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THE WEB'S PROMOTIONAL EFFECT

AND ARTISTS' STRATEGIES

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QUADERNO DI RICERCA n. 345

DIPARTIMENTO DI ECONOMIA

Luglio 2010

Abstract

The paper explores one of the new business models of the music market proposed by Varian (2005): the importance of the promotional effect of web-based diffusion. An indirect form of such investment consists in non-opposition by artists against the circulation of their music files online, or, likewise, their choice of permitting free downloads of their music albums. The profits lost from legal sales – online or on traditional supports – may be off-set by promotional advantages deriving from greater diffusion, with an increase in the artist's market share. The model assumes the existence of a strong network effect and an exchange of information, opinions and contents among web users. The model's results are determined by the initial conditions, i.e. by an artist's market share at an initial instant of time: or in other words, by his/her popularity. It is shown that emerging artists should make maximum investment in promotion, so that the diffusion of their work can be driven by the network effect and they can emerge from anonymity. Instead, for well-established artists, whose market shares are already large, the optimal strategy is to make the least promotional effort, given that the spontaneous diffusion of their work is already high.

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The web's promotional effect and artists' strategies

Francesco Balducci

1 Introduction

The music market has been traversed by profound changes in recent years. As well known, this has been due to the appearance and the rapid spread of the new digital technologies which have facilitated the retrieval of music files and their exchange on the internet. The phenomenon has undergone gradual and continuous evolution in its perception among consumers, recording companies, artists, hi-tech companies, internet service providers, etc.

The economic literature and legal science have sought to take account of this evolution, albeit with a certain delay due to the extreme speed of ongoing changes. The initial approach of both operators in the sector and economists was to denounce the phenomenon of file sharing as piracy. They conceived music files simply as innovations advantageous to a small number of users unwilling to pay for music. As a consequence, the main concern was to devise mechanisms with which to recover profits in the presence of piracy (Novos and Waldman, 1984; Chen and Png, 1999) through *indirect appropriation* (Boldrin and Levine, 2002; Besen and Kirby 1989), or to earn profits by taxing supports (Netanel, 2002; Fisher, 2004; Romer, 2002). The empirical literature likewise concentrated on the relationship between sales of original discs on traditional support and file sharing, the purpose being to identify and to quantify a possible displacement effect (Liebowitz, 2004; Oberholzer and Strumpf, 2007; Rob and Waldfogel, 2005; Session and Stevans, 2006; Zentner, 2006).

It was then realized that the new digital products were not solely illegal and low-level substitutes for the originals but had new and distinctive features, moreover, which were greatly ap-

preciated by consumers. This strand of analysis[1] comprised studies which focused on informational features of digital products (Halonen and Regner, 2004; Duchene and Waelbroeck, 2002; Peitz and Waelbroeck, 2003; Takeyama, 2002 and Zhang, 2002) and in particular on the network effect (Takeyama, 1994; Belleflamme, 2005 and 2007; Shy and Thisse, 1999; Gayer and Shy, 2003, Conner and Rumelt,1991). In the latter case, upon excluding the rather unrealistic hypothesis that consumers are autonomous in their choices, it was possible to take account of network effects, i.e. interdependence among consumer choices. On this basis, the idea was generally rejected that piracy necessarily reduces profits.

Today the digital market is no longer marginal and alternative with respect to that for physical supports. Rather, it has assumed an autonomous and significant role: a technological transition has come about among formats to which the supply side must necessarily respond. Numerous attempts have been made to deal with the change and a dominant model has not emerged, although *Apple*, by coupling *iTunes* with *iPod* has led the way in setting the music market's strategies in recent years. Varian (2005) lists a series of possible scenarios for the current digital world, in which it is increasingly difficult to earn revenues through the traditional channel of copyright payments. One of the main remedies proposed is that artists should advertise themselves through the diffusion of their music online. This promotional action may then impact on the live music market, which currently represents the prime source of income for artists.

This paper builds on the above considerations to propose a possible solution for artists and to develop a new scenario for the digital music market. It is based on the conviction that, following the technological transition, is necessary to devise new business models. In reality the economic literature has – at the time of writing – explored few of the scenarios proposed by Varian (2005).[2]

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[1] For an exhaustive survey see Peitz and Waelbroeck (2004).

[2] Krueger (2005) studies the concerts market, introducing the idea of substitutability between live shows and CDs and reprising Rosen's notion of the superstar effect (1981). Other studies concentrate on attempts at technological protection based on encryption or DRM (Jaisingh, 2004). Yet others propose, exploiting the benefits of the network effect, alternative strategies for recording companies, such as entry into the live music market, although they do not consider the interrelations between agents and markets (Gayer and Shy, 2006; Curien and Mureau, 2005; Becchetti and Eleuteri, 2007).

The following model attempts to explain some of the strategies recently adopted by artists. An example is the decision by the rock group *Radiohead* – then emulated by many other artists – to allow downloads of their latest album from their website for a price (even zero) decided by the consumer. Such a strategy was unimaginable until a few years ago, when artists and recording companies openly campaigned against file sharing, seeking technological or legal remedies against the online diffusion of music. But Radiohead's strategy brought it enormous profits deriving from the voluntary payments,[3] but above all from the impact on sales of the original album and attendance at the band's concerts.

The model proposed focuses on the effects of web-based promotion and *word-of-mouth*. Exploiting the network effect, artists can intervene in the dynamics of the spontaneous diffusion of their work by investing in web-based promotion. The frames of reference are clearly those of the network effect and the *peer group effect* (i.e. imitation among friends and acquaintances).

It is clear that artistic consumption choices are driven by a series of factors, many of which are of an emotional and strictly personal nature – and especially so in the case of avid music consumers, whose preference structures are very well defined. It is equally true, however, that the network aspect and the imitation effect are of great importance for the mass of music listeners, particularly those who frequent virtual communities, forums and discussion groups.[4] And it is all the more true in the present-day reality based on the virtual exchange of all kinds of information, and in which individual preferences seem highly malleable. Of this the media and, specifically the recording companies, are well aware.

In the following model the exchange of information comes about through contacts, with a mechanism similar to that of virus propagation (analysed in the literature on viral marketing and word-of-mouth: see e.g. Bass (1980). In this sense the modelling builds upon the work of Han, Hosanagar and Tan (2008).[5] Obvi-

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[3] For analysis of these voluntary payment systems see Regner and Barria (2007).

[4] Consider websites like *MySpace*, *FaceBook* or, specifically for music, *LastFM*, which works on the basis of information exchange, suggestions and reports among users. Indeed, there are automatic mechanisms which compile personalized music playlists on the basis of listening by the user, his/her friends, or other consumers of that genre of music.

[5] However, Han, Hosanagar and Tan's (2008) study deals with another problem: the choice of consumers whether or not to share downloaded content, in a static network.

ously, the model studies only one of ways in which the mechanism of information transmission can be analysed. But it seems well suited to the case of a peer-to-peer network whose members, tied together by a shared passion for music, are known to each other only by anonymous nicknames.

One of the results of the model is that, from a dynamic perspective, there are cases where it is convenient for artists to make maximum investment in promotion via the internet – for instance following Radiohead's example by granting free access to their work – in a first phase and then reap the promotional benefits propagated over time. This result depends closely on the initial conditions, that is, the market share possessed by the artist at the initial instant of time: in short, his/her popularity. It will therefore be emerging artists, who start with low market shares, that benefit from maximum investment in promotion via the web. Instead, for already-established artists, whose initial market shares are above a certain threshold value, it will be shown that the optimal choice is minimal investment in promotion, because the mechanism of their music's spontaneous diffusion is already sufficiently well-developed to guarantee high profits. This result is partly at odds with the observation that even very well-established artists opt for the free diffusion of their music. Evidently, the promotional effects are very marked. They impact on several sectors of the artists' activity, above all the live music sector, and they are not captured by the market share alone. It is precisely this that can represent an extension of the model presented and be an avenue for future research.

2 The process of diffusion of digital content

The network modelled is dynamic[6], i.e. the number of users N does not remain constant but grows in time according to an exogenous law of the type $N(t) = N_0 e^{nt}$, where $n = \dfrac{\dot{N}(t)}{N(t)}$ is the rate of exogenous expansion and N_0 is the number of consumers at the

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[6] The analysis is restricted to a flat P2P network, i.e. a network in which all the nodes (peers) are symmetrical in terms of hierarchical relationships and connectivity (Han, Hosanagar and Tan, 2008).

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initial time *t = 0*.[7] That the web is expanding is borne out by the empirical evidence, with regard to both the total number of users with internet access and the number of users of file-sharing or music information exchange websites, forums, discussion groups, and so on.[8]

Let *Q(t)* be the network members who know a particular artist and his/her work[9] and *q(t) = Q(t) / N(t)* their share in the total of web users. By definition *Q(t) ≤ N(t)* and $0 ≤ q(t) ≤ 1$. The share is the *state variable* and has its own dynamic, which will be discussed below.

The ability of artists to intervene in the dynamic of the state variable is translated by the variable *r(t)*, which represents the unit investment in web-based promotion. *P* is the unit price of the artistic product. The share *r(t)* is under the direct control of the optimizing artists, and it denotes their willingness or otherwise to allow their artistic products (e.g. music tracks or albums) to circulate on the internet. This share with respect to price can be interpreted in various ways: directly as investment in promotion via the web; indirectly as non-opposition to the circulation of one's products online.

Direct action can be immediately understood if *r(t)* is interpreted as investment in promotion via the internet, paying for the placement of advertising banners on specialized websites or in search engines, or paying commission or a proportion of sales proceeds to online retail websites. It is nevertheless possible to conceive *indirect* ways of investing in the new opportunities offered by the web: for instance, by not charging the highest possible prices for traditional supports but offering free or reduced-price downloads of unencrypted files from one's personal website. Also the strategy of not strenuously opposing the circulation of files through file-sharing networks – already adopted by many

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[7] For the sake of simplicity, in what follows N_0 will be set equal to 1.

[8] For example, in the past year the *Facebook* website has grown by 150%. In Europe alone, *Facebook* users have increased by 300% since June 2007. At world level, social networking sites were used by more than 580 million people in June 2007, with a 9% growth in North America, 35% in Europe, and 66% in the Middle East and Africa. *Facebook*, with 132 million contacts, for the first time outstripped *Myspace*, which attracted 117.5 million users (source: *LA7.it* - 14/08/2008).

[9] Here, the expression "knowing an artist" obviously does not denote the mere reception of information but direct, albeit superficial, artistic knowledge of an economic actor's output. The reference is to 'knowing with awareness of doing so': by way of example, consider the difference of meaning between the verbs *hearing* and *listening*.

artists at the time of *Napster* – or uploading videoclips or musical tracks to free websites like *MySpace* or *YouTube*, are examples of promotional initiatives.[10]

In other words, the advantages of internet promotion are so great that artists are willing to pay some sort of indemnity for them. Whilst it is possible to isolate oneself from the new technologies, it cannot be taken for granted that this will guarantee higher profits. Although it is probably possible to charge higher prices in this situation, the advantages of the network effect in terms of promotion would be entirely lost. This is all the more true in the case of the artistic sector, where the promotion of one's name, reputation, and recognizability assumes crucial importance. However, giving efficacy to web-based promotion requires obstructing the entirely selfish behaviour of consumers. One strategy in this regard has been to let the users of file-sharing websites download files provided that they in turn allow those files to be downloaded, without withdrawing them from the set of shared files (see Han, Hosanagar and Tan, 2005).

The share $r(t)$ ranges from zero to P: it will be zero if it is decided not to invest in internet promotion;[11] it will be instead equal to the price P if an amount equal to the artistic product's price is invested for each unit with no profit being made.[12] It is evident that the limit quantities $r(t) = 0$ and $r(t) = P$ are purely theoretical, in that they denote the intention to invest the minimum or the maximum possible in the short period with costs covered. It is unlikely that a situation will arise in practice where it is decided to invest a quantity $r(t) = P$ with nil profits, or a quantity $r(t) = 0$, with no expenditure on promotion.

The objective function of artists is profit, where for simplicity the unit costs are represented only by the share $r(t)$.[13] In what

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[10] Strenuous opposition against the circulation of files in P2P networks represents a cost for artists in terms of the constant struggle against piracy through DRMs and the encryption of files with innovative technologies. However, also letting one's work circulate freely online can be considered an indirect cost which consists in the forgoing of an optimal price due to the impossibility of stopping piracy and duplications.

[11] Not promoting oneself on the internet, when its spread seems unstoppable, also means opposing the circulation of files. In this sense, account should be taken of the direct costs incurred in fighting, and indirect ones due to not exploiting IT markets to acquire new customers.

[12] In the short period it is also possible to invest at a loss with $r(t) > P$.

[13] Consequently not considered are other types of costs like those of production, distribution and marketing.

follows, for the sake of simplicity the price *P* will be set equal to one: hence $0 \leq r(t) \leq 1$.

$$\Pi(t) = [P - r(t)]Q(t) = [1 - r(t)]q(t)N(t) \qquad (1)$$

The hypothesis of profit maximization is not self-evident; yet on considering a typical artist type, it seems the most plausible. An emerging artist, in fact, may be interested only in the promotion of his/her name and image, exploiting the economic advantages that derive therefrom in a second phase. However, it is difficult to imagine a situation in which the variable to be maximized is only image promotion (which is initially costly) without ever recouping the investments made. By contrast, well-established artists will probably seek to extract the maximum revenues from consumers during the predictable time-span of their success. The interval of time $[0;T]$, indeed, can be interpreted as an individual's artistic life-span, or, in the case of an emerging artist, the period that s/he self-assigns to become established. If in this lapse of time s/he fails to break through, s/he will give up the idea of pursuing a professional artistic career, or switch to the amateur circuit.

2.1 Diffusion online

A key hypothesis of this study is that the dissemination of information via the internet works by means of contact, in a way entirely similar to that already studied in the economic literature on the propagation of viruses or the effects of word-of-mouth. In fact, a particularly efficient way to acquire, circulate and exchange information is by word-of-mouth among the members of a network. Numerous successful websites, in fact, operate on the basis of the exchange of opinions, feedback and comments among users.

The dynamic of *Q(t)*, therefore, in the absence of action by artists, is as follows:

$$\dot{Q}(t) = \beta(N(t) - Q(t))\frac{Q(t)}{N(t)}$$

or, in terms of the growth rates of the shares,

$$\frac{\dot{q}(t)}{q(t)} = \frac{\dot{Q}(t)}{Q(t)} - n = \beta(1 - q(t)) - n$$

The meaning of the dynamic of *Q(t)* is as follows: a share *β* of users unaware of a particular artist (*N(t) - Q(t)*) may get to know about him/her if they come into contact with one of the network's members who already knows the artist (*Q(t) / N(t)*). The parameter *β* denotes, in a way, the *influenceability* of users, i.e. their predisposition to increase their knowledge of an artist recommended by other users or simply discussed with them.[14] Aside from its economic meaning, the equation for the dynamics of *q(t)* is entirely similar to that comprised in numerous growth theory models, where the parameter *β* measures the speed of convergence.

Having described the spontaneous dynamic of the diffusion process, the control variable *r(t)* is now introduced:

$$\dot{q}(t) = [\beta(1 - q(t)) - n + \gamma r(t)]q(t) \qquad (2)$$

Promotional action to induce diffusion via the web directly influences the rate of growth[15] of *q(t)* through the weighting of parameter γ. This parameter measures the effectiveness of the promotional action.

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[14] If the majority of users were passionate about music and had well-established preferences, *β* would be very low, or even zero. In this case, every individual would base his/her musical choices on personal motivations unaffected by the relationship with others, and the network effect would be annulled.

[15] There are other ways to intervene in the dynamic of *q(t)*. One can imagine that the artists' action directly affects the speed of convergence:

$$\dot{q}(t) = [\beta(r(t))(1 - q(t)) - n]q(t)$$

This way of inserting the control variable in the model is much used by growth theory scholars. However, it does not appear congruent with the meaning attributed to the exogenous parameter *β*, defined as the share of *non-Q(t)* users influenceable by contact with a user *Q(t)*. The exogeneity of *β* is evident, and it is unlikely to be modified by the promotional action of artists.

Alternatively, one could envisage a one-off promotional campaign (similar to a lump-sum tax) that influences the variation of *q(t)* through parameter γ but not its rate of growth:

$$\dot{q}(t) = [\beta(1 - q(t)) - n]q(t) + \gamma r(t)$$

 This variant is plausible in the case examined here and may represent an extension of the analysis conducted.

The following table summarizes the symbols used in the model.

Table 1: variables used in the model

t	time
$N(t)$	number of network users ($N_0 = 1$)
n	the exogenous expansion rate of the web
$Q(t)$	number of web users who know a particular artist
$q(t)$	share of users $Q(t)$ in the total of users $N(t)$ ($q(t) = Q(t) / N(t)$)
P	unit price of the artistic product ($P = 1$)
$r(t)$	promotional investment or unit cost
$\beta > 0$	speed of convergence or share of 'influenceable' users
$\gamma > 0$	measure of the impact of the promotional action on the dynamic of $q(t)$
$\rho > 0$	intertemporal discount rate; $\rho \geq n$

3 The dynamic optimization problem

3.1 The optimal problem

The optimization problem in a finite time horizon[16] ($t \in [0, T]$) of an artist whose works are diffused via internet and P2P networks can be expressed by the following equation. Intertemporal profit is actualized by the magnitude $e^{-\rho t}$, in which ρ denotes the intertemporal discount rate:

$$\max_{r(t)} \int_0^T e^{-\rho t} \Pi(t) dt \qquad (3)$$

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[16] The problem can also be studied, and with conclusions entirely similar to those reached here, in an infinite horizon. However, given the short-term perspective typical of artistic choices and the markets for new technologies, it has been preferred to present the finite-horizon solution.

$$s.to \begin{cases} \dot{q}(t) = [\beta(1 - q(t)) + \gamma r(t) - n]q(t) \\ q(0) = q_0 > 0 \\ q(T) = free \\ r(t) \in [0,1] \end{cases}$$

The equation for the dynamic of *q(t)* is that expressed by (2). The problem will be solved using the optimal control theory procedure according to Pontryagin's maximum principle. Control is limited, being between zero and one, and the problem is finite-horizon with final time (*T*) given but with free final state. Required in this situation is fulfilment of a transversality condition on the *co-state variable λ(t)*:

$$\text{TVC: } \lambda(T) = 0$$

That is to say, the co-state variable must assume nil value at the final time.

Optimal control theory envisages formulation of the so-called Hamiltonian equation to solve the intertemporal maximization problem:

$$H[r(t), q(t), \lambda(t), t] = e^{(n-\rho)t}[1 - r(t)]q(t) + \lambda(t)q(t)[\beta(1 - q(t)) + \gamma r(t) - n]$$

$$(4)$$

where the co-state variable multiplies the constraint expressed by the equation for the motion of the state variable. In formulating (4), moreover, the term *N(t)* has been substituted with its specification $N(t) = N_0 e^{nt}$, in which $N_0 = 1$. In what follows, it will be set $\rho = n$ for the sake of simplicity and for clearer exposition of the results. This simplification does not alter the results of the model, which are valid also with a generic $\rho > n$.

According to the maximum principle, the Hamiltonian function must be maximized with respect to the control; it is therefore necessary to annul the first derivative of (4) with respect to *r(t)*:

$$\frac{\partial H(\cdot)}{\partial r(t)} = 0$$

In this case, however, the Hamiltonian function is linear with respect to the control, which appears at the first degree in $H(\cdot)$. This is evident if we rewrite (4) in the following way:

$$H(\cdot) = r(t)q(t)[\gamma\lambda(t) - e^{(n-\rho)t}] + q(t)[e^{(n-\rho)t} + \lambda(t)\beta(1-q(t)) - n\lambda(t)]$$

Here we have a particular case in which the maximum of the function does not lie at a point within the interval but at one of the extremes of the control variable's range (values zero or one). This type of corner solution is known as the 'bang-bang solution'.

A context where $r(t) = 0$ or $r(t) = 1$ is, as said, clearly unfeasible in practice. But it assumes major theoretical importance by indicating a propensity to invest the maximum or the minimum possible. The values zero and one, therefore, must be instead conceived as r_{min} or r_{max}. For example, initially investing the maximum in promotion may enable an artist to emerge from anonymity and acquire a share of the market which enables him/her to continue making profits whilst reducing the promotional investment to the minimum.

Because this is a corner solution, optimal control will be maximum if the slope of the Hamiltonian is positive, and it will be minimum if the slope is negative, i.e. according to the sign of the expression $[\gamma\lambda(t) - 1]$:[17]

$$\begin{cases} r^* = r_{max} = 1 & if \quad \lambda(t) > \frac{1}{\gamma} \\ r^* = r_{min} = 0 & if \quad \lambda(t) < \frac{1}{\gamma} \end{cases} \qquad (5)$$

Optimal control will therefore be minimum or maximum according to the value of the co-state variable: it is necessary to deter-

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[17] Because $q(t) \geq 0 \ \forall t \in [0;T]$.

mine in what interval $\lambda(t)$ lies above or below the horizontal line $1/\gamma$.[18]

3.2 The steady-state solutions

Before studying the dynamics of the co-state variable, we continue with analysis of the equation of motion in the two possible cases. In fact, the optimal values for the control variable can be substituted for the general expression (2):

$$\begin{cases} \dot{q}(t) = [\beta(1 - q(t)) + \gamma - n]q(t) & \text{if} \quad r^* = r_{\max} = 1 \\ \dot{q}(t) = [\beta(1 - q(t)) - n]q(t) & \text{if} \quad r^* = r_{\min} = 0 \end{cases} \quad (6)$$

The above system describes the dynamic of $q(t)$.

Of major importance for characterization of the dynamic equilibrium are the so-called steady-state points to which the system may tend in the long period. Two[19] steady-state values for $q(t)$ can be obtained from system (6)

$$\begin{cases} q_H^* = \frac{\beta + \gamma - n}{\beta} & \text{if} \quad r^* = r_{\max} = 1 \\ q_L^* = \frac{\beta - n}{\beta} & \text{if} \quad r^* = r_{\min} = 0 \end{cases} \quad (7)$$

The subscripts H and L characterize the steady-state points as high and low respectively, in correspondence with the maximum and minimum values of the optimal control. It is evident, in fact, that q^*_H is greater than q^*_L of a quantity γ/β. From a comparative statics, both the steady-state points increase with an increase in parameter β and decrease with an increase in n. In other words, the market share that can be achieved in a steady-state condition is larger, the greater the speed of convergence (or the more web users are influenceable by word-of-mouth), and it is smaller, the greater the speed of the web's exogenous expansion. Obviously, a continuous expansion of the web erodes the market share

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[18] In the general case with $\rho > n$, the term [$\gamma\lambda(t)$ – 1] becomes $[\gamma\lambda(t) - e^{(n-\rho)t}]$. Therefore $\lambda(t)$ should be compared with the decreasing curve $\frac{e^{(n-\rho)t}}{\gamma}$ rather than with the horizontal line $1/\gamma$.

[19] A third steady-state point is the null one where there is no dynamic and which gives rise to a trivial result.

achieved instant by instant, because of a widening of the reference base.

Moreover, in the case of q_H*, the steady-state market share may also be increased by the exogenous increase of γ, that is, of the parameter that measures the impact of $r(t)$ on the dynamic of $q(t)$.

Proposition 1: *the steady-state values increase with an increase in parameter β (influenceability of users or speed of convergence) and they decrease with an increase in n (rate of exogenous expansion of the web).*

Proposition 2: *the high steady-state value ($q*_H$) increases with an increase in parameter γ, which measures the impact of the promotional action on the dynamic of the market share.*

Having characterized the steady-state equilibrium of the state variable, it is of interest to trace its optimal trajectory by solving the two differential equations of the first order and second degree of system (6). For calculation of these solutions, the initial condition $q(0) = q_0$ is used to determine the constant, and parameters $\frac{\beta+\gamma-n}{\beta}$ and $\frac{\beta-n}{\beta}$ are substituted out using their definitions.
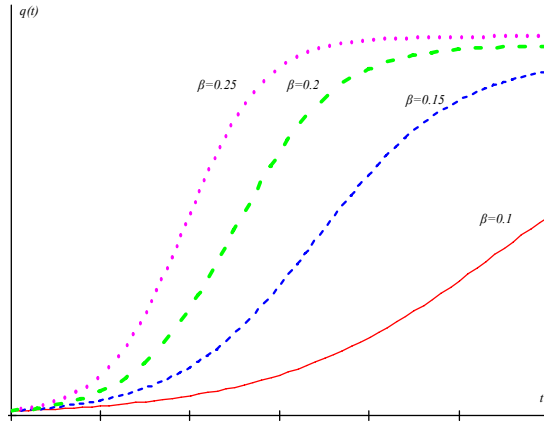
$$\begin{cases} q(t) = \frac{q_H^*}{1+(\frac{q_H^*}{q_0}-1)e^{-\beta q_H^* t}} & if \quad r^* = r_{\max} = 1 \\ q(t) = \frac{q_L^*}{1+(\frac{q_L^*}{q_0}-1)e^{-\beta q_L^* t}} & if \quad r^* = r_{\min} = 0 \end{cases} \tag{8}$$

The optimal trajectories of $q(t)$ have, by definition, an initial point at q_0 (for $t \to 0$), and in the long period (for $t \to \infty$) they converge on their steady-state values, more rapidly when they tend to the high steady state, and more slowly when they tend to the low one.
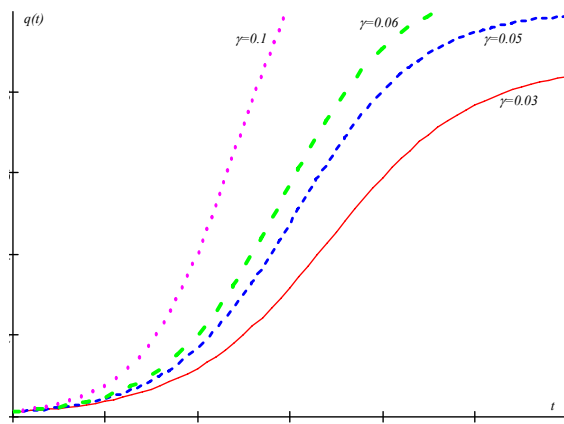
The following figures show the effect of variation of the parameters on the optimal trajectory with $r* = r_{max} = 1$ and $q*_H$. As previously said in regard to the steady-state values, to be noted in the simulations of figures 1 and 2 an increase in β and γ respectively cause an increase in the speed of convergence. An increase in the web's rate of expansion (n) instead causes a downward shift in the optimal trajectory (figure 3).
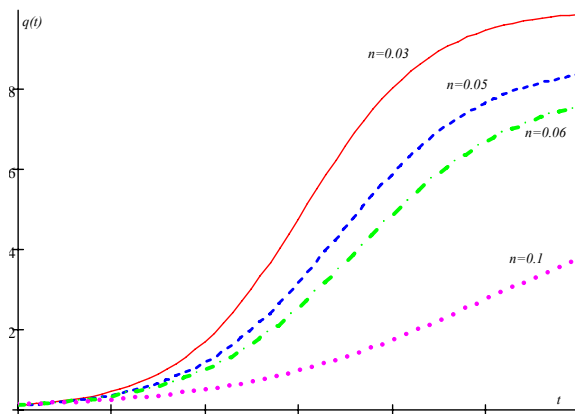
Figure 1: Impact of *β* on q*(t)*

q(t)

β=0.25   *β=0.2*   *β=0.15*

β=0.1

t

Figure 2: Impact of *γ* on q*(t)*

q(t)

γ=0.1   *γ=0.06*   *γ=0.05*

γ=0.03

t

Figure 3: Impact of *n* on *q(t)*

q(t)

n=0.03

n=0.05

n=0.06

8

6

n=0.1

4

2

t

14

3.3 The dynamic of the co-state variable

The optimal problem has not yet been solved: hence analysis of the co-state variable is now resumed. Pontryagin's maximum principle requires fulfilment of a further condition: the derivative of the Hamiltonian with respect to the state variable defines the dynamic of the co-state variable.

$$\frac{\partial H(\cdot)}{\partial q(t)} = -\dot{\lambda}(t) \Rightarrow \dot{\lambda}(t) = -(1 - r(t)) - \lambda(t)[\beta(1 - 2q(t)) + \gamma r(t) - n]$$

(9)

Solving the differential equation (9) yields an optimal trajectory for the co-state variable. From this trajectory, which identifies the intervals in which $\lambda(t)$ is greater or less than the horizontal line $1/\gamma$, then derives the attribution of the maximum or minimum value to the control. Consequently, after determining the optimal control, the high or low trajectory and the steady-state market share are selected. However, owing to the complexity of the systems of differential equations (9) and (6), it is not possible to obtain an explicit solution for the dynamic of $\lambda(t)$.

Although the equation for the co-state variable is not written in closed form, it is possible to solve the dynamic optimization problem by studying the information that derives from the slope of $\lambda(t)$ (equation 9) and from the transversality condition. Fulfilment of the TVC, in fact, ensures that, at the final instant of time (T), the value of $\lambda(t)$ is nil ($\lambda(T) = 0$). This information guarantees that $\lambda(t)$, by being zero in T, cannot be greater than the positive value $1/\gamma$ at that point. At the final point, therefore, the control will necessarily be $r^* = r_{min} = 0$ and the steady-state value will be q^*_L.

This information makes it possible to exclude 'explosive' dynamics for $\lambda(t)$, like those represented by Graph (b) in figure 4, because they do not fulfil the TVC.[20] The dynamics of $\lambda(t)$ depicted by

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[20] The convexity of the curves is purely indicative and represents only one of the possible patterns. However, because this concerns the dynamic of the co-state variable, study of the concavity of the curves it is not relevant to the present analysis. What is instead important is understanding whether the curve $\lambda(t)$ lies above or below the straight line $1/\gamma$.

Graph (a) are instead all compatible with the optimization condi-tions. However, analysis of cases in which the curve always lies below the straight line without ever intersecting with it is not of interest. In such cases, in fact, a true bang-bang situation does not arise, and the solution is obvious: the minimum promotional effort is always made, necessarily converging on the low steady state. The case in which $\lambda(t)$ is always negative, moreover, is plausible only for a narrow interval of the parameters.

**Figure 4: dynamics of $\lambda(t)$ (Graph a, left; Graph b, right)**
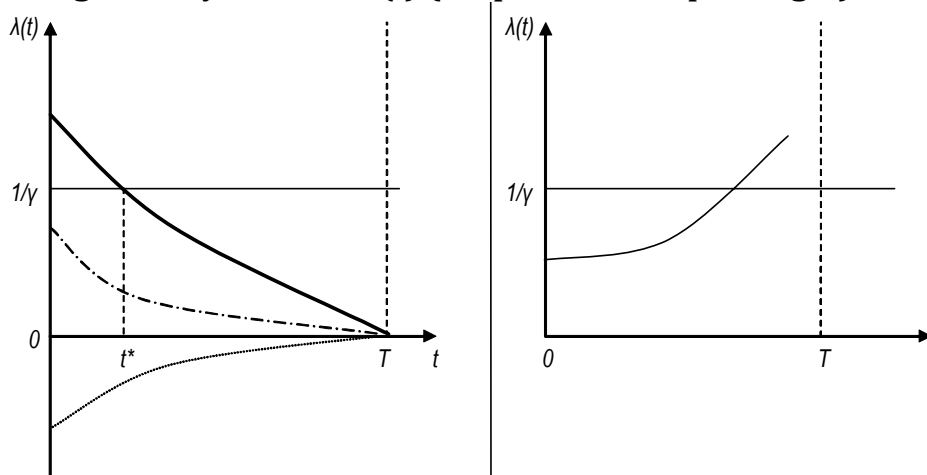
The truly significant case to analyse is the one in which, work-ing backwards from $\lambda(T) = 0$, $\lambda(t)$ intersects the straight line $1/\gamma$ at a point of the interval $[0,T]$. This point is denoted with t^*.

At the point of intersection t^* we have $\lambda(t^*) = 1/\gamma$ and

$$\dot{\lambda}(t^*) = -(1 - r(t^*)) - \tfrac{1}{\gamma}[\beta(1 - 2q(t^*)) + \gamma r(t^*) - n].$$ By definition, working backwards from the final point at which $\lambda(T) < 1/\gamma$, it is clear that in the right-hand neighbourhood of point t^* the curve will lie below the discriminant straight line and the optimal strat-egy will be the one with $r^* = r_{min} = 0$ and $q(t) = \frac{q_L^*}{1 + (\frac{q_L^*}{q_0} - 1)e^{-\beta q_L^* t}}$.

By contrast, in the left-hand neighbourhood of t^*, two cases may arise (figure 5): it is possible that the curve $\lambda(t)$ may move above the straight line $1/\gamma$, giving rise to a switch strategy (case A), or

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the curve may turn downwards again, reaching a maximum point in *t** (case B).

Figure 5: $\lambda(t)$ in the neighbourhood of point t^*
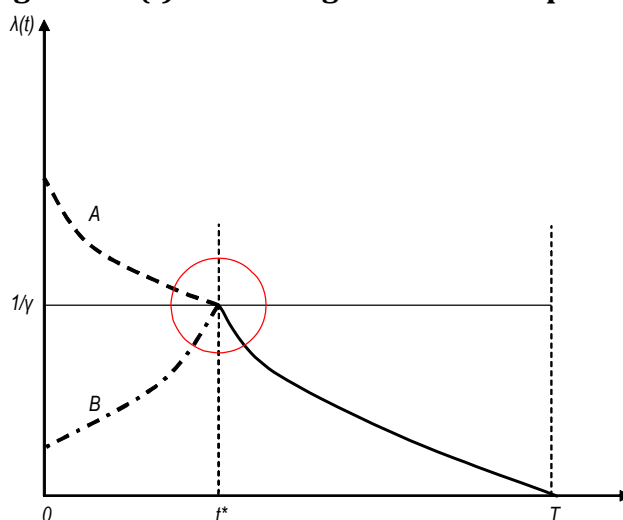
Once again, even without explicitly knowing the expression of $\lambda(t)$, it is possible to obtain significant information by analysing its slope. If in the left-hand neighbourhood of *t** the slope remains negative, the curve will cross the horizontal straight line, rising above it until it reaches an initial point $\lambda(0) = \lambda_0 > 1/\gamma$. If instead a change of slope occurs in this neighbourhood, such that to the left

of *t** it is $\dot{\lambda}(t)^- > 0$ and $\dot{\lambda}(t^*) = 0$, there will be a maximum in *t** and always a low-type strategy, i.e. minimum promotional effort.

Hence, to summarize, it is possible to have a switch strategy – i.e. make the maximum promotional investment in the first phase and minimum in the second – only if there exists an instant of time in which the co-state variable crosses the horizontal line $1/\gamma$. This can only happen if in the left-hand neighbourhood of the point *t** the slope of $\lambda(t)$ is still negative.

It will be shown in the next section that the slope of $\lambda(t)$ to the left of the instant of time *t** depends on the value reached by the market share at point *t**. This value ultimately depends on the initial value of the market share.

3.4 Dependence on the initial conditions

Figure 5 above illustrates two possible patterns of $\lambda(t)$. As said, the cases in which $\lambda(t)$ does not intersect the horizontal straight line are not of interest, because by definition the optimal control would always be $r(t) = 0$. It is instead useful to determine the values of the parameters that make it possible for a 'bang-bang solution' to come about. In particular, by analysing the slope of $\lambda(t)$ in the left-hand neighbourhood of t^*, it is possible to identify the values of $q(t)$ and q_0 that verify a switch strategy or a non-switch strategy.

It is therefore hypothesised that this is the case where a switch effectively takes place at point t^*. In a neighbourhood to the left of this point, therefore, $\lambda(t)$ will be greater than $1/\gamma$ and hence $r(t)$ will be at its maximum value, equal to one. The slope with $r^* = r_{max} = 1$ will be:

$$\dot{\lambda}(t^*)^- = -\frac{1}{\gamma}[\beta(1 - 2q(t^*)) + \gamma - n] \qquad (10)$$

It is now necessary to determine the values of $q(t^*)$ for which it is admissible that the slope expressed in (10) is negative as hypothesised. For some values of $q(t^*)$, in fact, this cannot be verified. Hence it follows that such values can be excluded from the cases studied. Put otherwise, in correspondence to this interval of values the slope expressed in (10) will be positive and not negative. In particular:

$$\dot{\lambda}(t^*)^- < 0 \quad if \quad q(t^*) < \frac{\beta + \gamma - n}{2\beta} = \frac{q_H^*}{2} \qquad (11)$$

It is therefore admissible that the slope of $\lambda(t)$ continues to be negative also in a left-hand neighbourhood of t^* – thus giving rise to a bang-bang solution – only if the value of the market share reached at that instant of time is still less than half the steady-state value ($q_H^*/2$). In other words, if during the time interval [0, t^* [the market share has reached a very high value, it is not admissible to think that a switch strategy has occurred between the

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maximum and minimum value of the control variable. The solution is also verified in the second set of cases considered: only some values of $q(t^*)$ (in particular for $q(t^*) \geq \frac{q_H^*}{2}$) render the slope of $\lambda(t)$ positive and therefore again generate a low-level strategy level with minimum promotional effort. To summarize in formal terms:

$$
if \quad q(t^*) < \frac{q_H^*}{2} \Rightarrow
\begin{cases}
r^* = r_{max} = 1; \quad q(t) = \frac{q_H^*}{1+(\frac{q_H^*}{q_0}-1)e^{-\beta q_H^* t}} \quad \forall t \in [0; t^*[\\
r^* = r_{min} = 0; \quad q(t) = \frac{q_L^*}{1+(\frac{q_L^*}{q_0}-1)e^{-\beta q_L^* t}} \quad \forall t \in [t^*; T]
\end{cases}
$$

(12)

$$
if \quad q(t^*) \geq \frac{q_H^*}{2} \Rightarrow r^* = r_{min} = 0; \quad q(t) = \frac{q_L^*}{1+(\frac{q_L^*}{q_0}-1)e^{-\beta q_L^* t}} \quad \forall t \in [0; T]
$$

But the characterization of the model is still not complete. Having understood that the discriminant between the possibility of adopting a switch strategy and continuing to make the minimum amount of promotional effort is the level reached in t^* by the market share $q(t)$, one can link this level back to the initial conditions of the model. In other words, one can relate the solution found for $q(t^*)$ to the initial value q_0, i.e. to an artist's starting level of fame as measured in terms of market share. This is possible precisely because the market share achieved in t^* depends on the initial conditions. In other words, one has to find the values of q_0 that effectively yield $q(t^*) < \frac{q_H^*}{2}$. Substituting the values of $q(t^*)$ and q^*_H with their expressions produces:

$$
q(t^*) < \frac{q_H^*}{2} \quad if \quad q_0 < \frac{q_H^*}{1+e^{\beta q_H^* t^*}} \qquad (13)
$$

A switch strategy will be practicable only if the initial level is not high, and in particular below the threshold $\frac{q_H^*}{1+e^{\beta q_H^* t^*}}$.[21]

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[21] Note, however, that the value q_0 cannot be equal to zero, otherwise there would be no dynamic in the model. Moreover, if q_0 were very small, or close to zero, this would constitute one of the cases not analysed in which $\lambda(t)$ is always below the discriminant straight line (see figure 4).

This circumstance seems reasonable for the large number of emerging artists who need to gain visibility by investing a great deal, to the limit of the maximum $r^* = r_{max} = 1$, in the first phase. Even if investment stops after a certain interval of time, the profits continue their optimal trajectory because of the beneficial effects of word-of-mouth and spontaneous diffusion via the web. Instead, for the smaller group of already-established artists – who have a q_0 greater than the threshold value – the process of automatic diffusion has been high since the outset. Hence their optimal strategy is not to invest much in promotion, maintaining a low $r(t)$ for the entire interval of time.

Proposition 3. *It is possible to invest the maximum in promotion to converge on the high steady state (q^*_H) in the interval $[0, t^*$ $[$ only if the initial market share is low (below a threshold: $q_0 < \frac{q^*_H}{1+e^{\beta q^*_H t^*}}$).*
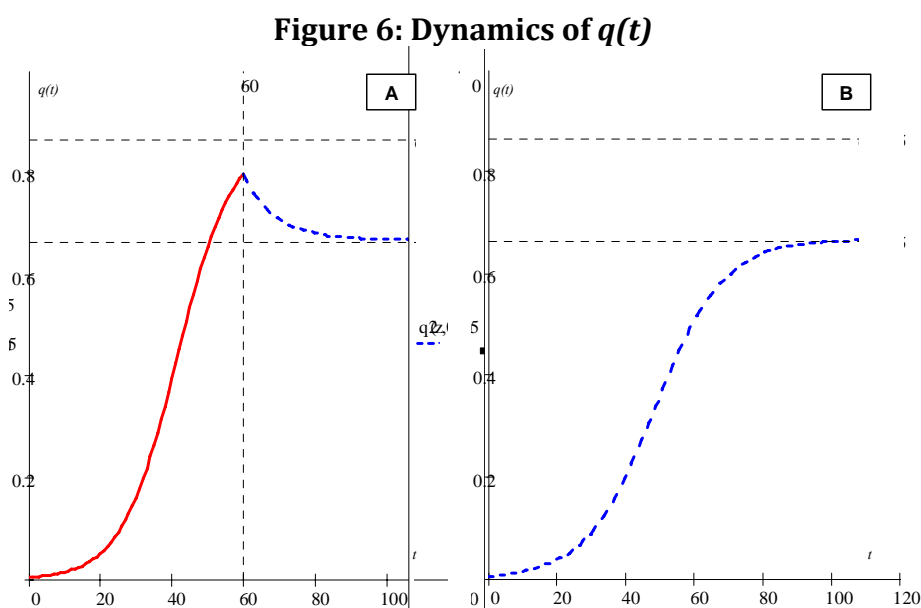
Figure 6: Dynamics of *q(t)*

Figure 6 illustrates the dynamics of *q(t)* in the presence of the bang-bang solution (graphic A) and otherwise (graph B). In each case the final result is convergence on the low steady state q^*_L. In the case of the switch, however, this convergence comes about

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top-downwards, after the convergence in the interval [0, t^*[has been towards the steady state q^*_H.[22]

3.5 The effects on profits

To complete the description of the model, having studied the dynamic of $q(t)$, it is now necessary to study that of the optimal profit, $\Pi(t) = [1 - r(t)] \, q(t)N(t)$. In the case where there is no switch strategy such description is straightforward: the profit will be equal, for the entire interval of time, to $q(t)N(t)$, because the value of the control variable is equal to zero. Hence:

$$if \quad q(t^*) \geq \frac{q_H^*}{2} \Rightarrow r^* = r_{\min} = 0; \quad \Pi(t) = e^{nt} \frac{q_L^*}{1 + (\frac{q_L^*}{q_0} - 1)e^{-\beta q_L^* t}} \quad \forall t \in [0;T]$$

(14)

Instead, in the case of a bang-bang solution, the profit will have the same formulation as before (equation 14) in the interval [t^*,T] , and it will be instead nil[23] in the first phase [0, t^* [. This is due to the fact that if it is decided to invest the maximum, the control variable is equal to one, so that $\Pi(t) = 0$:

$$if \quad q(t^*) < \frac{q_H^*}{2} \Rightarrow \begin{cases} r^* = r_{\max} = 1; \quad \Pi(t) = 0 \quad \forall t \in [0;t^*[\\ r^* = r_{\min} = 0; \quad \Pi(t) = e^{nt} \frac{q_L^*}{1 + (\frac{q_L^*}{q_0} - 1)e^{-\beta q_L^* t}} \quad \forall t \in [t^*;T] \end{cases}$$

(15)

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[22] Dependence on the initial conditions is also relevant to establishing another important result, namely the point of maximum expansion of an artist's market share. Converging on q^*_L from below, the maximum attainable value of the market share will necessarily be that of low steady state. If a switch strategy is adopted instead, it is possible that the market share will reach a higher value and then converge on q^*_L from above. The result is determined by the position of the instant of switching time t^*, which in turn is determined by the initial condition q_0.

[23] As already mentioned, the situation of nil profit must be interpreted in a purely theoretical context caused by a promotional investment equal only to the price. However, the intertemporal profit valued in the overall interval is positive and, as has been shown, is the optimal one.

Figure 7: the optimal profits
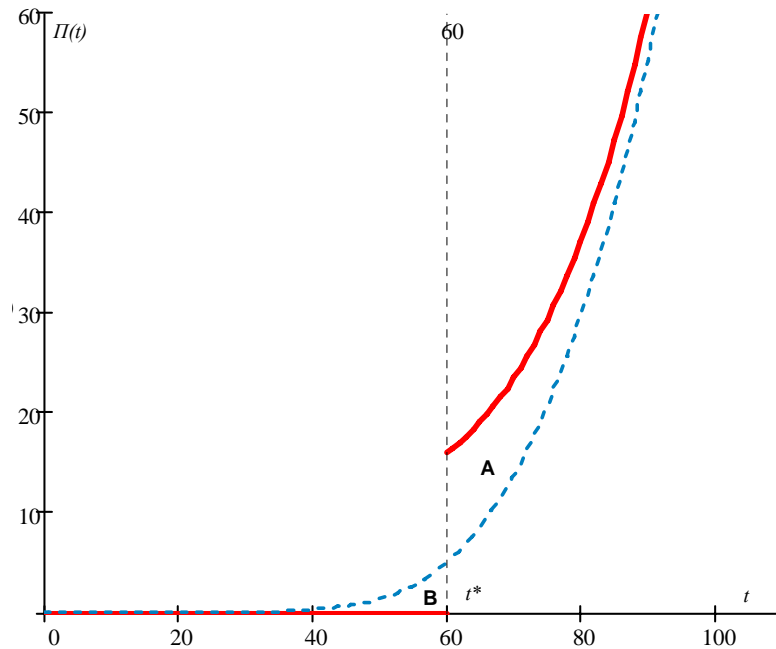
Note that, although they have the same analytical formulation, the profit expressed by (14) is different from that expressed by (15) for the interval $[t^*,T]$. This is due to the fact that the expressions of profit are characterized by different values of q_0. As shown in the previous section, in fact, the values of q_0 that make a switch strategy admissible (and therefore generate the profit expressed by (15)) are different from those that generate (14).

This conclusion can be depicted with Figure 7, which shows the profits in the case of a bang-bang solution (solid line) and otherwise (dashed line). It is evident from the graph that if area A (lying between the solid and dashed curves in the interval $[t^*,T]$) is larger than area B (lying underneath the dashed curve in the interval $[0, t^*[$), profits are higher in the case of switching. If the reverse is the case, the highest profits are obtained on pursuing a strategy of minimum promotional effort throughout the entire interval of time instead. It is the value of q_0 that alters the two areas: once again, different values of the initial market share give rise to different results.

Proposition 4. *If the area A lying between the solid and dashed curves in the interval* [*t*,T*] *is larger than the area underneath the dashed curve in the interval* [0, *t** [*(area B), profits will be greater if the maximum investment in promotion is made in the first phase and the minimum one in the second* (figure 7).

4 Conclusions

Technological innovation has brought profound changes to the traditional workings of the music market. After a first phase of bewilderment, artists, the recording companies, and operators in the sector are beginning to devise new business models in order to keep pace with the evolution of consumer preferences. Although numerous scenarios have been proposed, none of them seems to have prevailed. Solutions have ranged from the idea – not new – of technological protection by means of proprietary formats, encrypted files and DRM, through attempts at legal protection by enforcing the law on copyright and intangibles, to discounts on traditional supports. More convincing, however, seem actions to exploit the new opportunities offered by hi-tech and the internet. One thinks in this regard of *Apple*'s strategy of using legal sales through its *iTunes* software to the advantage of sales of hardware devices – portable PCs and *iPods* – and vice versa generating a profitable lock-in.

One of the main opportunities offered by the web is without doubt the promotion that it makes possible in terms of visibility and reaching a large number of consumers. In fact, due to the network effect widely studied in the economic literature, it is possible to earn profits even in the presence of piracy and without charging the highest possible price – or even by allowing free downloads.

The model proposed takes account of the promotional capacity of the internet and imagines a possible scenario for artists, who can foster the diffusion of their music by exploiting the network effect and word-of-mouth. It is imagined in fact that artists are able to intervene in the dynamic of spontaneous web-based diffusion by encouraging it – with direct or indirect promotional actions – or hindering it.

By means of the model it has been shown that investing in promotion generates a steady-state level for the market share which is greater than that which would be obtained if no investment were made (or in other words, if the online circulation of music files were opposed). There are two possible optimal results: no invest at all in promotion if the spontaneous diffusion dynamic is strong from the outset; or maximum investment in the first phase, to then once again converge on the low steady state. It has been shown that the choice between the former and the latter strategy is determined by the value of the market share that artists have at the initial instant of time.

In particular, a switch strategy – i.e. a shift from the maximum to the minimum investment – can be adopted only if the initial market share is below a threshold value. From this derives one of the main results of the model: namely that it is advisable for emerging artists to invest the maximum in the first phase, even accepting that they will operate at a loss, so that they can gain visibility and trigger the process of automatic diffusion through word-of-mouth. For those who start with a large market share (the established artists), the mechanism is already well under way, and the optimal strategy is not to invest in promotion at all.

In reality, also very well-established artists today opt for free diffusion. Evidently, the promotional effects are very marked. They impact on several sectors of artists' activity, above all live music, and are not captured by the model's state variable, namely the market share. From this derives an interesting extension and a direction for further research.

The model's bang-bang solution depends on the formulations (highly standard) adopted for profits, dynamic of the state variable, and the control variable: other formulations may not generate corner solutions. Yet the corner solution is interesting if it is conceived as the propensity to invest the maximum or the minimum in the phases of the artistic life-cycle. It is well known that emerging artists are often willing to invest the maximum in a first phase in order to achieve a breakthrough.

A further interesting extension could be the introduction of a stochastic element able to shift the initial market share. It would be thus possible to grasp the effect of the 'strokes of luck' or random occurrences that often launch the careers of artists, enabling

them to emerge from anonymity and achieve a substantial market share. Diffusion via internet, which is increasing unlikely to halt, would do the rest.

References

ALVISI M., ARGENTESI E., CARBONARA E. (2003), Piracy and production differentiation in the market for digital goods, *Rivista Italiana degli Economisti*, Il Mulino, vol. 2, (3 - August), 219-244.

BAE S.-H., CHOI J.P. (2003), A Model of Piracy, *Information Economics and Policy*, Elsevier, Volume 18 (3 - September), 303-320.

BALDUCCI F. (2008), *Music* or *Hi-Tech Lovers*? An Empirical Analysis of the Digital Music Market in Italy , Quaderni di ricerca del Dipartimento di Economia, Università Politecnica delle Marche, n.324, luglio.

BASS F. (1980) "The Relationship Between Diffusion Rates, Experience Curves, and Demand Elasticities for Consumer Durable Technological Innovations." *Journal of Business*, 53 July 1980, 551-67.

BAUMOL W.J., BOWEN, W.G. (1968), Performing arts: the economic dilemma, MIT Press.

BECCHETTI L., ELEUTERI S. (2007), Piracy repression and `Proustian effects' in popular music markets, *Quaderni CEIS* n.243.

BELLEFLAMME P. (2007), Piracy and competition, *Journal of Economics and Management Strategy*, Wiley-Blackwell, 16/2, 351-383.

BESEN S.M., KIRBY S.N. (1989), Private Copying, Appropriability, and Optimal Copying Royalties, *Journal of Law and Economics*, UChicago Press, vol. 32, 255-280.

BOLDRIN M., LEVINE D.K. (2002), The Case Against Intellectual Property., *American Economic Review*, May, 92 (2), pp.209-212.

CHEN Y., PNG. I.P.L. (1999), Software Piracy: an analysis of protection strategies, *Management Science*, 37, 125-139.

CONNER, K.R., RUMELT R.P. (1991), Software Piracy - An Analysis of Protection Strategies, *Management Science* 37, 125-139.

CURIEN N., MUREAU F. (2005), The Music Industry in the digital era: Towards new businnes frontiers, document de travail, CNAM, cnam-econometrie.com.

FAVARO D., FRATESCHI C. (2007), A discrete choice model of consumption of cultural goods: the case of music, *Journal of Cultural Economics*, Springer, vol. 31(3 - September), 205-234.

FISHER W. (2004), An Alternative Compensation System in Promises to Keep: Technology, law and the future of entertainment, Stanford University Press.

GAYER A., SHY O. (2006), Publishers, Artists, and Copyright Enforcement, *Information Economics & Policy*, Elsevier, 18(4), November, 374-384.

HALONEN A. M., REGNER T. (2004), Digital Technology and the Allocation of Ownership in the Music Industry in *Royal Economic Society Annual Conference*, n. 54.

HAN P., HOSANAGAR, K., TAN Y. (2008) Optimal Dynamic Referrals in Peer-to-Peer Media Distribution, *Marketing Science* (forthcoming).

HAN P., HOSANAGAR, K., TAN Y. (2008). "Di¤usion Models for P2P Content Distribution", Information Systems Research (forthcoming).

HARBAUGH R., KHEMKA R. (2001), Does copyright enforcement encourage piracy? , *Claremont colleges working paper* 2001-14.

HUI K.L., PNG I.P.L. (2003), Piracy and the demand for legitimate recorded music, *Contributions to Economic Policy and Analysis*, Volume 2(1), article 11.

IFPI (2008), Digital Music Report.

JAISINGH J. (2004), Piracy on file sharing networks: strategies for recording companies, *Journal of Organizational Computing and Electronic Commerce*, Lawrence Earlbaum, vol. 17(4), 329-348.

JOGIA H. (2005), Survival in the UK Market for Popular Music: An Alternative Test of Superstar Theory, *University of Warwick working paper*.

KRUEGER A.B. (2005), The Economics of Real Superstars:The Market for Rock Concerts in the Material World, *Journal of Labor Economics*, UChicagoPress, vol. 23(1), 1-30.

LERNER J., TIROLE J. (2005), The economics of technology Sharing: Open source and beyond, *Journal of Economics Perspectives*, vol. 19(2), Spring, 99-120.

LIEBOWITZ S.J. (2006), Creative destruction or just plain destruction?, *Journal of Law and Economics*, UChicago Press, 49(1-April), 1-28.

LITMAN J. (2004), Sharing and Stealing, *Hastings Communications and Entertainment Law Journal*, Vol. 27.

NETANEL N.W. (2002), Impose a Noncommercial Use Levy to Allow Free P2P File Swapping and ReMixing, University of Texas Law, *Public Law Research Paper*, n.14, Nov.15.

NOVOS I.E., WALDMAN M. (1984), The Effects of Increased Copyright Protection: An Analytic Approach, *Journal of Political Economy*, UChicago Press, vol. 92, 236-246.

OBERHOLZER F., STRUMPF K. (2007), The Effect of File sharing on Record Sales: an Empirical Analysis, *Journal of Political Economy*, vol. 115 (1-February), 1-42.

PEITZ M., WAELBROECK P. (2004), The Effect of Internet Piracy on CD Sales: Cross-Section Evidence, *CESifo working papers* series n° 1122.

PEITZ M., WAELBROECK P. (2006), Piracy of Digital Products: A Critical Review of the Theoretical Literature, *Information Economics and Policy*, Elsevier, vol. 18 (4-November), 449-476.

RICOLFI M. (2007), Individual and collective management of copyright in a digital environment, in Copyright Law. A handbook of contemporary research P. Torremans, Edward Elgar Publishing.

ROB R., WALDFOGEL J. (2006), Piracy on the high C's: Music downloading, sales displacement, and social welfare in a sample of college students, *Journal of Law and Economics*, UChicago Press, vol. 49(1-April), 29-62.

ROMER P. (2002), When Should We Use Intellectual Property Rights?, *American Economic Review*, May, 92 (2), pp.213-216.

ROSEN S. (1981), The economics of superstars, *American Economic Review*, vol. 71 n.5, 845-58.

SAMUELSON P. (2003), DRM, (And, Or, Vs.} the Law, *Communications of the AACM* 46, 41-45.

SHY O., THISSE J. (1999), A Strategic Approach to Software Protection, *Journal of Economics and Management Strategy*, vol. 8, 163-190.

STEVANS L., SESSIONS D. (2006), An Empirical Investigation into the Effect of Music Downloading on the Consumer Expenditure of Recorded Music: A Time Series Approach *Journal of consumer Policy*, 28, 311-24.

TAKEYAMA L.N. (1994), The Welfare Implications of Unauthorized Reproduction of Intellectual Property in the Presence of Network Externalities, *Journal of Industrial Economics*, vol. 42, 55-166.

TAKEYAMA L.N. (2002), Piracy, Asymmetric Information and Product Quality, in The Economics Of Copyright: Developments in Research and Analysis, Wendy J. Gordon and Richard Watt (Eds), Edward Elgar.

VARIAN H.R. (2005), Copying and Copyright, *Journal of Economics Perspectives*, vol. 19, n.2, Spring, 121-138.

ZENTNER A. (2006), Measuring the Effect of Music Downloads on Music Purchases, *Journal of Law and Economics*, UChicago Press, vol. 49(1-April), 63-90.