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MASS TERTIARY EDUCATION, HIGHER EDUCATION STANDARD AND UNIVERSITY REFORM: A THEORETICAL ANALYSIS

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Abstract

After the «3+2» University reform in Italy there has been a fast increase in the number of students. A common wisdom is that this result was partly achieved by reducing the standard of Higher Education (HE). In this paper we first build a theoretical model in which individuals decide whether to enrol in HE along with their optimal course quality, and whether to drop-out. Then, we use the model to analyse the effect of a reduction in the standards of HE courses available in the educational system on overall enrollment and drop-out. We show that a reduction in HE standard helps achieving a mass tertiary education by increasing both the number of students and that of university graduates but it does not necessarily increase the overall efficiency of the HE system measured in terms of drop-out or graduation rates.

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Mass tertiary education, higher education standard and university reform: A theoretical analysis*

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1 Introduction

The Italian system of Higher Education (HE, hereafter) has been recently targeted by an extensive reform that, among other things, introduced in 2001 a unitary two-tier system replacing the old one-tier architecture where most degrees duration was four years. In the new system, often called «3+2», secondary school leavers can enrol in a First level degree, whose duration is three years,[1] and after completing it they can decide to enrol in a Second level degree (i.e. graduate studies), whose duration is two years.

The primary objectives of the reform were to increase the number of graduates, since the Italian HE system was characterised by very high drop-out rates, and to reduce the age at graduation, given the excessive actual duration of university studies well above the legal one (the phenomenon of the so called *fuori corso* students). Indeed, most Italian students used to graduate in their late twenties before the reform.

The «3+2» reform has been accompanied by a complete rethought of university curricula. The reduction of one year of length determined the need to reduce degree contents in First level degrees and to move some undergraduate courses (often the most complex) to Second level degrees. Moreover, in the last few years there has been a proliferation of new university degrees, which have been created by HE institutions in the attempt

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[1] Except for degrees in a small number of fields.

to attract new students.

A first effect of the reform has been an increase in the number of students. Bondonio (2006), for instance, uses data from the Italian Ministry of University and Research (MIUR) and finds a strong effect of the reform on the number of enrolled students, which increased in the range of 8.3-9.6 percent points in the first year and in the range of 12.2-14.7 percent points in the second year of application of the reform. Moreover, Bondonio also finds a statistically significant effect of the increase in the supply of university degrees on enrolment rates: for each new degree set up by an institution, the enrolment increased by between 2.6 and 2.8 percent points.

The increase in enrolment determined by the reform per se (net of the increase in the supply of degrees) can be interpreted in several ways. HE institutions may have increased their marketing efforts (e.g., orientation activities) after the reform and raised in this way the number of students. Another possibility is that the reform has increased the enrolment of those students who were credit constrained before the reform, i.e. those coming from low social classes, by shortening degree length and the direct and opportunity costs of studying. Last but not least, the reform might have lowered the academic requirements of students, so as now also less academically oriented students are likely to succeed in HE studies. The lower requirements can be the result of a reduction in the standard of HE, which in our theoretical analysis is a general feature of HE courses that raises the cost of education.

Some evidence consistent with the last explanation is provided in Bratti, Broccolini and Staffolani (2006), who focus on first year students in the Economic Faculty of Marche Polytechnic University, and show a huge reduction in course workloads required to pass exams and a sizeable increase in student performance after the «3+2» reform. These effects are very unlikely to be totally explained by gains in universities' efficiency after the reform, especially because the reform did not change the overall organization of the didactic activity apart from reducing course contents. Evidence that is consistent with a reduction in HE standards after the reform is also provided, in our opinion, by Di Pietro and Cutillo (2006). The authors make a decomposition of the impact of the reform on drop-out rates between changes of the characteristics of the student intake and change in what they call 'student behaviour'. They find that while the change in students' characteristics after the reform (mainly a reduction in academic ability) would have increased drop-out rates, the change in 'student behaviour' (i.e. in the coefficients estimated from the model) has more than compensated the former and determined a net decrease in drop-out rates. While the authors mantain an 'optimistic' interpretation for this result, such as an increase of the matching quality between HE courses and students (through pre-university guidance or the increase in course supply) or a relaxation of liquidity constraints, the same effects might also have been produced by a reduction in HE standard.[2]

Indeed, the wisdom that the reform was accompanied by a reduction in the standard of HE is common in the academic profession (see Ranieri, 2006).

The concern that a stronger competition among HE institutions may have produced incentives towards the reduction of HE standards, in particular grading standards, is also expressed in Bagüés, Sylos-Labini and Zinovyeva (2006) who use data on Italian graduates, although before the reform,[3] and find that grade standards decreased (i.e. grades rose) in those Departments were enrolment fell. This may be a perverse incentive created by the Italian HE funding system, where public funds are partly allocated to HE institutions on the basis of the number of students and universities were facing declining student numbers in the early 90s (see Perotti, 2002, and Bratti, Broccolini and Staffolani, 2006).

In the light of this empirical evidence, in the current paper we build a theoretical model in which forward-looking secondary school leavers choose whether to enrol in a HE course and the 'quality' of the enrolled course. Course quality raises both the costs and the returns to HE. The standards of university courses (or HE) is centrally set by the government and educational costs are increasing in standards. This model enables us to examine the effects in terms of enrollment, drop-out and graduation of a change in overall HE standard. Our analysis shows that a reduction in HE standard, although raises enrollment rates, may have perverse effects on drop-out rates, reducing rather than increasing universities' efficiency, defined as the fraction of students who complete HE courses.

Therefore, mass tertiary education, i.e. an increase in the number of graduates (and the fraction of the population with a university degree), can be achieved by a reduction in university standards. However, this reduction does not necessarily raise the efficiency of the HE system: drop-out rates might indeed increase. The predictions of the theoretical model are consistent with the empirical evidence observed in Italy after the reform, that is an increase in university enrolment that was not coupled with a substantial reduction in drop-out rates (see Di Pietro and Cutillo, 2006).

[2]Unlike Bondonio (2006), Di Pietro and Cutillo are not able to disentangle the separate effect of the increase in the supply of degrees from the other characteristics of the reform (i.e., a reduction in the length of studies).

[3]Data on graduates after the 2001 reform have not been released by the Italian National Statistical Institute (ISTAT) yet.

The structure of the paper is as follows. Section 2 describes the main features of a theoretical model of university enrolment and choice of course quality, which allows us to analyse the effects of an overall reduction in the standard of HE determined by the central government. Section 3 analyses an individual's enrollment and drop-out decisions while section 4 examines the changes in overall enrollment and drop-out determined by a change in HE standard that may be induced by a HE reform. Section 5 summarises the main findings.

2 The model

In this section we introduce a simple theoretical model to analyse an individual's choice (under uncertainty) of enrolling in HE, after completing secondary schooling.

The assumptions of the model are the following:

- in the HE system there is a continuum of university courses with different quality (α).[4] Therefore each course is uniquely identified by its level of quality;
- course quality (α) is rewarded in the labour market through wage premia;
- individuals are differentiated according to several characteristics (family background, type of secondary school, age, talent and so on), although we will consider only *ability* hereafter; the probability density function of ability (θ) in the population of secondary school graduates is $f(\theta)$ with support $[0, +\infty)$. θ is known to an individual and can be interpreted as an individual's assessment of her ability, which she can infer, for instance, from secondary school performance;
- ability does not affect wages, which are determined only by the educational level. Hereafter, graduates will be defined as "skilled" workers and w^U, w^D, $w^S(\alpha)$ will represent the wages of unskilled individuals, individuals who drops out from HE courses and graduates, respectively. In our model only the wage of graduates is an increasing function of α, that is the HE course quality;[5]

[4]This can also be interpreted either as the quality of different faculties or of different HE institutions.

[5] We are implicitly assuming that differences in earnings of unskilled, drop-out and graduate students can be explained both by the human capital theory (in that case, we

- the HE courses duration is two periods;
- if an individual decides not to enrol in HE and assuming that utility is temporally separable and linear in income (i.e. individuals are risk neutral), her utility is given by the discounted flow of unskilled wages (w^U) over the life cycle:

$$V^U = \frac{w^U}{r} \quad (1)$$

  where r is the discount rate;
- if an individual decides to enrol, the period cost of education, paid at the end of each of the two periods, is given by $c(\theta_i, \alpha, \gamma, x)$, where i stands for the individual and where:
  - γ is an idiosyncratic stochastic shock, with known density function $g(\gamma)$ and distribution function $G(\gamma)$, whose realization will be known to each individual at the end of the first period of enrolment. This shock affects the cost of education. Several interpretations are possible, γ may for instance represent the "toughness" of teachers or an imperfect self-assessment of one's own ability;
  - x is the standard required to all the educational institutions by the central goverment (i.e. the minimum number of exams to be passed, the minimum number of credits). We assume that x does not have a direct effect on wages (but only an indirect effect through the optimal choice of course quality α).

  We assume that $\frac{dc}{d\theta} < 0$, $\frac{dc}{d\alpha} > 0$, $\frac{dc}{d\gamma} > 0$ and $\frac{dc}{dx} > 0$. Thus, educational costs are decreasing in ability (e.g. abler individuals benefit from fee waivers), are increasing in both course quality (e.g. higher fees) and the standard required by the central government.[6] Costs are also increasing in the level of the shock γ.

should assume that $w^D > w^U$ because of the accumulation of human capital in the first period of studies) and by the signalling theory (in that case, we should assume that $w^D < w^U$ because of the bad signal arising from dropping out, which could be however excluded if employers do not observe drop-out and drop-out students can cheat on them). Furthermore, both theories explain that the highest wage is the one of graduates enrolled in the course with the highest quality.

[6]For instance, a higher number of exams may determine higher costs both in terms of books and costs borne to attend lectures for students.

Given the above assumptions, the utility of individual i at time $t = 0$ from enrolling in HE is given by:

$$U^E_{0,i}(\alpha) = \int_{-\infty}^{\bar{\gamma}(\theta_i,\alpha,x)} \Big[\frac{w^S(\alpha)}{r(1+r)^2} - \frac{c(\theta_i,\alpha,\gamma,x)}{(1+r)} - \frac{c(\theta_i,\alpha,\gamma,x)}{(1+r)^2}\Big] g(\gamma)d\gamma + \\ + \int_{\bar{\gamma}(\theta_i,\alpha,x)}^{\infty} \Big[\frac{w^D}{r(1+r)} - \frac{c(\theta_i,\alpha,\gamma,x)}{(1+r)}\Big] g(\gamma)d\gamma \qquad (2)$$

where, for the sake of simplicity, we have assumed that capital markets are perfect and individuals can borrow against their future incomes. In this case the only thing that matters to individuals is the discounted value of lifetime wealth. The first integral represents the expected utility of obtaining the HE degree and the second one the expected utility of dropping out. Both depend on $\bar{\gamma}(\theta_i, \alpha, x)$ that is the endogenous minimum level of the shock that pushes the individual i, who has chosen course quality α, to drop out (see section 3). Indeed, since the net benefit of enrolling in education is decreasing in γ whereas the benefit of not enrolling is independent of it (see equation 1), at time $t = 1$ for some realizations of the shock γ the individual will decide to drop out from HE. Utility depends on the HE course quality α, that is the choice variable for the individual.

Our model describes the demand side of HE while, as to the supply side, we just assume that universities will offer a continuum of courses with different qualities which meet the standard centrally set by the government and for which there is a positive demand. Also firms' behaviour is not modelled, we just assume that firms pay higher wages to graduates for whatever reason (human capital or signalling) and that wage premia are an increasing function of course quality. We do not model here possible dynamic effects such as those produced on wages by the evolution of the demand and supply of graduates in the labour market. Indeed, the main aim of the model is to analyse the short-term effects of a university reform that reduces the standard required to HE courses and, in such time span, we think that both the evolution of supply and that of the demand of graduates in the labour market should play only a minor role.[7]

In what follows, we assume that the cost of education $c(\theta_i, \alpha, \gamma, x)$ can be written as $c(\theta_i, \alpha, \gamma, x) = C(\theta_i, \alpha, x)G(\gamma)$ so that the shock enters multiplicatively in the cost function troughout its probability distribution function. Note that the above assumptions imply that the expected cost of

[7]Indeed, supply-side effects will be relevant only when more and more cohorts of new graduates enter the labour market, while demand-side effects are generally of a long-term nature (e.g., skill biased technological change).

education is bounded both upward and downward, and that, for $-\infty \leq \gamma \leq +\infty$, the cost is always between 0 and $C(\theta_i, \alpha, x)$.[8]

Equation 2, after some manipulations (see Appendix), can be written as:

$$\begin{aligned} U^E_{0,i}(\alpha) \quad &= \tfrac{1}{1+r}\left[\tfrac{w^D}{r} - \tfrac{C(\theta_i,\alpha,x)}{2} + \tfrac{W(\alpha)G(\bar{\gamma}(\theta_i,\alpha,x))}{1+r}\right] + \\ &\qquad - \tfrac{C(\theta_i,\alpha,x)}{(1+r)^2}\tfrac{[G(\bar{\gamma}(\theta_i,\alpha,x))]^2}{2} \end{aligned} \tag{3}$$

where:

$$W(\alpha) = \frac{w^S(\alpha) - w^D(1+r)}{r} \tag{4}$$

is the lifetime expected wage premium of continuing studies in course α after the first period.

Individuals enrol by comparing the utilities of equation 3 and equation 1, so that the condition to enrol in HE is:

$$V^E_{0,i}(\alpha^*_i) > V^U$$

where[9], as we will se later, α^*_i represents the optimal HE course quality for individual i.

3 An individual's enrollment decision

In this section we determine :

- the level of the shock $\bar{\gamma}(\theta_i, \alpha, x)$ that induces a student to drop-out;
- her optimal course quality (α^*_i);
- her indirect utility ($V^E_{0,i}$);
- her decision whether to enroll or not by comparing $V^E_{0,i}$ with V^U.

The first point can be addressed by considering that at time $t = 1$, individuals will continue in higher education if $U^E_{1,i} \geq V^U_1$, that is equivalent to saying that, once the realisation of the shock is known (γ_R), the individual

[8] This specification of the cost function, as we will see later, greatly simplifies the analytical solutions of the model.

[9] The letter V indicates the indirect utility derived from the maximisation of the utility function U.

will continue HE studies if the lifetime expected wage premium of continuing in course α after the first period (see equation 4) is higher than the cost of one further year of education:

$$W(\alpha) \geq C(\theta_i, \alpha, x)G(\gamma_R). \tag{5}$$

Solving equation 5 with the equal sign, we obtain the maximum level of the shock inducing the individual i to continue studies in the HE course α, that is $\bar{\gamma}(\theta_i, \alpha, x)$ in equations 2 and 3.

From equation 5, we can easily obtain:

$$G(\bar{\gamma}(\theta_i, \alpha, x)) = \frac{W(\alpha)}{C(\alpha, \theta, x)}. \tag{6}$$

Substituting $G(\bar{\gamma}(\theta_i, \alpha, x))$ in equation 3, we obtain the expected utility of enrolling:

$$U^E_{0,i} = \frac{w^D}{r(1+r)} + \frac{1}{2C(\alpha, \theta_i, x)}\left(\frac{W(\alpha)}{1+r}\right)^2 - \frac{C(\alpha, \theta_i, x)}{2(1+r)} \tag{7}$$

that, according to the specific functional forms of $W(\alpha)$ and $C(\alpha, \theta_i, x)$, can have a maximum in α, which is the optimal HE course quality for individual i.[10]

Thereafter, we use the following cost function:

$$C(\theta_i, \alpha, x) = \frac{\alpha + x}{\theta_i} \tag{8}$$

which respects the above assumptions, and we assume that the skilled wage is given by:

$$w^S(\alpha) = w^D(1+r) + \mu\alpha^{\frac{1}{2}}$$

with $\mu > 0$, that is graduates receive a wage premium, compared to individuals who drop out, which is increasing and concave in course quality α.[11] Therefore, given equation 4 we have:

$$W(\alpha) = \frac{\mu\alpha^{\frac{1}{2}}}{r}. \tag{9}$$

[10] Given the above utility function, a positive drop out rate exists only if the wage of individuals who dropped out is higher than the one of individuals that did not enroll multiplied by $(1+r)$ so that human capital theory must hold (see note 5). In fact, to have $U^E_{0,i} > \frac{w^u}{r}$, after having substituted $W(\alpha) = C(\theta_i, \alpha, x)G(\bar{\gamma}(\theta_i, \alpha, x))$ we obtain $[G(\bar{\gamma}(\theta_i, \alpha, x))]^2 > \left[1 - 2\frac{w^D - w^U(1+r)}{rC(\theta_i, \alpha)}\right](1+r)$. But since the completion probability must not be greater than one, i.e. $G(\bar{\gamma}(\theta_i, \alpha, x)) \leq 1$, a necessary condition is $w^D > w^U(1+r)$.

[11] This is an *ad hoc* specification which allows us to find analytical solutions.

These assumptions make the expected utility of equation 7 concave in α, so that we can calculate the optimal course quality for each individual substituting equation 8 and 9 into equation 7. The analytical form of the optimal HE course quality for individual i is the following:

$$\alpha_i^*(x) = \left(\frac{\mu}{r}\right)\left(\frac{x}{1+r}\right)^{\frac{1}{2}}\theta_i - x \tag{10}$$

so that more talented individuals (higher θ_i) will sort themselves into the courses with higher quality.

Remark 1. *If the "standard " required to all HE institutions grows ($dx > 0$), more talented individuals will choose higher course quality with respect to the past, less talented individuals will choose lower course quality.*

Proof. The first derivative of $\alpha_i^*(x)$ with respect to x is positive if $\theta_i > 2\frac{r}{\mu}[(1+r)x]^{\frac{1}{2}}$. Therefore, the optimal course quality increases with x for individuals endowed with high ability whereas it decreases for less talented individuals. □

Substituting α_i^* in equation 8 and the result of this substitution in equation 7, we obtain the maximum expected utility of enrolling for individual i:

$$V_{0_i}^E = \frac{1}{r(1+r)}\left[w^D + \frac{\mu^2\theta_i}{2r(1+r)} - \mu\left(\frac{x}{1+r}\right)^{\frac{1}{2}}\right] \tag{11}$$

which is increasing in θ_i and decreasing in x.

In order to decide whether to enrol or not, individuals will compare the indirect utility of enrolling in their optimal course quality (equation 11) with that of not enrolling (equation 1). Given that the former is increasing in θ_i whereas the latter is not dependent on it, there must exist some θ_i that separates the population of secondary school leavers among those who enroll and those who do not enroll. Solving with the equal sign equations 11 and 1 we obtain this threshold level for θ, which we will define θ_m:

$$\theta_m(x) = 2\frac{r(1+r)}{\mu}\left[\left(\frac{x}{1+r}\right)^{\frac{1}{2}} - Z\frac{r}{\mu}\right] \tag{12}$$

where $Z = \frac{w^D - w^U(1+r)}{r}$ is the expected premium of enrolling and dropping out and must be positive (see foonote 10). $\theta_m(x)$ identifies the ability of the least talented individual who decides to enroll; we define her as the "marginal" student.

Using this last result we can also define the minimum quality of the courses offered by academic institutions (and demanded by students). In fact, plugging $\theta_m(x)$ into equation 10, we obtain the optimal course quality for the marginal student which also represents the "worst" course available in the HE system:

$$\alpha_m^*(x) = x - 2Z\frac{r}{\mu}[x(1+r)]^{\frac{1}{2}}. \tag{13}$$

Remark 2. *An increase in the HE standard set by the central government, raising the ability required to the "marginal" student, also raises the "worst" course quality available in the HE system.*

Proof. Differentiating equation 13 with respect to x and rearranging terms we obtain:

$$\frac{d\alpha_m^*(x)}{dx} = \frac{1}{x}\left[x - Z\frac{r}{\mu}[x(1+r)]^{\frac{1}{2}}\right] = \frac{1}{2x}\left[x + \alpha_m^*(x)\right] > 0.$$

$\square$

Only students with ability higher than $\theta_m(x)$ will enroll in HE. Once each student has chosen her optimal HE course, the probability of completing studies can be easily calculated using equations 6, 8 and 10:

$$G(\bar{\gamma}(\theta_i, \alpha_i^*(x), x)) \equiv G(\theta_i, x) = \left(\frac{\mu}{r}\left(\frac{1+r}{x}\right)^{\frac{1}{2}}\theta_i - (1+r)\right)^{\frac{1}{2}}. \tag{14}$$

However, not all students will risk to drop out. In fact, individuals with $W(\alpha_i^*) > C(\theta_i, \alpha_i^*, x)$ will never drop out (see equation 5). Solving this inequality,[12] the maximum level of θ for which individuals are at risk of dropping out is:

$$\theta_M(x) = \frac{r}{\mu}(2+r)\left(\frac{x}{1+r}\right)^{\frac{1}{2}}. \tag{15}$$

Therefore, if $\theta_i > \theta_M(x) \Rightarrow G(\theta_i, x) = 1$.

We can restrict the analysis of enrollment to those students with $\theta_i > \theta_m(x)$ and restrict the analysis of drop-out to those students whose θ_i is such that $\theta_m(x) \leq \theta_i \leq \theta_M(x)$.[13] According to our results, secondary school leavers can therefore be divided in three groups: individuals who do not enrol, individuals who enroll and have a positive probability of dropping out and individuals who enroll and are sure of finishing studies.

[12] This is also equivalent to computing the value of θ_i for which $G(\theta_i, x) \leq 1$.

[13] An implicit parameter restriction in order to have students who drop out (i.e. $\theta_m < \theta_M$) is $Z > \frac{\mu}{2x(1+r)^{\frac{1}{2}}}$.

4 Overall enrollment and graduation

We are now able to evaluate the enrollment rate. Assuming that the ability θ is distributed according to the density function $f(\theta)$, the number of students enrolled in HE is:

$$E(x) = 1 - F(\theta_m(x)) = \int_{\theta_m(x)}^{\infty} f(\theta)d\theta. \tag{16}$$

Remark 3. *The number of enrolled individuals depends negatively on the HE standard required by the government to HE institutions (x).*

Proof. This result is derived immediately by considering that $\frac{d\theta_m(x)}{dx} > 0$. $\square$

If we assume a population of secondary school leavers of unitary mass the above equation also provides the overall university enrollment rate.

Let us now consider the number of graduates $\Gamma(x)$, i.e people enrolled in HE that complete their studies. It is given by the sum of the number of students that are sure to finish studies and students who risk to drop out weighted by the probability of finishing studies:

$$\Gamma(x) = \int_{\theta_M(x)}^{\infty} f(\theta)d\theta + \int_{\theta_m(x)}^{\theta_M(x)} G(\theta,x)f(\theta)d\theta \tag{17}$$

where $\theta_M(x)$ is defined in equation 15, $\theta_m(x)$ in equation 12 and $G(\theta,x)$ in equation 14.

Remark 4. *The number of graduates depends negatively on the minimum standard required by the government to HE institutions (x).*

Proof. Equation 17 can be differentiated using the Leibniz's rule for the second integral, obtaining:

$$\begin{aligned} \frac{d\Gamma(x)}{dx} &= -\frac{d\theta_M(x)}{dx} f(\theta_M(x)) + \\ + \int_{\theta_m(x)}^{\theta_M(x)} \frac{dG(\theta,x)}{dx} d\theta + \frac{d\theta_M(x)}{dx} & f(\theta_M(x))G(\theta_M(x),x) + \\ &-\frac{d\theta_m(x)}{dx} f(\theta_m(x))G(\theta_m(x),x) \end{aligned}$$

that, considering that $G(\theta_M(x),x) = 1$ by the definition of $\theta_M(x)$ (see equation 15), simplifies to:

$$\frac{d\Gamma(x)}{dx} = \int_{\theta_m(x)}^{\theta_M(x)} \frac{dG(\theta,x)}{dx} f(\theta)d\theta - \frac{d\theta_m(x)}{dx} f(\theta_m(x))G(\theta_m(x),x) \tag{18}$$

where the first term is negative (see equation 14) and the second is positive (see equation 12) but with the minus sign. Therefore, the derivative is negative. □

The number of individuals who drop out ($D(x)$) is given by the difference between the number of enrolled individuals and the number of graduates, so that:

$$D(x) = E(x) - \Gamma(x).$$

Unfortunately, it is not possible to obtain unambigous results for the relationship between the number of drop out and the HE standard. Both $E(x)$ and $\Gamma(x)$ are decreasing in x, and, following the same procedure as the previous proof, we obtain:

$$\frac{dD(x)}{dx} = -\frac{d\theta_m(x)}{dx} f(\theta_m(x)) - \frac{d\Gamma(x)}{dx} \tag{19}$$

and, substituting equation 18:

$$\frac{dD(x)}{dx} = -\int_{\theta_m(x)}^{\theta_M(x)} \frac{dG(\theta,x)}{dx} f(\theta) d\theta - \frac{d\theta_m(x)}{dx} f(\theta_m(x)[(1 - G(\theta_m(x),x)] \tag{20}$$

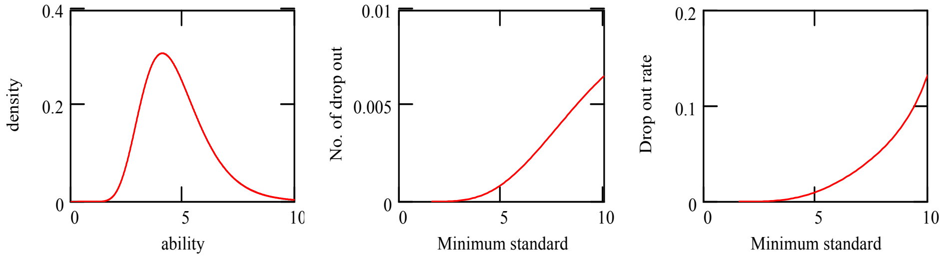
that is the sum of two terms, the first negative and the second positive. Therefore the sign is undefined.

In fact, the increase in the HE standard required by the government:

- decreases the number of enrolled people; by this channel, the number of drop-out should decrease;
- affects the number of students at risk of dropping out; indeed, a higher HE standard raises the ability required to the "marginal" student ($\theta_m(x)$) faster than the ability required for not being at risk of dropping out $\theta_M(x)$ (see equations 12 and 15); the effect on the number of drop out depends on the distribution $f(\theta)$;
- increases the risk of dropping out for each of the enrolled students with ability between $\theta_m(x)$ and $\theta_M(x)$ (in fact, $\frac{dG(\theta_i,x)}{dx} < 0$ as emerges from equation 14) and the number of drop out should increase.

The sign of the net effect cannot be determined in our theoretical model. Therefore, there is no way to define a general result for the relationship between the number of drop out and the HE standard. This is not a surprising result: a higher HE standard pushes a lower number of more motivated students to enroll but they will incur in a higher probability of dropping out because of the higher standard.

Figure 1: Lognormal distribution ($\sigma = 0.30$), Number of drop out, Drop-out rate



From the above equations, we can define the drop-out rate as $d(x) = \frac{D(x)}{E(x)}$ and the graduation rate, that is simply $1 - d(x)$. When we evaluate the derivative of the drop out rate (or the one of the graduation rate) with respect to x, for a general density $f(\theta)$ we are not able to define its sign, as in the case of the number of drop out.

$$\frac{dd(x)}{dx} = \left(\frac{dD(x)}{dx} - \frac{dE(x)}{dx} \frac{D(x)}{E(x)} \right) \frac{1}{E(x)} \tag{21}$$

and, given the indeterminacy of $\frac{dD(x)}{dx}$ we can state:

Remark 5. *If a higher HE standard raises the number of drop out students, the drop out rate must increase and the graduation rate must decrease. If a higher HE standard decreases the number of drop-out students, the drop-out rate and the graduation rate can both be increasing or decreasing with the HE standard.*

Proof. The results can be obtained from equation 21 once we consider that all the variables in level are positive and that $\frac{dE(x)}{dx}$ is negative. □

We can highlight the theoretical results by using numerical simulations.

Let us consider a standard lognormal distribution of the ability in the population, with σ being the standard deviation.

The value of σ defines different shapes for the number of drop-out and for the drop-out rate.[14]

For instance, with $\sigma = 0.30$ we obtain that both the number of drop-out students and the drop-out rate are always increasing in the HE standard x (see figure 1).

[14]Simulations are based on the following values for the parameters of the model: $w^D = 1$; $w^U = 0.8$; $r = 0.10$; $\mu = 0.20$; the average of the f distribution is 1.5.

Figure 2: Lognormal distribution ($\sigma = 0.60$), Number of drop out, Drop-out rate

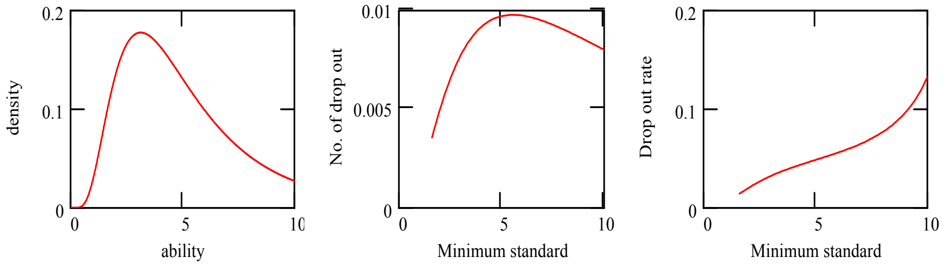
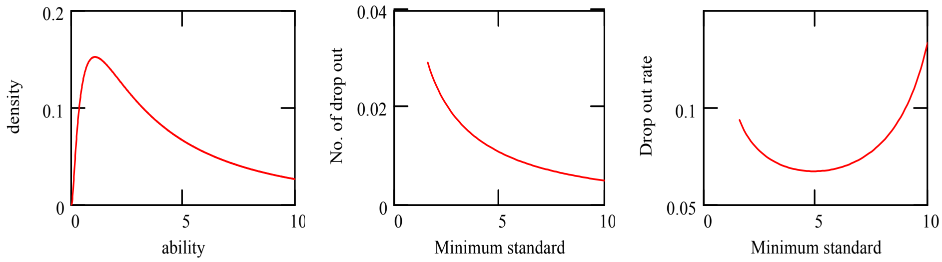


Figure 3: Lognormal distribution ($\sigma = 1.20$), Number of drop out, Drop-out rate



For higher values of the standard deviation of the lognormal distribution the shape of the above variables change. For $\sigma = 0.60$, the number of drop-out students shows a maximum in x whereas the drop-out rate is always increasing (see figure 2).

For $\sigma = 1.20$, the number of drop out is always decreasing with respect to the HE standard x whereas the drop out rate shows a minimum in x (see figure 3).

5 Concluding remarks

In this paper we present a theoretical model in which secondary school leavers, who are differentiated by their ability levels, have to decide whether to enroll in HE or not in an educational system in which two-period courses of different qualities are available. Both educational costs and graduate wages are increasing with course quality. Moreover educational costs are affected by the standard of HE set centrally by the government. Costs are also increasing in an idiosyncratic stochastic shock whose realisation is known only at the end of the first period. In the case the cost of education becomes too high individuals have an exit option: to drop out from the

HE course after the first period. Therefore, we define the optimal course quality for each individual, her decisions to enroll or not and, in the second period, the choice to drop out or complete the HE course. Then we investigate how a change in the HE standard affects overall enrollment and graduation. Our model shows that an increase in the HE standard:

- reduces the enrollment rate and the number of students who graduate;
- depending on the values of the parameters of the model and on the distribution of ability, may reduce or increase the number of students who drop out and the drop-out rate.

Therefore, according to our analysis a reduction in HE standard helps achieving a mass tertiary education by increasing both the number of students and that of university graduates but it does not necessarily increase the overall efficiency of the HE system measured in terms of drop-out or graduation rates.

Appendix

In order to simplify notation, we define $\bar{\gamma} \equiv \bar{\gamma}(\theta_i, \alpha, x)$ and $G(\bar{\gamma}) \equiv G(\bar{\gamma}(\theta_i, \alpha, x))$.

Considering that $c((\theta_i, \alpha, x, \gamma) = C(\theta_i, \alpha, x)G(\gamma)$, we can write equation 2 as follows:

$$
\begin{aligned}
U^E_{0,i}(\alpha) \quad = \quad & \int_{-\infty}^{\bar{\gamma}} \Big[\frac{w^S(\alpha)}{r(1+r)^2} - \frac{C(\theta_i, \alpha, x)G(\gamma)}{(1+r)} - \frac{C(\theta_i, \alpha, x)G(\gamma)}{(1+r)^2}\Big] g(\gamma) d\gamma + \\
& + \int_{\bar{\gamma}}^{\infty} \Big[\frac{w^D}{r(1+r)} - \frac{C(\theta_i, \alpha, x)G(\gamma)}{(1+r)}\Big] g(\gamma) d\gamma.
\end{aligned}
$$

Since $G(\bar{\gamma}) = \int_{-\infty}^{\bar{\gamma}} g(\gamma)d\gamma$, $\int_{\bar{\gamma}}^{\infty} g(\gamma)d\gamma = 1 - \int_{-\infty}^{\bar{\gamma}} g(\gamma)d\gamma$ and taking out of the integral all the terms not depending on γ, we obtain:

$$
\begin{aligned}
U^E_{0,i}(\alpha) \quad = \quad & \frac{G(\bar{\gamma})}{r(1+r)} \left(\frac{w^S(\alpha) - (1+r)w^D}{(1+r)} \right) + \frac{w^D}{r(1+r)} + \\
& - \frac{C(\theta_i, \alpha, x)}{1+r} \int_{-\infty}^{\infty} G(\gamma)g(\gamma)d\gamma - \frac{C(\theta_i, \alpha, x)}{(1+r)^2} \int_{-\infty}^{\bar{\gamma}} G(\gamma)g(\gamma)d\gamma
\end{aligned}
$$

Using the definition of $W(\alpha)$ given in equation 4, and considering that:

$$
\int G(\gamma)g(\gamma)d\gamma = \int G(\gamma)\frac{dG(\gamma)}{d\gamma}d\gamma = \frac{[G(\gamma)]^2}{2} + k
$$

where k is a constant, we obtain equation 3.

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