LOVE FOR VARIETY AND NON MARKET ALLOCATION MECHANISMS IN PUBLIC PROVISION OF GOODS

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In this note we modify a Dixit-Stiglitz’s classical framework to consider the love for variety argument in public provision of goods and services. Since in the supply of public goods and services the allocation cannot be driven by market, we analyse which is the optimal allocation mechanism.

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**Keywords**: Dixit-Stiglitz’s framework; non market allocation; public provision.
1 Introduction

The aim of this paper is to reconsider the classical Dixit - Stiglitz’s framework on love for variety (Dixit e Stiglitz, 1977) in a public economics context.

In public economics, the issue of differentiation is common and it is an important topic on fiscal federalism studies. Public goods and services, like private ones, can often be produced in many forms or variety; each form can be considered a mix or a bundle of different characteristics (Lancaster, 1991b). Citizens have different tastes on public goods and services, thus a differentiated supply can better fit different sets of preferences. Oates’s theorem (1972) states that local governments supplying goods different in quantity and in quality can lead to a welfare improvement with respect to a central government supply. In this case each citizen prefers a particular set of goods or a particular bundle of characteristics (Lancaster, 1991a). Hence, Oates’s idea of differentiation is not far from the idea used by models which deal with spatial product diversity (Hotelling, 1929): the diversity of localisation of the consumers symbolizes consumers’ different tastes. When each variety correspond to different consumers’ tastes, producing different varieties has no cost if there are not scale economy. If scale economy appears there is a trade-off between efficiency in production and diversity in tastes. This trade-off in Hotelling’s monopolistic competition model is the same as in Oates’s theorem.

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1Theory of differentiation or theory of product variety in economics began as an incidental byproduct of analysis primarily concerned with deviations from the competitive model in prices and numbers of firms (Hotelling, 1929; Chamberlin, 1933). Lancaster (1990) surveys the Economics of Product Variety in a monopolistic competition set up.

2Less resources can be used if we concentrate the production on few varieties, thus each consumer has to choose between a greater quantity of goods which does not fit perfectly his preferences and lower quantity of his preferred good.
The main difference consists in the allocation mechanism: in monopolistic competition the allocation mechanism which determines the number of varieties and the quantity of each variety is the market, in Oates’s theorem local governments allocate local public goods differentiating them because of quantity or because they are different bundles of characteristics (Lancaster, 1976).

Lancaster (1991a) mentions another reason of product diversity: the fact that every consumer may have a preference for diversity. Having a vast variety of products is a value in itself. *Love for variety* can be thought of as a positive consumption externality: the more goods are different, the more consumers feel satisfied.

These justifications describe not only the complexity of demand for market goods, but also the correspondent complexity of demand for goods supplied by government (both public and merit goods). It is not surprising that individuals, which referes at the same time to different systems of values, may also require public services very specific and differentiated. In effect they require very sophisticated sets of social, educational and health services, where individuals consider the possibility to receive different varieties of public services a value in itself. Using Lancaster’s terminology they prefer to have many small different bundles of characteristic (lots of varieties) instead of a larger quantity of the few bundles of characteristic (few varieties). In effect, in public utilities provision an ignorance argument must be considered: citizens do not express a quantifiable demand but some needs which have to be translated into demand and this translation is a difficult task not only for the government but also for the individuals which express the public needs. For this reason citizens, instead of a standardize public provision, prefer to have a individually differentiated set of public goods and services.

In spite of this, individual love for variety is not investigated in public economic theory. In the next section we review the classical Dixit - Stiglitz’s framework on *love for variety* (Dixit e Stiglitz, 1977) in order to adapt it to public goods

Moreover, we may assume that consumer’s tastes are randomly chosen at every period and they are not stable. In this case, the more diverse the set of products, the more consumer is likely to be fully satisfied. Suen (1991) assumes that the value of any good from the consumers’ point of view may be decomposed in a fixed (private) value proper to the customer and a variable (social) one that depends on the context of the use or the consumption of the good. For example, someone may prefer to use their car in order to go to work but when it rains, they would rather take the train. The consequence is that the larger the number of different products, and, most important, the larger the set of available products, the better for consumers. Product diversity may help not only in case of risk but, more generally, any time there is uncertainty, which are cases in which there are not even probabilities on the appraisal of events.

On the contrary, it is a common issue in lots of literature of industrial organization (Tirole, 1988), international trade (Krugman, 1979), economic geography (Krugman, 1991)

Such a framework, actually, is ambiguous on the diversity explanation, they state: “Product diversity can then be interpreted either as different consumers using different varieties or as
and services provision. Obviously monopolistic models based on Dixit-Stiglitz’s framework cannot mechanically applied when we deal about public provision of goods and services. If we want to use this framework to analyse the provision of local public goods, the issue of allocation mechanism naturally arises.

In the next section the effect of different non market allocation in Dixit-Stiglitz’s love for varieties framework are considered, in particular we analyze the result of a benevolent government allocation comparing with the ones determined by market allocation. The third section considers the problem of government enlargement comparing it with the traditional market enlargement argument. The last section concludes.

2 The Dixit-Stiglitz’s framework and non market allocation mechanisms

The well known Dixit-Stiglitz’s model (1977) describes a monopolistic competition framework in which q consumers choose among many goods (or varieties) which are not perfect substitutes, the utility function is

\[ u = \left( \sum_{i} x_i^\alpha \right)^{\frac{1}{\alpha}} \]

thus the substitution elasticity \( \left( \frac{1}{1-\alpha} \right) \) gives a monopolistic power\(^6\), decreasing with \( \alpha \), to the firms which produces each variety \( i \) facing a cost function which presents scale economy due fixed cost \( b_i \).

\[ c(qx_i)qx_i = a_iqx_i + b_i \]

For this reasons each firm sets its monopolistic price at a level equal to:

\[ p_i = \frac{a_i}{\alpha} \]

assuming symmetry in production function \( (a_i = a \) and \( b_i = b \), free entry condition permits to calculate the quantity of each good produced \( x \) and the number of goods \( K \):

\[ qx = \frac{b}{a} \frac{\alpha}{1-\alpha} \quad , \quad K = \frac{qM}{b}(1-\alpha) \]

\(^6\)It is also a love for variety index (Benassy, 1996).
where $M$ is the amount of resources that each consumer devotes to buy the variety $x_i$.

The allocation mechanism which Dixit and Stiglitz assumed is the market [MK]$^7$, but it is clear that other allocation mechanism can be considered. In particular we may investigate three alternative allocation mechanisms which a benevolent government can adopt$^8$:

LS A benevolent government which decides both quantity and number of goods setting prices to marginal cost $p_i = a_i$, and applying a lump sum on $M$ to pay any losses $\sum_i b_i$.

SB A benevolent government which decides both prices and number of goods, setting prices at second best point $p_i = c(qx_i)$.

BM A benevolent government which leaves firms to act as monopolist in setting prices, but receives the whole profit which it reinvests for producing new variety. Let us define it a benevolent monopoly.

**Proposition 2.1** All the benevolent government allocation (LS, SB, BM) are equivalent market allocation

This equivalence proposition states that in a Dixit-Stiglitz’s framework a benevolent government regulated allocation produces the same results as the market allocation does in classical model. **Proof.** See the appendices ■

### 3 Government enlargement

A very general result of Dixit - Stiglitz’s framework is that a market enlargement permits to exploit scale economy in a better way and thus it increases welfare. The underlying assumption on market allocation mechanism drives the results: the disposal of resources increases, each firm is able to produce more, but free entry leads each firm to produce the same quantity they produce before market enlargement, thus each consumer receives less of each variety, but the number of varieties increases since more firms enter the market, the assumption on love for variety grants that welfare increases$^9$.

$^7$The result holds if firms have no strategic interaction, or if $K_i = \frac{p_i x_i}{M}$, the units of variety $i$ which can be bought using all the income, are large enough. Otherwise, the exact monopolistic price is $p_i = \frac{K_i - a_i}{K_i - a_i} a_i$ (Yang e Heijdra, 1993).

$^8$Obviously a large number of non benevolent allocation mechanism can be assumed.

$^9$Recalling the monopolistic competition results, if market enlarges $n$ times, the overall resources is $nqM$, in symmetric equilibrium each firm produces $X_{MK} = \frac{b_i a_i}{n (1 - \alpha)}$ (the same quantity it
In a non market context, we can analyze an association of \( n \) governments (city councils) (Bartolini e Fiorillo, 2007). Let us assume that each government is equal and has the same number of citizens-consumers \( q \). All the non market allocation mechanism described in the previous section can be adopted by a benevolent government which is globally responsible towards \( nq \) citizens.

**Proposition 3.1** Market allocation and benevolent global government allocations are equivalent.

**Proof.** The proof follows the one proposed in the previous section. 

The benevolent monopoly solution may drive to a different allocation if governments are local and not global. In this case all the local governments remain responsible towards their own citizens, when the number of councils increases, local government may constitute a joint benevolent monopoly [JM] where each council can decide to produce with its own firms, in this case they receive the entire profit, or with a joint firm which produces for every council, in this case each council receives a profit share. The aggregate utility function for each council is:

\[
\max_{\{sc, x\}} U = q \left[ \sum_{i=1}^{s} x_i^{\alpha_i} + \sum_{s+1}^{K} \left( \frac{x_i}{nq} \right)^{\alpha_i} \right]^{\frac{1}{\alpha}} \\
\text{s.t. } q \left( \sum_{i=1}^{s} p_i x_i + \sum_{s+1}^{K} p_i y_k \right) = qM
\]

The total number of goods is \( K = s + sc \), where \( s \) is the number of variety directly produced by the council and \( sc \) is the number of goods produced jointly. When \( sc = 0 \) the local council opted for no cooperation, all goods are self-produced. When \( s = 0 \), all goods are produced jointly. For any variety \( s \), the quantity self-produced is \( x_i \), while for any good \( sc \), \( X_i \) is the aggregate demand for each jointly produced good and \( y_k = \frac{X_k}{nq} \) is the demand of each consumer. Each council’s objective is to maximize its utility under a budget constraint.

Let us assume that each local government starts from an autarky position and that it is in an equilibrium as the one described in the previous section, produces before market enlargement), so each consumer receives \( y_{MK} = \frac{X_{MK}}{nq} \), and the aggregate demand for each country is \( qy_{MK} = \frac{X_{MK}}{n} \) (only the \( n \) ratio of the previous quantity), but the number of varieties produced are \( K_{MK} = nq\left(\frac{MK}{1-\alpha}\right) \), \( n \) times the previous varieties. Assuming symmetries among producers and among consumers, the love for variety function assures that the aggregate utility \( (U(n)) \) of each country when the market is characterized by \( n > 1 \) countries is higher than the utility for an autarky country \( U(1) = qu \).

\[
U(1) = qK(1)^{\frac{1}{\alpha}} x < qU_{MK}(n) = n^{\frac{1-\alpha}{\alpha}} U(1)
\]
the benevolent joint monopoly allocation permits to a council to produce jointly some goods. Firms produce these services as monopolists and the profits are distributed among the councils. Since, in a symmetric equilibrium, monopolist price is constant for all the varieties $i$ ($p_i = \frac{a}{\alpha}$), the quantity demanded for any goods does not depend on whether it is self or jointly produced. Moreover the self produced goods has to be produced up to zero profit point, thus in one council the aggregate demand for each good $i$ is $q_{x_i} = q_{y_i} = \frac{b}{a} \frac{n}{1-\alpha}$.

For each service joint production is $X_{JM} = nqy = n \frac{b}{a} \frac{\alpha}{1-\alpha} > X_{MK}$. Thus the profit from a joint production is $\pi_i = (n-1)b$.

We assume that this surplus is equally distributed among councils. Therefore, the amount of resources available to each council for self-production, $R_s$, is obtained by subtracting to the resources $qM$ the amount transferred for joint production $(sc + qy)$ and adding the share of profit earned $(sc \frac{(n-1)b}{n})$. After some manipulation we get the following expression,

$$R_s = qM + sc \frac{n}{b}(\frac{n-2}{n} - \frac{\alpha}{1-\alpha})$$

The number of varieties self produced is obtained by dividing the resources spared by the cost of self-producing a variety, $c(qx_i)qx_i$, then the total number of variety produced is

$$K \equiv s + sc = \frac{qM}{b} (1-\alpha) + 2(1-\alpha) \frac{n-1}{n} sc$$

it increases with $sc$, where $\frac{qM}{b}(1-\alpha)$ is the autarky number of varieties.

The maximum number of goods is achieved by devoting all resources to the joint production:

$$K_{JM}(n) = \frac{n}{n(2\alpha-1) + 2(1-\alpha)} \frac{qM}{b}(1-\alpha)$$

The utility function is at a maximum when a council allocates all varieties to a joint production:

$$U_{JM}(n) = q[K_{JM}(n)]^{\frac{1}{2}}y_{JM}$$

$$= \left(\frac{n}{n(2\alpha-1) + 2(1-\alpha)}\right)^{\frac{1}{2}} * U(1)$$

**Proposition 3.2** Joint benevolent monopoly is not equivalent either to market allocation solution or to global benevolent government solution. Moreover, for $\alpha > \hat{\alpha}$ with $\hat{\alpha} \in [0, 0.5]$, it exists a value $\hat{n} \in [1, \infty]$, such that for $n \leq \hat{n}$, $U_{JM}(n) \geq U_{MK}(n)$, and vice versa. $\hat{n}$ is decreasing with $\alpha$. 

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Proof. See JM solution in the appendices. ■

Such a proposition states that if the enlargement is not large enough, global benevolent allocations (and also the market allocation) equilibrium leads to a suboptimal allocation while the allocation we called joint benevolent monopoly may display an higher welfare. In effect for a small enlargement, the increase in the number of varieties, which is higher in global allocations than in joint benevolent monopoly, does not compensate the reduction on the quantity of each variety produced in a global allocations$^{10}$.

Proposition 3.2 is also useful for analyse the effect of consumers coalitions in a Dixit - Stiglitz’s framework. Let us define $\overline{y_i}$ the aggregate demand of each variety in a market of $Q$ consumers. If these consumers form $N$ equal coalitions (thus in each coalition there are $\frac{Q}{N}$ consumers) and each group produces in autarky, the aggregate demand for each variety is $N\overline{y_i}$. The following proposition holds:

**Proposition 3.3** If coalitions can cooperate forming a benevolent joint monopoly, welfare increase is higher than the one consumers can reach without forming coalitions$^{11}$ until $1 < N < \hat{N}$.

Proof. The proof follows the one proposed for the previous proposition. ■

4 Conclusion

As we demonstrated, in Dixit-Stiglitz’s framework, a non market allocation may produce a better welfare result than a market-equivalent allocation. This result can be useful if we analyse government cooperation or government agglomeration or in general can be an helpful tool in fiscal federalism studies.

The relevance of this result depends on the institutional setting we analyze with this framework. Obviously, when we are analyzing a setting as the ones described in international trade theory (Krugman, 1979) this result does not apply. A joint benevolent monopolist cannot exist in an international context where many firms exist and compete and where free entry is possible. On the contrary this result can easily be applied in a public economics context, where utilities are produced in a regulated market because of market failure or because they are merit goods. In this context government states both prices (or quantities) and number of firms, moreover government can exploit monopolistic rent and can use it for its objectives. Thus is not difficult for a government to join other government in producing some services for scale economy exploitation, and actually local governments do it.

$^{10}$The results of propositions 3.2 and 3.3 depend on a violation of the shrinking core theorem assumptions both on cost function and on utility function shapes.

$^{11}$Note that if consumers do not form a coalition $N = 1$. 

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Furthermore, citizens present to local government a very complex and differentiated demand of services, this can justify the adoption of a love for variety function also in public economics theory.

For this reason in this institutional setting, in particular when public behavior is analyzed, Dixit-Stiglitz’s framework can be useful modified assuming joint monopoly allocation.

References


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Appendices

LS solution

In this case government maximizes the utility function, $qu$, under the balance constraint $(q \sum_{i=1}^{K} a_i x_i = qM - \sum_{i=1}^{K} b_i)$, the lagrangian of the problem is:

$$\Lambda_{LS}(x, K) = q \left( \sum_{i=1}^{K} (x_i)^{\alpha} \right)^{\frac{1}{\alpha}} - \lambda \left( q \sum_{i=1}^{K} a_i x_i + \sum_{i=1}^{K} b_i - qM \right)$$

It is easy to demonstrate that the aggregate demand of a variety $qx_i$ is:

$$qx_i = \left( qM - \sum_{i=1}^{K} b_i \right) \frac{a_i^{\frac{1}{\alpha-1}}}{\sum_{i=1}^{K} a_i^{\frac{1}{\alpha-1}}}$$

and assuming symmetry $qx = \frac{qM - Kb}{Ka}$.

Let me rewrite the maximum problem as:

$$\Lambda_{LS}(x, K) = q \left[ E(x^{\alpha}) \right]^{\frac{1}{\alpha}} - \lambda \left( qKE(ax) + KE(b) - qM \right)$$

thus deriving for $K$

$$\frac{1}{\alpha} K^{\frac{1-\alpha}{\alpha}} E(x^{\alpha})^{\frac{1}{\alpha}} = \lambda E(ax + b)$$

with $\lambda = \frac{\partial V}{\partial qM - KE(b)} = \left( \sum_{i=1}^{K} a_i^{\frac{1}{\alpha-1}} \right)^{\frac{1-\alpha}{\alpha}}$

assuming symmetry:

$$\lambda = \frac{K^{\frac{1-\alpha}{\alpha}}}{a} \frac{qM}{b(1-\alpha)}$$
SB solution

In this case government maximizes the aggregate inverse utility function of citizens, \( qV(p, M) \), choosing \( p_i \) and \( K \), under cost recovery constraint

\[
\sum_{i=1}^{K} qM \frac{p_i^{\alpha-1}}{\sum_{j=1}^{K} p_j^{\frac{\alpha}{\alpha-1}}} = \sum_{i=1}^{K} qMa_i \frac{p_i^{\frac{1}{\alpha-1}}}{\sum_{j=1}^{K} p_j^{\frac{\alpha}{\alpha-1}}} + \sum_{i=1}^{K} b_i
\]

in which the demand function is \( x_i(p, M) = M \frac{p_i^{\alpha-1}}{\sum_{j=1}^{K} p_j^{\frac{\alpha}{\alpha-1}}} \). The lagrangian is:

\[
\Lambda_{SB}(p, K) = q \frac{M}{\left(\sum_{i=1}^{K} p_i^{\frac{\alpha-1}{\alpha}}\right)^{\frac{\alpha-1}{\alpha}}} - \mu \left[ qM \sum_{i=1}^{K} \left( (p_i - a_i) p_i^{\frac{1}{\alpha-1}} \right) - \sum_{i=1}^{K} b_i \cdot \sum_{j=1}^{K} p_j^{\frac{\alpha}{\alpha-1}} \right]
\]

Deriving for \( p_i \)

\[
- \left( \sum_{i=1}^{K} p_i^{\frac{\alpha-1}{\alpha}} \right)^{\frac{1-\alpha}{\alpha}} \frac{\alpha-1}{\alpha} + \alpha \frac{K}{qM} = \frac{\alpha}{\alpha-1} p_i
\]

thus the mark-up is constant \( \left( \frac{a_i}{p_i} = \frac{a_i}{p_i} = h \right) \), under the constraint \( \sum_{i=1}^{K} b_i = q(1 - h) \sum_{i=1}^{K} p_i x_i = (1 - h) qM \), hence \( \mu = - \left( \sum_{i=1}^{K} p_i^{\frac{\alpha-1}{\alpha}} \right)^{\frac{1-\alpha}{\alpha}} \frac{1}{h} \).

Let us rewrite the problem as:

\[
\Lambda_{SB}(p, K) = q \frac{M}{E\left(p^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha}{\alpha}}} K^{\frac{1-\alpha}{\alpha}} - \mu \left[ qM K E\left( (p - a) p^{\frac{1}{\alpha-1}} \right) - K^2 E(b) \cdot E\left(p^{\frac{\alpha}{\alpha-1}}\right) \right]
\]

deriving for \( K \)

\[
\frac{1 - \alpha}{\alpha} q \frac{M}{E\left(p^{\frac{\alpha}{\alpha-1}}\right)^{\frac{\alpha}{\alpha}}} K^{\frac{1-\alpha}{\alpha}} = \mu \left[ qM K E\left( (p - a) p^{\frac{1}{\alpha-1}} \right) - 2 K E(b) \cdot E\left(p^{\frac{\alpha}{\alpha-1}}\right) \right]
\]

substituting \( h \), \( \mu \) and \( KE(b) = (1 - h) qM \), we obtain \( h = \alpha \) and \( K = \frac{qM}{h}(1 - \alpha) \).

BM solution

Government let firms set monopoly price \( p = \frac{a}{\alpha} \), profit is

\[
\pi(K) = \frac{1 - \alpha}{\alpha} qx(K) - b
\]
while the aggregate consumption of a variety when $K$ varieties are produced is:

$$qx(K) = \frac{qM}{K} \alpha$$

The profit is collect by government and reinvested, thus the resources for producing the $K$ variety are the overall resources - minus the costs to produces the $K$ varieties and the profit which government receives from them.

$$R(K) = qM - (aqx(K) + b)K + \pi(K)K = 2qM - 2Kb - 2\alpha qM$$

thus government goes on producing until all the resources are employed.

$$K = \frac{qM}{b}(1 - \alpha)$$

**JM solution**

Note that to have $K_{JM}(n) > \frac{qM}{b}(1 - \alpha) = K(1)$ it is sufficient\(^{12}\) to assume $\alpha \geq 0.5$, in this case the aggregate demand of each variety is the same of autarky case while the number of varieties increases.

Since $U_{MK}(n) = n^{\frac{1-\alpha}{\alpha}} U(n)$ and since $n = 1$ $U_{MK}(1) = U_{JM}(1) = U(1)$.

As $n \to \infty$ we have

$$\lim_{n \to \infty} U_{JM} = \left(\frac{1}{2\alpha - 1}\right)^{\frac{1}{2}} U(1)$$

and

$$\lim_{n \to \infty} U_n = +\infty$$

Since both $U_{JM}$ and $U_{MK}$ are concave and always increasing in $n$, the existence of $\hat{n} > 1$ depends on the value of the derivative $\frac{\partial U_{JM}(1)}{\partial n}$ and $\frac{\partial U_{MK}(1)}{\partial n}$. Given

$$\frac{\partial U_{JM}(1)}{\partial n} > \frac{\partial U_{MK}(1)}{\partial n}$$

we have $U_{JM}(1 + \gamma) > U_{MK}(1 + \gamma)$ with $\gamma > 0$ however small; this implies that $\hat{n} > 1$ always exists and that for $n < \hat{n}$, $U_{JM}(n) > U_{MK}(n)$, while for $n > \hat{n}$, $U_{JM}(n) < U_{MK}(n)$.

\(^{12}\)If $K > 0$ then it is easy to demonstrate that $K_{JM} > \frac{qM}{b}(1 - \alpha)$. The value of $\alpha$ which guarantees $K > 0$ depends on the number of members. As the number of members increases the value of $\alpha$ must increase as well, up to 0.5. The condition to be satisfied is $\left[\alpha > \frac{1}{2} \left(\frac{n-2}{n-1}\right)\right]$ which asymptotically converges to 0.5 as $n \to \infty$. This condition also implies $\frac{\partial B(\infty)}{\partial n} < 0$, hence the resources allocated to the joint production of each good are greater than the profits (resources) received.
Let us define the implicit function \( F(\hat{n}, \alpha) \equiv U_{JM}(\hat{n}, \alpha) - U_{MK}(\hat{n}, \alpha) = 0 \). If \( n < \hat{n} \), \( F(n, \alpha) > 0 \), while if \( n > \hat{n} \), \( F(\hat{n}, \alpha) < 0 \), so \( \frac{\partial F(\hat{n}, \alpha)}{\partial \hat{n}} < 0 \).

Since it is possible to show\(^{13}\) that the derivative \( \frac{\partial F(\hat{n}, \alpha)}{\partial \alpha} < 0 \), applying the implicit function theorem we get \( \frac{\partial \hat{n}}{\partial \alpha} < 0 \).

\(^{13}\)Demonstration is based on a sufficient condition \( \frac{\partial^2 F(n, \alpha)}{\partial n \partial \alpha} < 0 \) \( \forall n \geq 4 \) and \( \forall \alpha \in (0.5, 1) \).