporary workers more opportunities of moving away from their temporary positions, reduces the expected length of temporary contracts and lowers the asset value of temporary jobs. Therefore, it pushes firms to keep more workers on a permanent basis.

Substituting equation 7 into equation 8, we obtain the implicit definition of the wage setting function:

$$w = \alpha \frac{\int_{\bar{x}(w,\theta)}^{\infty} x f(x) dx}{1 - F(\bar{x}(w,\theta))} + (1-\alpha)B \tag{12}$$

Remark 2. *The wage setting function can be both decreasing or increasing in labour market tightness. A higher union bargaining power, α, a higher difference between average and marginal productivity of permanent workers, $\gamma(\bar{x}) - \bar{x}$, and a lower probability of being hired on a permanent basis, $1 - F(\bar{x})$, imply that the positive sign for $\frac{dw}{d\theta}$ is more likely. Therefore, we cannot exclude a negatively sloped wage setting function in the space w, θ.*

Proof. By defining $T = w - \left[\alpha\gamma^P(\bar{x}(w,\theta)) + (1-\alpha)B\right]$, $\frac{dw}{d\theta} = -\frac{\frac{\partial T}{\partial \theta}}{\frac{\partial T}{\partial w}}$. We obtain

$$\frac{dw}{d\theta} = -\frac{-\alpha \frac{d\gamma^P}{d\bar{x}} \frac{d\bar{x}}{d\theta}}{1 - \alpha \frac{d\gamma^P}{d\bar{x}} \frac{d\bar{x}}{dw}}$$

The numerator must be positive because the average productivity of permanent workers increases with the *keeping productivity* $\bar{x}$ and because $\bar{x}$ decreases with θ (see remark 1).

Therefore, the sign depends on the opposite of the denominator of the previous equations, so that

$$sign\left(\frac{dw}{d\theta}\right) = sign\left(\alpha \frac{d\gamma}{d\bar{x}} \frac{d\bar{x}}{dw} - 1\right)$$

Given that $\frac{d\gamma^P}{d\bar{x}} = \frac{f(\bar{x})}{1-F(\bar{x})}[\gamma^P(\bar{x}) - \bar{x}]$ from equation 7, where $f(\bar{x})$ is the probability density function of $\bar{x}$, and that $\frac{d\bar{x}}{dw} = \kappa(\theta)$ from equation 11, we obtain:

$$sign\left(\frac{dw}{d\theta}\right) = sign\left[\alpha \frac{f(\bar{x}(w,\theta))}{1 - F(\bar{x}(w,\theta))} \left[\gamma(\bar{x}(w,\theta)) - \bar{x}(w,\theta)\right] \kappa(\theta) - 1\right]$$

□

The indeterminacy of the slope of the wage setting function stems from the existence of temporary contracts. Indeed, higher labour market tightness induces firms to hire more workers on a permanent basis (see equation 11) and, in this way, to reduce the average productivity of permanent workers. Given that the bargaining process takes into account the productivity of the average worker, this can also lead to a lower bargained wage rate.

Remark 3. *The wage rate of permanent workers increases in union bargaining power if it decreases in labour market tightness.*

Proof. We can compute the sign of $\left(\frac{dw}{d\alpha}\right)$ following the same steps seen above in proof 2.3. Hence, we obtain that

$$\frac{dw}{d\alpha} = -\frac{-[\gamma^P(\bar{x}) - B]}{1 - \alpha \frac{d\gamma^P}{d\bar{x}} \frac{d\bar{x}}{dw}}$$

Given that $\gamma(\bar{x}) > B$ and that the denominator is the same as that of $\frac{dw}{d\theta}$, the sign of $\frac{dw}{d\alpha}$ is the opposite of that of $\frac{dw}{d\theta}$. If the wage rate increases with union bargaining power, it must also decrease with labour market tightness. □

In conclusion, if higher union bargaining power increases the wage rate, as one would expect, the wage rate must depend negatively on labour market tightness.

2.4 Job Creation

Firms enter the market until the asset value of a vacancy, given by[6]:

$$rV = -c + \eta \left\{ [F(\bar{x}) - F(\underline{x})](J^T - V) + [1 - F(\bar{x})](J^P - V)] \right\} \tag{13}$$

is positive. c is the search cost, η is the probability of a match and J^T, J^P stand for the expected average asset values of temporary and permanent positions, in turn given by:

$$rJ^T = \gamma^T(\bar{x}, \underline{x}) - B + [\lambda + \phi + \eta\theta](V - J^T) \tag{14}$$

because $w^T = B$ as shown above. $\gamma^T(\bar{x}, \underline{x})$ is the average productivity of temporary workers, depending on the endogenous thresholds of the *hiring* and *keeping* productivity, and it is defined as:

$$\gamma^T(\bar{x}, \underline{x}) = \frac{\int_{\underline{x}}^{\bar{x}} x f(x) dx}{F(\bar{x}) - F(\underline{x})} \tag{15}$$

[6]To simplify the notation, we write $\bar{x}$ for $\bar{x}(w, \theta)$

In order to compute the job creating condition, the asset value J^T computed from equation 13 and that computed from equation 14 must be equal for $V = 0$.

We can greatly simplify the model without losing in generality by assuming the x distribution is such that $x_i > B$ for all i holds. This assumption obviously gives $F(\underline{x}) = 0$, so that every match ends in a job (temporary or permanent). After substituting J^P computed from equation 6, given that $w^T = B$, using $\kappa(\theta) = 1 + \frac{R}{\phi+\eta\theta}$, and defining $C \equiv \frac{cR}{\eta}$ we obtain the *job creation condition*:

$$C = \left(\mu - w[1 - F(\bar{x})] - \left(\frac{\gamma^T(\bar{x}, \underline{x}) - B}{\kappa(\theta)} + B \right) F(\bar{x}) \right) \tag{16}$$

where $\bar{x}(w, \theta)$ is defined in equation 11 and μ is the mean of the x distribution.

Remark 4. *The relationship between the wage rate of permanent workers, w, and the labour market tightness, θ, alongside the* Job creation condition *is negative.*

Proof. The relationship can be computed using the same tools seen in proof 2.3. We obtain the following result:

$$\frac{dw}{d\theta} = \frac{F(\bar{x})}{1 - F(\bar{x})} \frac{\frac{d\kappa}{d\theta}}{\kappa(\theta)^2} [\gamma^T(\bar{x}, \underline{x}) - B]$$

which, according to the results of traditional search models, is negative. Indeed, $\frac{d\kappa}{d\theta} < 0$ (see equation 11) and $\gamma^T(\bar{x}, \underline{x}) > B$ (if not, temporary workers would have been paid more than their average productivity). □

The main conclusion obtained in the previous section is that if the wage setting function is increasing in union bargaining power it must be decreasing in labour market tightness and vice versa whereas alongside the job creation function the relationship between the wage rate and labour market tightness is always negative.

The whole solution of the model is given by the wage setting function of equation 12 and the job creation condition of equation 16.

2.5 The steady state in a simplified case

In order to obtain meaningful results, let us now propose some specific assumptions to simplify the model. Obviously, the results proposed in this section represent simply a particular case whose results can nevertheless not be ignored.

In particular, we assume that x is uniformly distributed in the range [0,1]. This assumption gives $\gamma(\bar{x}) = \frac{1+\bar{x}}{2} = \frac{1+w\kappa(\theta)+(1-\kappa(\theta)B}{2}$ and allows us to solve in w the wage setting function defined in equation 12:

$$w(\alpha, \theta, B) = B + \frac{\alpha}{2 - \alpha\kappa(\theta)}(1 - B) \tag{17}$$

The derivative of $w(\alpha, \theta, B)$ with respect to B and α is positive, whereas its derivative with respect to θ is negative[7].

We can conclude that the existence of a dual labour market where temporary workers are not covered by centralized bargaining makes the wage setting function decreasing in labour market tightness.

This puzzling result can be readily explained by considering that

- higher tightness increases the share of permanent workers, because even if labour market tightness does not influence the probability of firms finding workers, it increases the probability of temporary workers employed in the firm of finding another firm. In this way, permanent workers become more suitable.

- searching for a higher number of permanent workers, firms reduce their keeping productivity (the productivity of the worst permanent worker). Hence the average productivity of permanent workers decreases and, according to equation 8, the wage rate decreases too.

To compute the "simplified" job creation condition, given that in equation 16 we had assumed $x_i > B \ \forall i$, we need now to assume $B = 0$. This simplification gives $\underline{x} = w^T = 0$ and, from equation 11, $\bar{x} = w\kappa(\theta)$. Consider that, under the assumption that x is uniformly distributed in the range [0,1], $F\bar{x} = \bar{x}$ and $\gamma(\bar{x}, 0) = \frac{\bar{x}}{2}$. By substituting the above results in equation 16, we obtain a simplified version of the *job creation condition.*

$$\kappa(\theta) = \frac{2w - \xi}{w^2} \qquad JCC \tag{18}$$

where $\xi \equiv 1 - 2C$.

The above simplification also gives raise to a further simplified version of the wage setting function shown in equation 17:

$$w(\alpha, \theta) = \frac{\alpha}{2 - \alpha\kappa(\theta)} \qquad WSF \tag{19}$$

Equations 18 and 19 define the steady state equilibrium of the simplified model. As shown in appendix A, (see also figure 4, which also shows that

[7] Note that $0 < \frac{\alpha}{2-\alpha\kappa(\theta)} < 1$ must always hold in order to have $B < w < 1$.

two equilibria can exist) it can be demonstrated that the stable steady state equilibrium gives:

$$w^* = \frac{\alpha}{4}\left[3 + \Gamma(\alpha, \xi)\right]$$

$$\theta^* = \frac{R}{\eta}\left[\frac{2}{\alpha}\frac{1 + \Gamma(\alpha, \xi)}{3 + \Gamma(\alpha, \xi)} - 1\right]^{-1} - \frac{\phi}{\eta} \tag{20}$$

$$\bar{x}^* = \frac{1}{2}\left[1 + \Gamma(\alpha, \xi)\right] \tag{21}$$

where:

$$\Gamma(\alpha, \xi) = \sqrt{9 - 8\frac{\xi}{\alpha}}$$

In order to have real solutions, $\alpha \geq \frac{8}{9}\xi$ must hold. Moreover, in order to have $\bar{x} < 1$ (otherwise all workers wold be employed on a temporary basis), $\Gamma(\alpha, \xi) < 1$ should hold. This implies $\alpha < \xi$. Real and significant solutions therefore exist only if $\alpha < \xi \leq \frac{9}{8}\alpha$. Therefore, $0 \leq \Gamma(\alpha, \xi) \leq 1$ always holds. These strong restrictions derive from the simplification of the model presented above.

We can now compute the effects of a variation in union bargaining power on the economic system.

Remark 5. *Stronger unions raise the wage rate and the* keeping productivity *and reduce labour market tightness. A greater difference in the probability of termination of the contract between temporary and permanent contract (ϕ) reduces labour market tightness without affecting the wage rate or the* keeping productivity.

Proof. Differentiating equations 20 we can write:

$$\frac{d\theta}{d\alpha} = \frac{1}{\alpha}\frac{(3\alpha - 2\xi)\Gamma(\alpha, \xi) + (9\alpha - 10\xi)}{[(3\alpha - 2) - (2 - \alpha)\Gamma(\alpha, \xi)]^2} \quad \frac{8R}{\eta\Gamma(\alpha, \xi)}$$

whose sign is positive if $(3\alpha - 2\xi)\Gamma + (9\alpha - 10\xi) > 0$. Solving for α, we obtain $\alpha > \xi$. However, as we have seen above, significant solutions of the model exist only if $\alpha < \xi$. Therefore, we conclude that $\frac{d\theta}{d\alpha} < 0$ must always hold.

Differentiating equation 21, we obtain:

$$\frac{d\bar{x}}{d\alpha} = \frac{2\xi}{\alpha\Gamma} > 0$$

the same sign holds for $\frac{dw}{d\alpha}$. □

3 Flow, unemployment and productivity in a simplified case

In steady state, the stocks of unemployed, u, temporary workers, L^T, and permanent workers, L^P, depend on the flow conditions.

The number of unemployed people remains constant if:

$$\eta\theta(1-\underline{x})u = \lambda L^P + (\lambda+\phi)L^T \tag{22}$$

where the left hand side represents the number of unemployed who find a job[8] whereas the right hand side represents the number of permanent and temporary workers who lose a job.

The number of workers employed on a temporary basis is constant if:

$$[\lambda+\phi+\eta\theta(1-\bar{x})]L^T = \eta\theta(\bar{x}-\underline{x})u \tag{23}$$

where the right hand side represents the number of workers who leave the temporary position, and the right hand side represents the number of workers entering the temporary position[9].

The number of permanent workers is constant if:

$$\lambda L^P = \eta\theta(1-\bar{x})(L^T+u) \tag{24}$$

Given that we are mainly interested in the effects of α, we set, as before, $B = \underline{x} = 0$ and we further simplify the model by assuming $\phi = 0$, since we know that in equilibrium it does not affect the wage rate and the keeping productivity. Solving equation 23 on L^T, substituting it in equation 22 and considering $L^P = 1-u-L^T$, we obtain the usual definition of the equilibrium unemployment rate:

$$u(\theta) = \frac{\lambda}{\lambda+\eta\theta} \tag{25}$$

Remark 6. *The unemployment rate is decreasing in the probability of finding a job such that it is increasing in union bargaining power.*

Proof. The negative relationship between θ and α was shown in remark 5. □

[8] Given by the probability of an unemployed person finding a job ($\eta\theta(1-\underline{x})$) times the number of unemployed (u).

[9] The former is given by the temporary workers who lose their job because of negative shocks ($(\lambda+\phi)L^T$) plus those who leave the temporary position finding a permanent job ($\eta\theta(1-\bar{x})L^T$) and the latter by the number of matchings whose productivity is between $\bar{x}$ and $\underline{x}$.

Given the unemployment rate, we can solve for L^T using equation 23 and for L^P using equation 24. The *shakiness index*, defined as $\frac{L^T}{L^P+L^T}$, is given by:

$$s = \frac{\lambda\bar{x}}{\lambda + \eta\theta(1-\bar{x})}$$

Remark 7. Shakiness *in the labour market depends positively on $\bar{x}$ and negatively on θ such that it depends positively on union bargaining power, α.*

Proof. s depends positively on $\bar{x}$ and negatively on θ. By considering remark 5, the proof comes immediately. □

With the aim of evaluate the effects of union bargaining power on labour productivity[10], we define the average worker's productivity as:

$$y = \frac{\frac{1+\bar{x}}{2}L^P + \frac{\bar{x}}{2}L^T}{L^P + L^T}$$

Substituting equations 23 and 24 and rearranging, we obtain:

$$y = \frac{1}{2}\frac{\lambda + \eta\theta(1-\bar{x}^2)}{\lambda + \eta\theta(1-\bar{x})} \tag{26}$$

Remark 8. *The average per worker productivity is increasing in θ and increasing in $\bar{x}$ if $y > \bar{x}$ holds.*

Proof. Defining $D = \lambda + \eta\theta(1-\bar{x})$, and differentiating equation 26 with respect to θ, we obtain $\frac{dy}{d\theta} = \frac{\eta\lambda\bar{x}(1-\bar{x})}{2D^2} > 0$. The derivative of equation 26 with respect to $\bar{x}$ can be written as follows: $\frac{dy}{dw} = \frac{\eta\theta(y-\bar{x})}{D}$. □

Remark 9. *There exists a given level of union bargaining power that maximizes workers' productivity.*

Proof. In equation 26 both θ and $\bar{x}$ are functions of union bargaining power α (see equations 20 and 21. Differentiating equation 26 with respect to α, we obtain:

$$sign\left(\frac{dy}{d\alpha}\right) = sign\left(2\theta(y-\bar{x})\frac{d\bar{x}}{d\alpha} + (1-\bar{x})(1+\bar{x}-2y)\frac{d\theta}{d\alpha}\right) \tag{27}$$

[10] Recent analysis on this issue (Hirsch, 2004) argues that the average effect of unions on labour productivity is negligible. Freeman (2005) concludes that the union effect on productivity is certainly not negative, even if it is difficult to establish a positive one. A meta-analysis on the relationship between unions and productivity shows that there is a negative association in the United Kingdom and a positive one in the United States (Doucoliagos and Laroche, 2003).

where $\frac{d\bar{x}}{d\alpha} > 0$ and $\frac{d\theta}{d\alpha} < 0$, as shown in remark 5.

Substituting the two derivatives displayed in remark 5 in the right hand side of equation 27 we obtain a cumbersome result whose sign is, in general, not definable.

We also know that, in order to have meaningful solutions, $\alpha < \xi < \frac{9}{8}\alpha$ must hold. We follow the strategy of evaluating equation 27 for the minimum and the maximum value of x_i.

For $\alpha = \xi$, we have $\Gamma = 1$. It is easy to compute that $\frac{d\theta}{d\alpha} = 0$ and that $\frac{d\bar{x}}{d\alpha} = \frac{2}{\alpha}$. Furthermore, from equation 21 we obtain $\bar{x} = 1$ and from equation 26 we obtain $y = \frac{1}{2}$. Therefore, $y - x < 0$ and $\frac{dy}{d\alpha} < 0$. When α takes its maximum value, the average workers' productivity is decreasing in α.

For $\alpha = \frac{8}{9}\xi$, or, more precisely, taking the limit for ξ tending to $\frac{9}{8}\alpha$, Γ tends to zero and as does $\frac{d\theta}{d\alpha}$. Therefore, $\frac{dy}{d\alpha} > 0$ if $y > x$. But, in that case, $\bar{x} = \frac{1}{2}$ (see equation 11). We must show that, conditionally to $\alpha = \frac{8}{9}\xi$, $y > \frac{1}{2}$ holds, so that $\frac{1}{2}\frac{\lambda+\frac{3}{4}\eta\theta}{\lambda+\frac{1}{2}\eta\theta} > \frac{1}{2}$.This condition is always respected.

We can therefore conclude that for α at its minimum acceptable value the per worker productivity increases with union bargaining power and that for α at its maximum value per worker productivity decreases with union bargaining power. □

4 Conclusions

The model developed above highlighted the role played by unions in a segmented labour market divided between permanent workers, covered by contractual arrangements, and temporary workers, paid at their reservation wage. In this setting, assuming that higher union bargaining power leads to a higher wage rate, the wage setting function must be downward sloping with respect to labour market tightness. This result is in contrast with the theoretical literature and can be explained by the fact that higher tightness makes it more difficult to replace temporary workers (who are the more likely to leave jobs) and drives firms to keep more workers on permanent basis. In this way, the marginal and average productivity of permanent workers decrease and the bargained wage rate, which is a weighted sum of the average productivity and the reservation wage, must decrease as well.

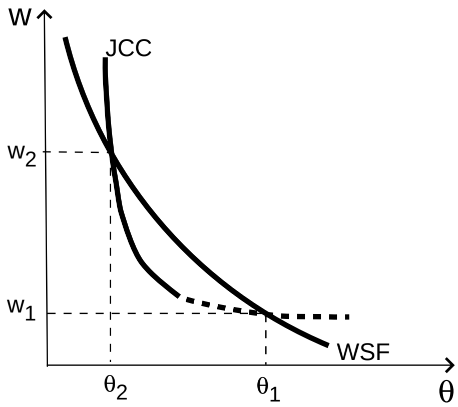
In the stable equilibrium, stronger unions raise the permanent workers' wage rate, unemployment and the ratio of temporary to total workers and reduce labour market tightness, as expected. By allocating the most productive matching to a permanent position, unions allow an increase in workers' productivity. Indeed, there is a given level of union bargaining power which maximizes per-capita product. Therefore, even when dealing with a decreasing wage setting function, we obtain results that are consistent with the economic literature.

The result concerning workers' productivity is probably the most interesting. In the setting presented above, the way in which workers are allocated between temporary and permanent jobs determines average workers' productivity. The two extreme cases consistent with the model hypotheses are the one where half of the workers are employed on a temporary basis and the one where all workers are employed on a temporary basis. The first case happens when unions have minimum bargaining power, the second coincides with the strongest unions. In the two cases, the average productivity of workers coincides with the average productivity of the average match. Firm evaluation of matching productivity and the choice of offering a permanent or a temporary position to each worker is crucial in determining the average productivity. Stronger unions, pushing firms toward less permanent jobs (because the wage rate of permanent workers becomes higher), allow a "better" allocation of workers and higher average productivity. Nevertheless, they increase unemployment. The price to pay for having higher productivity is higher unemployment and shakiness.

The reforms aimed to raise productivity may explain the contemporaneous reduction in unemployment rates and productivity and the increase in the ratio of temporary to permanent workers in many European Countries.

Further developments of the model should remove some hypotheses which, even if they allow us to solve the model analytically, are probably too restrictive, namely: a) the absence of firing costs; b) the wage rate of temporary workers equal to the reservation wage.

Figure 1: The wage setting function (WSF) and the job creation condition (JCC)



Appendix A

The whole solution of the model is represented by the system of two equations (18 and 19) in two endogenous variables ($\kappa(\theta)$, w).

By solving the WSF and the JCC defined above in $\kappa(\theta)$ and by equating the two solutions, defining as in the main text $\Gamma(\alpha,\xi)\sqrt{9-8\frac{\xi}{\alpha}}$, we end up with an equation in w that gives the following two roots:

$$w_i^* = \frac{\alpha}{4}\left[3 \pm \Gamma(\alpha,\xi)\right] \qquad for \quad i=1,2 \quad (i=1: minus \quad i=2: plus)$$

Given this result, we can compute the value of $\kappa(\theta)$:

$$\kappa^*(\theta)_i = \frac{2}{\alpha}\left(\frac{1 \pm \Gamma(\alpha,\xi)}{3 \pm \Gamma(\alpha,\xi)}\right) \qquad for \quad i=1,2 \quad (i=1: minus \quad i=2: plus)$$

and, given $\bar{x^*}_i = w_i^*\kappa^*(\theta)_i = 2$ the value of the *keeping productivity*:

$$\bar{x^*}_i = \frac{1 \pm \Gamma(\alpha,\xi)}{2} \qquad for \quad i=1,2 \quad (i=1: minus \quad i=2: plus)$$

The two roots give real solutions if $\xi \leq \frac{9}{8}\alpha$ where, if the equal sign holds, the roots coincide. The *keeping productivity*, $\bar{x}^*$, is included in the $0-1$ interval if $\alpha < \xi$, so that $\alpha < \xi < \frac{9}{8}\alpha$.

Assume now that two significant solutions exist. The selection between these two solutions can be obtained considering the dynamic of the model.

In particular, assume that the wage rate of equation 19 adjusts to labour market tightness with one period lag, so that $w_t = \frac{\alpha}{2-\alpha\kappa(\theta_{t-1})}$. Computing $\frac{w_t - w_{t-1}}{w_{t-1}}$, we obtain:

$$\frac{w_t - w_{t-1}}{w_{t-1}} = -\frac{2w_{t-1}\left(w_{t-1} - \frac{3}{2}\alpha\right) + \alpha\xi}{2w_{t-1}\left(w_{t-1} - \alpha\right) + \alpha\xi}$$

where both the numerator and the denominator show a minimum in w_{t-1}. The denominator is always positive because the minimum of w_{t-1} is attained for $w_{t-1} = \frac{\alpha}{2}$ and the value of w_{t-1} calculated at the minimum gives $w_{t-1}^{MIN} = \alpha(\xi - \frac{\alpha}{2}) > 0$ because $\xi > \alpha$.

Therefore the sign of the variation rate of wages depends on the opposite of the sign of the numerator, so that $\frac{w_t - w_{t-1}}{w_{t-1}} > 0$ if $2w_{t-1}\left(w_{t-1} - \frac{3}{2}\alpha\right) + \alpha\xi < 0$. Solving this quadratic form for w_{t-1}, we obtain that $\frac{w_t - w_{t-1}}{w_{t-1}} > 0$ for $w_1^* < w_{t-1} < w_2^*$, where w_1^* and w_2^* are the two roots of the system presented at the beginning of this appendix (see also figure 4) and that it decreases for external values of the two roots. We can therefore conclude that the wage rate varies until the only stable equilibrium w_2^* is attained and that, for the value of $w_{t-1} < w_1^*$, the system has no solutions.

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