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Multivariate GARCH Models and Black-Litterman Approach for Tracking Error Constrained Portfolios: An Empirical Analysis

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Abstract

In a typical tactical asset allocation setup managers generally make their choices with the aim of beating a benchmark portfolio. In this context the pure Markowitz strategy does not take two aspects into account: asset returns often show changes in volatility and managers' decisions depend on private information.

This paper provides an empirical model for large scale tactical asset allocation with multivariate GARCH estimates, given a tracking error constraint. Moreover, the Black and Litterman approach makes it possible to tactically manage the selected portfolio by combining information taken from the time varying volatility model with some personal "view" about asset returns.

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Multivariate GARCH Models and Black-Litterman Approach for Tracking Error Constrained Portfolios: An Empirical Analysis*

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1 Introduction

Nowadays, the task of beating a benchmark portfolio in terms of return given a superior limit on tracking error is the key objective the crucial point if the manager wants to increase the value of her investment. Tactical asset allocation (hereafter TAA) strategies are based on an approach according to which the manager is induced to maximise her active return, also known as "alpha", taking its volatility under control. This intuition moves from the traditional optimisation proposed by Markowitz (1959) and shifts the problem from global mean-variance trade-off to the space spanned by active risk and active return.

The performance of tactically managed portfolios is obviously strictly related to the one of a prespecified benchmark: the fundamental assumption is that the optimal portfolio is composed of three separate components, being minumum variance, strategic and tactical portfolios[1]. Given that the strategic mix, or benchmark, is the sum of the first two components, the tactical component derives from the manager's perception about expected returns that can be different from equilibrium. This leads her to maximise the expected utility by selecting a portfolio that is a function of her degree of relative risk tolerance, the covariance matrix and the deviations of expected returns from their equilibrium.

The aim of this work is to show how it is possible to make a portfolio optimisation, in presence of a large number of assets, by combining two different types of information: the first is given by the estimation of a time varying volatility, and the second is private information that derives from the manager's bets about the evolution in time of asset excess returns.

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[1] A tactical approach to asset allocation allows for opportunistic moves between various asset classes in an attempt to provide additional return by taking advantage of changing market conditions. For a detailed review about the TAA see for example Lee (2000).

From an analytic point of view Roll (1992) has shown that the active portfolio is not a global mean-variance efficient one, because it has a sistematically higher element of risk compared to the benchmark, from which it is also independent; this leads one to consider an additional constraint on his correlation with the benchmark termed "tactical beta". This fact is also supported by the empirical work of Jorion (2002). Subsequently, Jorion (2003) tries to solve the problem by inserting a constraint on the total portfolio risk and Corvalán (2005) summarizes some literature contributions by suggesting a model in which the TAA portfolio is the sum of an alpha portfolio, selected to have excess return on the benchmark, and a beta portfolio to hedge total risk.

The reminder of the paper is organized as follows: section 2 is a short summary on the mean-variance efficient frontier and the restricted frontier achieved by imposing a tracking error constraint, while section 3 reviews the main aspects of the Black and Litterman approach. The empirical model is the subject of section 4: in sections 4.2 and 4.3 the attention is focused on the estimation and forecasting of the expected returns vector and the covariance matrix, while section 4.4 is dedicated to optimal portfolio allocation. Section 4.5 consists of an application of the Black and Litterman model. Finally, section 5 concludes and provides some suggestions for further research.

2 Portfolio frontiers

This section reviews some useful results about the portfolio frontiers in the space spanned by the absolute expected return and its variance. Given the n-dimensional vector of asset excess returns R and the covariance matrix Ω, the manager's portfolio P is the portfolio selected. Its expected excess return and its variance are given by the scalars R_P and σ_P^2, while the vector ω contains all the portfolio weights. The symbols R_B, σ_B^2 and ω_B refers to the benchmark portfolio B.

In sections 2.1 and 2.2, the absence of a riskfree asset is assumed to preserve the traditional hypherbolic form of the efficient frontier. Another important hypothesis is that regarding the possibility of net-short sales; this assumption guarantees a closed-form solution for the manager's optimisation problem, even if it allows for negative portfolio weights[2].

2.1 Mean-variance efficient frontier

The mean-variance efficient frontier in the total return and absolute risk space is derived from the traditional Markowitz (1959) framework. When

[2] In a recent paper, for example, Jagannathan and Ma (2003) argue that imposing non-negativity constraints to weights surprisingly improves the efficiency of optimal portfolios constructed using sample moments.

there is no riskfree asset, for each value of expected portfolio return R_P, its equation solves the problem

$$\begin{cases} \text{Min} & \sigma_P^2 = \frac{1}{2}\omega'\Omega\omega \\ \text{s.t.} & R_P = \omega' R \\ & \iota'\omega = 1, \end{cases} \tag{1}$$

where ι is a $n \times 1$ vector of ones. The efficient frontier equation is given by

$$\sigma_P^2 = -\frac{1}{d}[aR_P^2 - 2bR_P + c], \tag{2}$$

with $a = \iota'\Omega^{-1}\iota$, $b = \iota'\Omega^{-1}R$, $c = R'\Omega^{-1}R$ and $d = b^2 - ac$. In the space spanned by the portfolio mean and variance it represents a parabola, thus it is an hyperbola in the (σ_P, R_P) space.

2.2 Constant TE frontier

If managers want to impose a fixed tracking error constraint to portfolio optimisation in the mean-variance space, the problem of asset allocation implies the decomposition of the vector ω in the sum of the strategic mix portfolio (q) and the tactical portfolio (x). Following Jorion (2003), the optimisation problem is

$$\begin{cases} \text{Max} & x'R \\ \text{s.t.} & x'\iota = 0 \\ & x'\Omega x = TE \\ & \sigma_P^2 = (q+x)'\Omega(q+x), \end{cases} \tag{3}$$

where the constraints respectively set the sum of tactical portfolio weights to zero, impose the fixed value TE to the tracking error and finally force the portfolio variance to a given value of σ_P^2.

The solution of the model (3) is given by those portfolios which satisfy the equation[3]:

$$k\sigma^2 + 4\Delta_2 R_T^2 - 4\Delta_1 \sigma R_T - 4TE(k\Delta_2 - \Delta_1^2) = 0. \tag{4}$$

The variable R_T of equation (4) is defined as the tactical portfolio return $R_T = R_P - R_B$ and σ is given by the difference $\sigma = \sigma_P^2 - \sigma_B^2 - TE$. The parameters

$$\Delta_1 = R_B - \frac{b}{a} = R_B - R_C$$

and

$$\Delta_2 = \sigma_B^2 - \frac{1}{a} = \sigma_B^2 - \sigma_C^2,$$

[3] See Jorion (2003) for a detailed analytical derivation of the tracking error constrained efficent frontier.

are respectively the differences between the benchmark and the minimum variance (portfolio C) regarding the means and the variances, where $k = -d/a$ and a, b, c, d are those defined in section 2.1.

The parameter Δ_2 is always positive by definition, with the only exception provided by the case $B \equiv C$, while $\Delta_1 \geq 0$ should be true for portfolios located under the efficent frontier in the (σ_P, R_P) space.

The quadratic equation (4) thus shows the relationship between the expected return and the variance for a fixed value for TE, and it represents an ellipse when the condition

$$4(k\Delta_2 - \Delta_1^2) > 0$$

is satisfied[4]. This ellipse gets somewhat distorted in the absolute expected return-risk space as Figure 1 shows.

Moreover, Jorion (2003) provides the following properties and theorems about the elliptical frontier in (σ_P^2, R_P) space:

1. R_B is the vertical center and $\sigma_B^2 + TE$ is horizontal center of the ellipse. If the value of TE is augmented, the center moves to higher risk regions;

2. maximum and minimun expected excess returns are given by
$$R_P = R_B \pm \sqrt{k \cdot TE}; \tag{5}$$

3. maximum and minimum risk are given by
$$\sigma_P^2 = \sigma_B^2 + TE \pm 2\sqrt{TE(\sigma_B^2 - \sigma_C^2)}; \tag{6}$$

4. the ellipse and the efficient frontier may intersect. The necessary condition to have curves that pass through at least one common point in the (σ_P^2, R_P) space is
$$(R_P - R_B)^2 = k \cdot TE - k \cdot \Delta_2 + \Delta_1^2. \tag{7}$$
Setting $\Psi = k \cdot TE - k \cdot \Delta_2 + \Delta_1^2$, there are three possibilities: first, if $\Psi < 0$, curves do not have common points because equation (7) has no solutions. Second, when $\Psi = 0$, the first contact occurs if
$$TE = \Delta_2 - \Delta_1^2/k. \tag{8}$$
From equation (7) it is evident that this is true for $R_P = R_B$. Third, when $\Psi > 0$, there are always two contact points given by
$$R_P = R_B \pm \Psi^{1/2}.$$

[4]The proof of this result derives from the property of equations of the type $Ax^2 + By^2 + Cxy + D = 0$; if the term $AB - (1/4)C^2$ is strictly greater than zero, such relationship represents an ellipse.

As TE increases Ψ is augmented and R_P moves along the hyperbola, hence all the constrained tracking error portfolios lie inside the area between the efficient frontier and the right arc formed by the two intersections;

5. the minimum possible risk of the constant tracking error frontier is σ_C^2, and is achievable only for $TE = \Delta_2$;

6. when $TE = 4(\Delta_2 - \Delta_1^2/k)$ the ellipse passes through B;

7. when $TE = 4\Delta_2$ the risk of the benchmark portfolio is the minimum level risk achieved by the ellipse.

3 The Black and Litterman model

The Black and Litterman (1991, 1992) approach (hereafter BL) was introduced to make portfolio optimisation more useful in practical investment situations. As shown in Michaud (1989), the mean-variance model often leads to irrelevant portfolios because errors are optimised[5], and it can suffer from instability due to the fact that small changes in inputs dramatically change portfolio weights. Black and Litterman (1991, 1992) also try to solve the problem of negative portfolio weights, especially in situations in which managers can not take short positions.

The BL model is a way to incorporate investor's views into the asset allocation process; it uses a Bayesian method to combine the investor's views about expected asset returns with the prior information given by the vector containing the implied equilibrium returns[6]; the posterior information is provided by a distribution whose mean is the mixed estimate of expected returns, and whose variance is a function of the covariance matrix of implied returns and of a diagonal matrix in which the confidence in the manager's views are set.

The starting point of the model are the equilibrium returns defined as

[5] The critique in Michaud (1989) is based on the use of sample means in place of expected returns which contribute to the generation and the maximisation of errors. Following the Markowitz (1959) approach managers tend to overweight assets with high expected returns and negative correlation, without taking into account the uncertainity associated with the estimated inputs.

[6] The implied equilibrium returns vector is the neutral starting point of the model. See, for example, Idzorek (2002) or He and Litterman (1999) for details. In Bevan and Winkelmann (1998) this vector is given by the benchmark.

the market-clearing returns[7], while expected returns follow the equation

$$\mu \sim N(\bar{R}, \gamma\Omega). \tag{9}$$

The scalar γ is the weight-on-views parameter used to make covariances proportional to the matrix Ω. The investor's views about the market are expressed according to the equation

$$P\mu = V + \eta, \tag{10}$$

with $\eta \sim N(0, S)$. The $(k \times n)$ matrix P contains the weights of the assets of the investor's views, the column vector V represents the estimated expected returns in each view, and k is the number of views. The subjective probability excess returns vector is provided by the Theil (1971) estimator

$$\hat{R}_{BL} = [(\gamma\Omega)^{-1} + P'S^{-1}P]^{-1}[(\gamma\Omega)^{-1}\bar{R} + P'S^{-1}V], \tag{11}$$

where S is a diagonal covariance matrix about the uncertainity of the views which are assumed to be mutually independent.

The aim of the BL model is to insert uncertain personal views into the equilibrium returns to modify portfolio weights in the direction of the manager's hypothesised scenarios.

Nowadays this approach has been revised by taking two drawbacks into account: one empirical and the other conceptual. The first problem is due to the joint normality assumption of the prior information and the investor's views, and this is in contrast with the empirical regularities about asset returns[8]. In a recent work of Fabozzi, Giacometti, Bertocchi and Rachev (2005), the standard hypothesis of Gaussian distribution of asset returns is relaxed in favor of heavy-tailed distributions such as α-stable and t-Student: they find that information depends on how the different distributions impact the optimal portfolio. This is true for marginal distributions of expected returns. As it will be shown in section 4.2, this paper deals with distributions conditional on the information set $\mathcal{F}_{t-1}$, so the normality assumptions can be mantained.

The second problem depends on the Bayesian nature of the model, according to which the manager's views invest the market parameters instead of the market realisations: Meucci (2005) solves this problem by using a copula and opinion-pooling methodology to determine the posterior market distribution. Moreover, he claims that his extension to BL model can be applied to any market distribution and non-normal views.

[7]In this paper the vector $\bar{R}$ will be estimated via the model of section 4.2. In the original contribution of Black and Litterman (1992), it is obtained by solving the unconstrained maximization problem in which the investor utility function is quadratic with constant risk aversion and normally distribuited returns. See also He and Litterman (1999) for details.

[8]Especially for high frequency data, excess returns are very often characterized by leptokurtosis, skewness or other properties that could make the normality assumption too restrictive.

4 The model

4.1 The data

The dataset is given by series included in the composition of the DJ Euro Stoxx 50 index at the beginning of 2006; asset returns time series are calculated as 100 times the log difference transformation[9]. Table 6 in Appendix A contains the complete list of the variables used in the model, with the related specifications about the country and the sector to which they belong, and their weight in the DJ Euro Stoxx 50 itself. Considering this index as the benchmark portfolio, the total number of variables is 50. Each time series has 870 daily observations, taken from a sample which goes from the 3^{rd} March 2003 through the 30^{th} June 2006. The last week, corresponding to 5 observations, is kept out of sample for forecasting.

4.2 The time-varying volatility model

It is well known that daily asset returns volatility often shows some empirical regularities[10], thus a time-varying volatility model should be chosen to estimate and forecast returns and covariances to insert in the TAA process[11]. The use of this approach yields two benefits: first, modelling heteroskedasticity explicitly leads to increased efficiency in the estimation of the parameters of the conditional mean; moreover, forecasting the covariance matrix itself for different time horizons would be useful for TAA, especially because the forecast of the conditional covariances is likely to be the main object of interest.

For this reason several choices are available from the wide literature about multivariate GARCH models: the first attempt to model multivariate conditional covariances is the Vech Model introduced by Bollerslev, Engle and Wooldridge (1988) together with its restricted formulation known as Diagonal GARCH. The BEKK model proposed by Engle and Kroner (1995), is a good choice to achieve a reasonable level of generality, but its counterpart is represented by the total amount of parameters which becomes very large for high dimensions of the number of time series n; in practice, from the computational point of view the model estimation is rather prohibitive for $n \geq 7$, therefore this choice would be inappropriate for the present work. Other relevant contributions are Factor GARCH by Engle and Ng (1993)

[9]Source for data: DATASTREAM. Series of Munch.Ruck (XET) is not available.

[10]A lot of stylised facts emerged from the empirical research in asset returns: the most important are thick tailed distributions, volatility clustering, common movements and persistence in volatilities. See Bollerslev, Engle and Nelson (1994) or Palm (1996) for details.

[11]Litterman and Winkelmann (1998) provide a detailed survey about the covariance matrices estimation, especially for situations such as asset allocation or risk hedging. Voev (2004) instead compares the forecasting performances of different suitable models for estimating large dimensional covariance matrices.

and Constant Conditional Correlations (CCC model) by Bollerslev (1990). Most recently models like O-GARCH (Alexander and Chibumba, 1996) or GO-GARCH (Van der Weide, 2002) based on principal components have been suggested to solve the problem of estimation in presence of a great number of time series and to achieve computational feasibility.

In this paper the forecast model used is the Flexible Dynamic Conditional Correlations (FDCC) by Billio, Caporin and Gobbo (2006), a useful generalisation of Engle's (2002) DCC model.

Given the n-dimensional vector y_t, the standard FDCC model has the following representation

$$\begin{cases} y_t = \mu + \Pi y_{t-1} + \varepsilon_t \\ E(\varepsilon_t|\mathcal{F}_{t-1}) = 0 \\ E(\varepsilon_t \varepsilon_t'|\mathcal{F}_{t-1}) = \Omega_t \\ \Omega_t = D_t^{-1/2} R_t D_t^{-1/2}, \end{cases} \tag{12}$$

where $\mathcal{F}_{t-1}$ is the information set available at time $t-1$, Ω_t is the conditional covariance matrix, and μ is a $(n \times 1)$ vector of constants. In the present work the matrix Π has the form

$$\Pi = \begin{bmatrix} 0 & 0 & \dots & \theta \end{bmatrix}, \tag{13}$$

where each element is a $(n \times 1)$ column of zeros, and θ contains all the coefficients of the equations

$$y_{i,t} = \mu_i + \theta_i \mathrm{BMK}_{t-1} + \varepsilon_t, \tag{14}$$

where $i = 1, 2, \dots, n$ and BMK_{t-1} is the lagged value of the DJ Euro Stoxx 50 variable[12].

The matrix D_t, estimated during the first step estimation of the model, is diagonal and each element is given by the conditional variances h_{it}, evaluated via the standard GARCH(1,1) model,

$$h_{i,t} = \omega + \alpha \varepsilon_{i,t-1}^2 + \beta h_{i,t-1}. \tag{15}$$

Several choices of univariate GARCH models are availables from the literature for the conditional variances estimation[13]. In this paper Bollerslev's

[12] Even if in theory asset returns should be unpredictable using past information, the use of the lagged value of the benchmark return as regressor in (14) is justified by results in Table 8 in which BMK_{t-1} surprisingly captures the dynamics of several expected returns. Moreover this allows to use the VAR(1) formulation in equation (12). Other values for lags are been tried, but coefficients related to explanatory variables are not statistically significant.

[13] In a previous version of their work, Billio, Caporin and Gobbo (2003) use the Glosten, Jagannathan and Runkle (1993) or GJR specification in the first step estimation to take into account for asymmetries.

(1986) GARCH(1,1) has been selected[14].

The block parameters structure of the FDCC is the main innovation: it allows a more general model than the standard DCC, where the correlation dynamics are simply given by[15]

$$Q_t = (1 - a - b)\bar{Q} + a u_{t-1} u'_{t-1} + b Q_{t-1}, \tag{16}$$

where the parameters a and b are scalars. This formulation is a very restricted version of Engle (2002)

$$Q_t = (\iota\iota' - A - B) \odot \bar{Q} + A \odot u_{t-1} u'_{t-1} + B \odot Q_{t-1}, \tag{17}$$

in which $\bar{Q}$ is the historical correlation matrix of the standardised innovations u_t, A and B are square $n \times n$ matrices, ι is a vector of ones and the symbol $\odot$ represents the Hadamard product; the imposed scalar restrictions solve the identification problem due to the fact that in equation (17) the number of parameters becomes very large when the dimension is augmented.

As in the case of the standard DCC, the estimation of the FDCC model proceeds in two stages: in the first step, parameters of the first equation in (12) and those of (15) are estimated, while in the second step the subject of inference is the dynamic correlations matrix.

Hence, the second stage equations are

$$R_t = \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1} \tag{18}$$

and

$$Q_t = cc' + aa' \odot u_{t-1} u'_{t-1} + bb' \odot Q_{t-1} \tag{19}$$

The matrix $\tilde{Q} = diag(\sqrt{q_{11,t}}, \sqrt{q_{22,t}}, \ldots, \sqrt{q_{nn,t}})$ guarantees that R_t satisfies the property of a correlation matrix, while u_t contains all the standardised innovation estimated by the equation (15). Note that also the law of motion of Q_t follow a GARCH(1,1) as in the first step estimation.

The FDCC model instead generalises the model introduced by Franses and Hafner (2003) and it can be easily estimated using the same two stages approach of the standard DCC.

Focussing on the parameter structure in equations (19) and (17), it is evident that $cc' = (\iota\iota' - aa' - bb') \odot \bar{Q}$, while $A = aa'$ and $B = bb'$: the peculiarity of the FDCC is the way by which the n-dimensional column vectors a and b are partitioned.

Assuming that n assets can be grouped upon their belonging to k different sectors with $k < n$, the dynamics of correlations are imposed to be

[14] Different alternative univariate GARCH models were used in first step estimation: EGARCH (Nelson, 1991), APARCH (Ding, Engle and Granger, 1993) and IGARCH (Engle and Bollerslev, 1986) sometimes fail to achieve convergence, while GJR model shows substantial asymmetries in a few cases.

[15] See for example Engle and Sheppard (2001).

the same among variables of the same sector, while this in not true for the whole correlation matrix. As a consequence, the vector a is partitioned as follows[16]

$$a = [\ a_1\iota_1' \quad a_2\iota_2' \quad \ldots \quad a_k\iota_k'\ ]' \tag{20}$$

where the vectors ι_j, with $j = 1, 2, \ldots, k$, are vectors of ones with the number of rows equal to the number of assets belonging to sector j. In the present work the $n = 50$ variables are divided into $k = 12$ macro-sectors listed in Table 7 of Appendix A. Given the equations (12), (15) and (19), $5n$ and $2k$ parameters have to be estimated in the two step estimation; the total number of parameters in the whole model is therefore $5n + 2k = 274$.

The use of the FDCC follows from the need to take into account different important purposes: first, this is a parsimonious model because it allows to use a large number of series without implying that the number of parameters becomes explosive. Second, it is an efficient model for estimating and forecasting time varying covariance matrices which are the fundamental input required during the TAA process. Third, the FDCC specification makes it possible to mantain the same GARCH dynamics of the DCC correlation structure, but it relaxes the contraint of equation (16) for which all the correlations have to follow the same pattern; from this point of view this model represents a good generalisation of standard DCC. Finally, Q_t is positive definite by construction if the contraint

$$a_i a_j + b_i b_j < 1 \qquad\qquad \text{for } i, j = 1, 2, \ldots, k \tag{21}$$

holds: according to Billio, Caporin and Gobbo (2006) this is a sufficient condition to avoid explosive patterns.

Estimation results for the estimated FDCC model are reported in Appendix B: Table 8, shows that GARCH effects occur, because parameters of the variance equation are statistically significant for each time series.

4.3 Forecasts

The model also provides the daily forecasts for the expected returns and for all the unique elements of the Ω_t. Once the manager has selected the number τ of periods in which she would tactically manage her portfolio, these results may be used to evaluate forecasts about total returns and conditional covariances matrix at time τ.

The forecast return of asset i at time $T + \tau$ is given by the equation:

$$\hat{y}_{i,T+\tau} = E\left(y_{T+\tau} | \mathcal{F}_T\right) = \hat{p}_{i,T+\tau} - p_{i,T}, \tag{22}$$

where $\hat{p}_{i,T+\tau}$ is the τ step-ahead forecast of the logarithm of the i-th asset price.

[16]The partitions of vector b are exactly the same, therefore the related equation is omitted for brevity.

The forecasts of the covariance matrix of (22) for the periods, which goes from time $T+1$ to time $T+\tau$, is provided by the following equation:

$$\hat{\Omega}_{T+\tau} = (I-\hat{\Pi})^{-1}\left[\sum_{j=1}^{\tau}(I-\hat{\Pi}^{\tau-j+1})\hat{\Omega}_{T+j}(I-\hat{\Pi}^{\tau-j+1})'\right](I-\hat{\Pi}')^{-1}, \quad (23)$$

where $\hat{\Omega}_{T+j}$ is the daily forecast conditional covariance matrix estimated through the last equation in (12) and equation (15). The proof of equation (23), which represents the forecast of the covariance matrix and not the covariance matrix of forecasts, is in Appendix C.

4.4 Portfolio selection results

Given the estimates of FDCC, the empirical analysis uses the forecasts about expected returns and covariance matrices[17] to build a few representative portfolios.

In order to follow the definition of TAA as a short term strategy to enhance a better return and/or less risk than the benchmark, the forecast horizon is set to $\tau = 5$ days. Short sales are allowed to make possible the optimisation process without using any numerical method, and there are no riskfree assets to preserve the traditional hyperbolic form of the efficent frontier.

Table 1 shows the evolution in each period of the performances related to five portfolios given respectively by

- the minimum variance portfolio (C),
- the Sharpe-optimal portfolio[18] (M),
- the efficient portfolio (E) which has the same risk as the benchmark and lies on the efficient frontier,
- the efficient constrained portfolio[19] (J) which has a fixed tracking error of 2%,
- the benchmark portfolio (B).

These performances, obtained from FDCC estimates, are evaluated in terms of absolute and relative expected return-risk perspectives: portfolio

[17] Forecast portfolios, expected returns and covariance matrices are available upon request.

[18] This portfolio is the one for which the Sharpe (1994) ratio index is maximised.

[19] As shown in equations (5) and (6) the model by Jorion (2003) selects two portfolios which lie on the constrained TE frontier. In this analysis only the more efficient is considered.

alpha index is the excess return on the benchmark and its volatility, or tracking error, is obtained using the formula

$$TE_P = x'\Omega x, \tag{24}$$

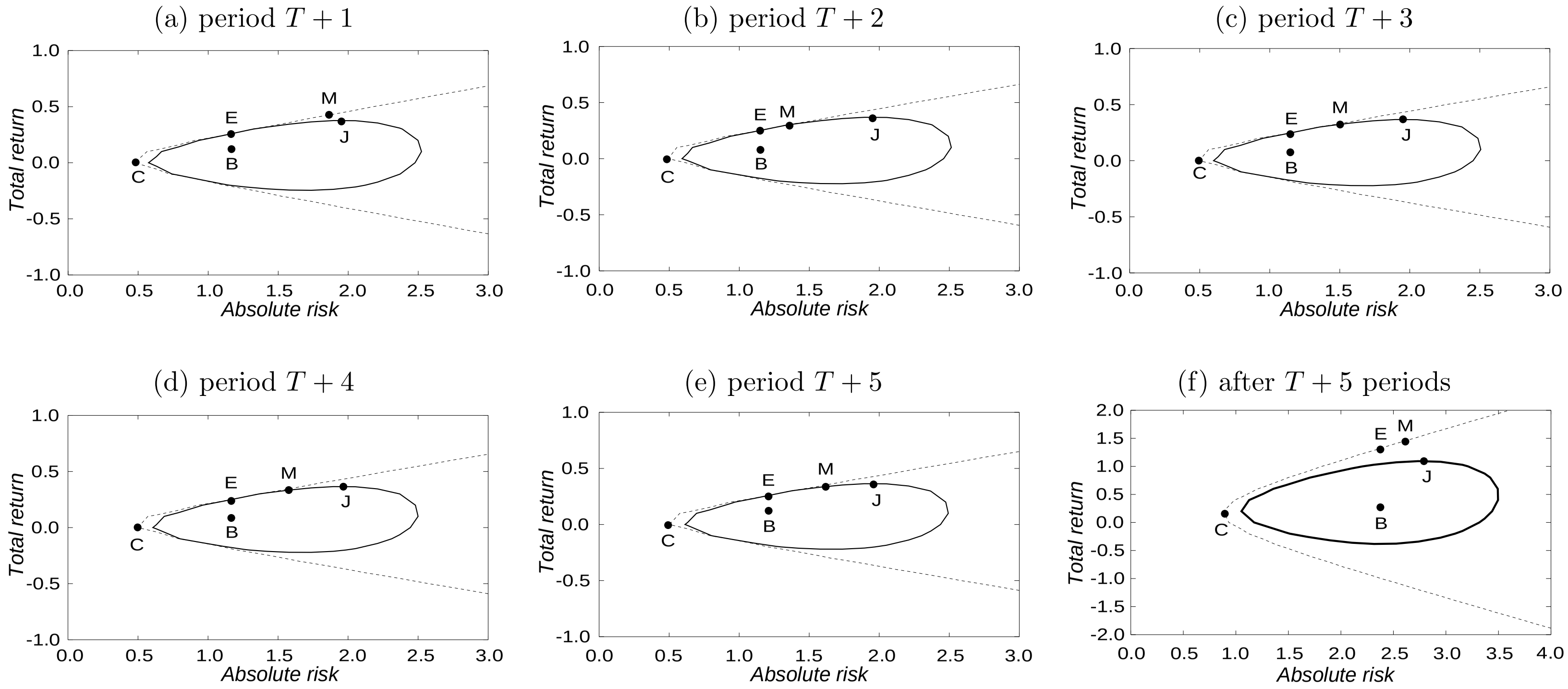
where x is the active portfolio introduced in the last equation of (3). Given that the manager can invest directly on the benchmark, x is the difference between the vector of portfolio weights ω_P and the n-dimensional vector q, a basis in which the one is associated to the benchmark. The Information Ratio is the natural counterpart of the Sharpe ratio in the relative return-risk space and it is simply given by the ratio of alpha to tracking error. All values in Tables are expressed as percentages.

For each period the tangency tracking error value (T_A) is evaluated. It is the minimum value for TE that provides the first intersection between the efficient frontier and the constrained frontier, according to equation (8).

Table 1: Forecast portfolio performances

Portfolios:	**C**	**M**	**E**	**J**	**B**
period T+1 (T_A: 1.1094)					
Return	0.0256	0.4376	0.2637	0.3804	0.0652
Risk	0.4607	1.9053	1.1633	1.9635	1.1633
Sharpe Ratio	0.0555	0.2297	0.2267	0.1937	0.0560
Alpha	-0.0396	0.3725	0.1985	0.3152	-
Tracking Error	1.1409	3.9022	1.9025	2.0000	-
Information Ratio	-0.0347	0.0955	0.1043	0.1576	-
period T+2 (T_A: 1.0903)					
Return	0.0348	0.3192	0.2599	0.3737	0.0733
Risk	0.4684	1.4183	1.1587	1.9635	1.1587
Sharpe Ratio	0.0743	0.2250	0.2243	0.1903	0.0633
Alpha	-0.0385	0.2458	0.1866	0.3004	-
Tracking Error	1.1232	2.4298	1.8619	2.0000	-
Information Ratio	-0.0343	0.1012	0.1002	0.1502	-
period T+3 (T_A: 1.0728)					
Return	0.0333	0.3349	0.2559	0.3719	0.0728
Risk	0.4736	1.5030	1.1542	1.9651	1.1542
Sharpe Ratio	0.0702	0.2228	0.2217	0.1893	0.0631
Alpha	-0.0396	0.2621	0.1830	0.2991	-
Tracking Error	1.1079	2.6086	1.8217	2.0000	-
Information Ratio	-0.0357	0.1005	0.1005	0.1495	-
period T+4 (T_A: 1.0570)					
Return	0.0326	0.3430	0.2526	0.3704	0.0729
Risk	0.4779	1.5510	1.3221	1.9656	1.1498
Sharpe Ratio	0.0682	0.2212	1.1498	0.1885	0.0634
Alpha	-0.0403	0.2702	0.2197	0.2976	-
Tracking Error	1.0937	2.7056	1.1797	2.0000	-
Information Ratio	-0.0368	0.0999	1.7867	0.1488	-
period T+5 (T_A: 1.0420)					
Return	0.0320	0.3502	0.2497	0.3691	0.0729
Risk	0.4818	1.5933	1.1455	1.9656	1.1455
Sharpe Ratio	0.0665	0.2198	0.2180	0.1878	0.0636
Alpha	-0.0408	0.2773	0.1769	0.2963	-
Tracking Error	1.0800	2.7943	1.7549	2.0000	-
Information Ratio	-0.0378	0.0992	0.1008	0.1481	-

Figure 1: Forecast frontiers



Note that in Table 1 the tracking error of portfolios C and E is less than 2%. This happens because of the property 4 of section 2.2 and the independence of these portfolios from the desired TE; portfolio M instead lies at the right of the curve, thus its tracking error is greater than 2%.

Figure 1 shows graphically all the forecast frontiers and portfolios of Table 1. The forecast frontiers after $T + 5$ periods, obtained using expected returns and covariance matrix evaluated from equations (22) and (23), is represented in Figure 1 (f); all the evaluated portfolios are those in Table 2 where the frontiers do not intersect because T_A=3.7977. In this case T_A is greater than 2%, hence all the tracking errors are also greater than 2%, with the only obvious exception being the benchmark.

Table 2: Forecast portfolios after $T + 5$ periods

Portfolios	**C**	**M**	**E**	**J**	**B**
Return	0.1642	1.4843	1.2070	1.1008	0.3571
Risk	0.8854	2.6618	2.1717	2.7845	2.1717
Sharpe Ratio	0.1855	0.5576	0.5558	0.3953	0.1644
Alpha	-0.1929	1.1272	0.8499	0.7437	-
Tracking Error	3.9323	8.3922	6.4100	2.0000	-
Information Ratio	-0.0490	0.1343	0.1326	0.3718	-

Tangency tracking error (T_A): 3.7977

Table 3: Contacts of frontiers for different value of TE

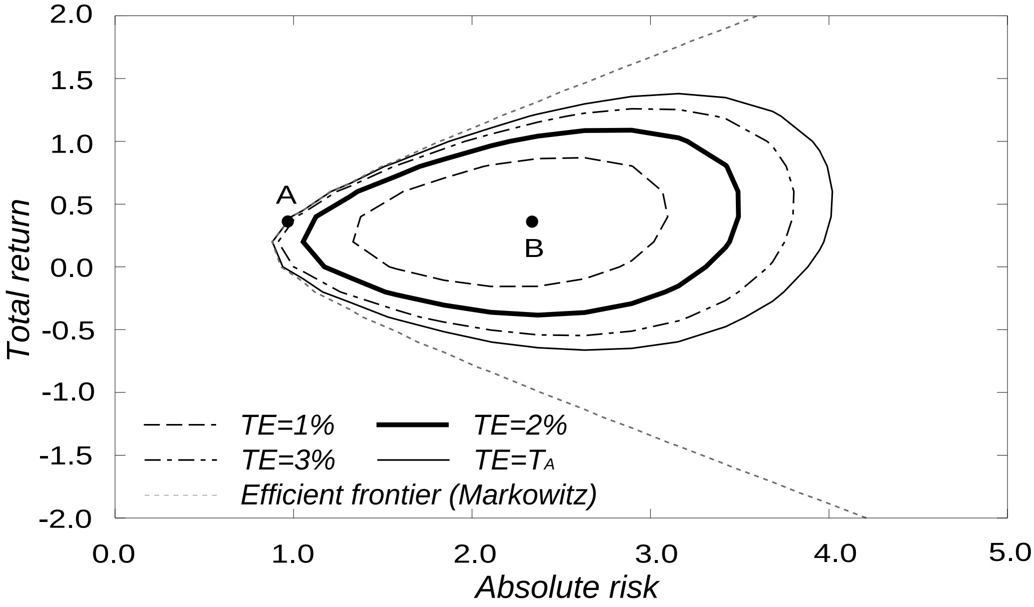
TE	**1**	**2**	**3**	$\boldsymbol{T_A}$
Contacts	0	0	0	1
Ψ	-0.7737	-0.4971	-0.2206	0.0000
Return	-	-	-	0.3571
Risk	-	-	-	0.9583
Sharpe Ratio	-	-	-	0.3726
Alpha	-	-	-	0.0000
Tracking Error	-	-	-	3.7977
Information Ratio	-	-	-	0.0000

TE	**4**		**8**		**10**	
Contacts	2		2		2	
Ψ	0.0559		1.1621		1.7152	
Portfolios	F	F'	G	G'	H	H'
Return	0.5936	0.1206	1.4351	-0.7209	1.6667	-0.9526
Risk	1.2044	0.8892	2.5738	1.9018	2.9912	2.3009
Sharpe Ratio	0.4929	0.1356	0.5576	-0.3791	0.5572	-0.4140
Alpha	0.2365	-0.2365	1.0780	-1.0780	1.3096	-1.3097
Tracking Error	4.0000	4.0000	8.0000	8.0000	10.0000	10.0000
Information Ratio	0.0591	-0.0591	0.1348	-0.1348	0.1310	-0.1310

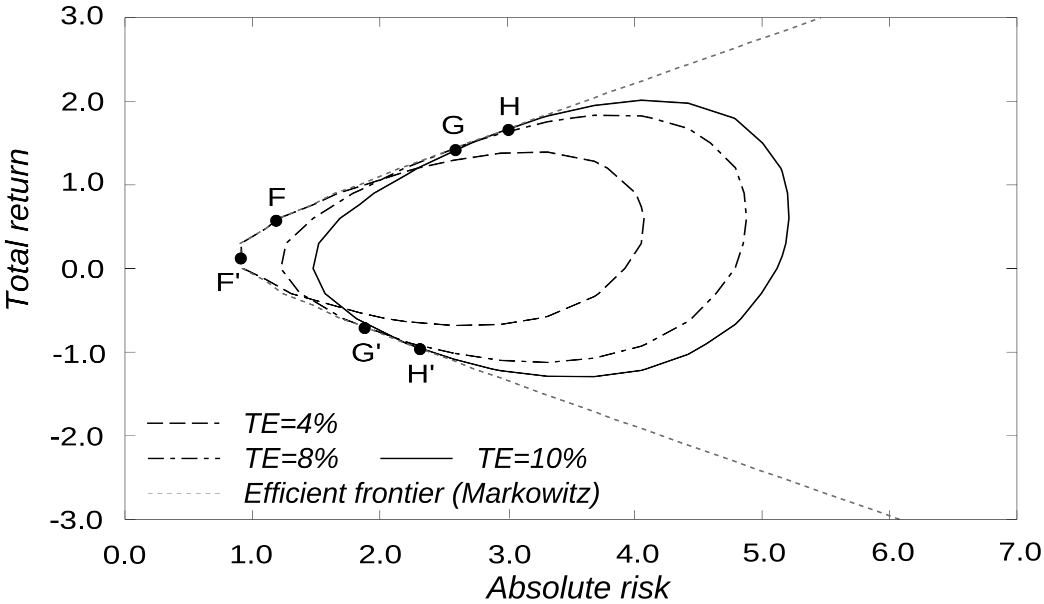
Figure 2 shows some curves corresponding to constrained frontiers evaluated for different values of desired TE, together with the Markowitz efficent set. This Figure refers to Table 3 which presents some scenarios; according

Figure 2: Constrained frontiers for different TE values

(a) $TE \leq T_A$



(b) $TE > T_A$



to equation (7), when $TE < T_A$, it follows that $\Psi < 0$ and contact portfolios do not exist because the constrained frontiers lie inside the hyperbola. The curves are somewhat distorted in (σ_P, R_P) space around the benchmark portfolio they are graphically represented by concentric areas.

The first contact ($\Psi = 0$) occurs in portfolio A which has the same expected return as the benchmark and a tracking error of T_A.

For TE greater than the tangency value, the curves have two intersections and $\Psi > 0$; the region between the efficient set and the right arc defined by those common points, contains all possible constrained portfolios. Figure 2 (a) illustrates the $TE \leq T_A$ case, while Figure 2 (b) shows curves obtained for 4% and non conventional 8% and 10% desired tracking errors. For example, when $TE = 4\%$, the frontiers have two contact points, F and F', therefore portfolio C lies within the surface given by the efficient set and right $\widehat{FF'}$ in which a tracking error of 3.7977 is allowed.

4.5 Blending the views

The empirical Bayesian nature of the BL approach leads to an estimate of the vector of expected returns as a weighted average of equilibrium returns and views, where weights depend upon differences of expected returns from the equilibrium and on the manager's confidence in views[20].

When the investor has some views about the expected returns, she can combine her private information with the information available from the forecast model. In this analysis the following views about expected returns are those formulated by the manager:

- all returns of assets belonging to *Chemicals* will change to 3% (3 absolute views),
- all returns of assets belonging to *Utilities* will change to 2% (6 absolute views),
- given the above scenario 3, the return of ENI equals that of DEB, the return of REP equals that of BBV and the return TOT equals that of AIB (3 relative views),
- AXA and ING outperform BMK by 2% (2 relative views).

Once the views are selected, implementing the BL approach requires the specification of, on one hand, a suitable weight-on-views to calibrate the confidence level of the prior belief $\gamma\Omega$ and, on the other, the matrix S containing the uncertainty of the views. Black and Litterman (1992) and Lee (2000) suggest the first solution to this practical problem by imposing

[20]For different approaches on asset return predictability with incremental information see for example Pesaran and Timmermann (1995), Avramov (2002, 2004) or the most recent papers by Aiolfi and Favero (2005) or Rodriguez and Sosvilla-Rivero (2006).

γ to be close to zero, because the uncertainty of the means is less than the uncertainty of the expected returns. On the other hand Shi and Irwin (2005) demonstrate that theoretically the parameter has to be equal to T^{-1}, where T is the number of observations of asset returns. Conversely, Satchell and Scowcroft (2000) provide an analytical method which often sets $\gamma = 1$. In all these contributions the uncertainty of views is given by the matrix S whose diagonal elements are the inverses of each investor's confidence in views.

In this paper the calibration used is that of He and Litterman (1999) in which the covariance matrix S is assumed to be proportional to the variance of the view portfolios, according to the equation

$$\frac{s_i}{\gamma} = p_i \Omega p_i' \qquad i = 1, 2, \ldots, k. \tag{25}$$

Variable s_i is defined as the i-th diagonal element in matrix S, $p_i \Omega p_i'$ is the variance of the view portfolio and p_i is the i-th row in matrix P.

The above specification leads to the following expression for the new combining expected returns vector:

$$\hat{R}_{BL} = [\Omega^{-1} + P' \langle P \Omega P' \rangle^{-1} P]^{-1} [\Omega^{-1} \bar{R} + P' \langle P \Omega P' \rangle^{-1} V], \tag{26}$$

where $\langle P \Omega P' \rangle$ is a diagonal matrix whose diagonal elements are the same as those of the product $P \Omega P'$.

The advantage of this assumption is that γ does not affect the new combined vector $\hat{R}_{BL}$ because only the ratio (25) enters into its evaluation. This implies that it is not necessary to assign any explicit confidence level to views.

4.6 Portfolios

The optimistic views expressed in the previous section make $\hat{R}_{BL}$ greater than FDCC forecast expected returns, so they determine the surface enlargement of both the efficient set and the constrained frontier. This means that, after the blending process, the manager can invest in a higher number of portfolios. Table 4 and Figure 3 respectively show the resulting portfolios and frontiers updated according to mean-variance paradigm and the manager's views, with fixed $TE = 2\%$.

Moreover, as shown in Table 10, the MSE associated to the BL model is less than the MSE evaluated on both FDCC forecasts and sample mean estimates.

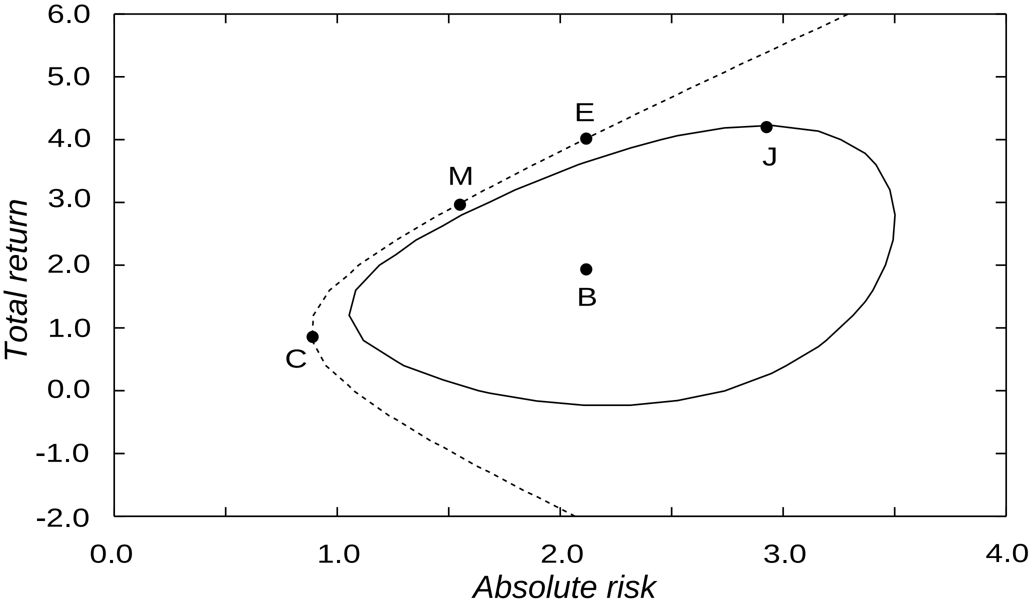
Finally Table 5 reports expected returns and Alpha evaluated for portfolios C, M, E, J and B using ω_{FDCC} and ω_{BL} which are respectively the portfolio weights evaluated after the FDCC estimation and after the BL blending. The first two columns compute performances using realized returns taken from the out of sample period which goes from the 26^{th} through the 30^{th} June 2006, while in the last two columns such returns are modified

Table 4: Forecast portfolios using BL approach

Portfolios	**C**	**M**	**E**	**J**	**B**
Return	0.9671	3.0078	4.1137	4.2384	1.9944
Risk	0.8854	1.5614	2.1717	2.9236	2.1717
Sharpe Ratio	1.0923	1.9264	1.8943	1.4497	0.9184
Alpha	-1.0273	1.0134	2.1193	2.2440	-
Tracking Error	3.9323	3.9210	5.2969	2.0000	-
Information Ratio	-0.2612	0.2585	0.4001	1.1220	-

Tangency tracking error (T_A): 3.5131

Figure 3: Portfolios and frontiers after using the BL approach



by the equation (26). This Table also shows that portfolios C and B are independent from manager's choices and therefore from the vectors of weights used.

Table 5: Out of sample real performances

		Real		Blending	
Porfolios		**Return**	**Alpha**	**Return**	**Alpha**
	C	0.8446	-1.8994	0.9671	-1.0273
	B	2.7440	0.0000	1.9944	0.0000
ω_{FDCC}	**M**	4.4639	1.7199	2.7628	0.7684
	E	3.7038	0.9598	2.3857	0.3913
	J	4.7831	2.0391	3.0061	1.0117
ω_{BL}	**M**	2.4203	-0.3237	3.0079	1.0135
	E	3.2742	0.5302	4.1137	2.1193
	J	4.4767	1.7327	4.2384	2.2440

5 Concluding remarks and further research

This paper proposes a portfolio optimisation for large scale TAA that has two key features: the first is that the model takes the changing volatility of asset returns over time into account and the secondis that it provides the possibility of using private information in the mean-variance paradigm. An empirical work is proposed to tactically manage some portfolios of interest in the space spanned by absolute risk and total expected return, using data taken from the DJ Euro Stoxx 50 index.

The FDCC model by Billio, Caporin and Gobbo (2006) is useful for solving the practical problems of forecasting the expected asset returns and their covariance matrix; the ability of the model to group variables among sectors permits the analysis of the persistence in volatility in a parsimonious way and does not involve any computational drawbacks. Moreover, the BL approach can instead present a good method for incorporating the manager's views about asset returns into the asset allocation process.

The whole analysis is carried out on different portfolios located along the mean-variance efficient set (Markowitz, 1959) and the fixed tracking error constrained frontier introduced by Jorion (2003).

This work is based on different assumptions which can be relaxed in future research: the absence of a riskfree asset, the possibility of short positions and finally the estimation of a GARCH(1,1) model in the first step of FDCC.

It is well known that the efficient frontier is not an hyperbola when a riskfree asset is included into the optimisation, while the form and the properties of the constrained tracking error frontier have to be explored. This can dramatically modify the portfolio allocations in the (σ_p, R_P) space.

The consequence of relaxing the second assumption is that the vector of weights has its elements $\omega_i \geq 0$; even if managers can not often make short positions, this constraint implies that equations (1) and (3) may return corner solutions or solutions wich have no closed-form. Hence, some numerical algorhithms are required and this can represent a drawback from the computational point of view.

The last hypothesis is about first step estimation of the FDCC model; the GARCH(1,1) in conditional variance equations does not take some aspects into account, such as asymmetries, unit roots or varying exponents (see, for example, APARCH model by Ding, Engle and Granger, 1993). The wide literature provides many solutions which can lead to forecasts of expected returns far from those obtained in this work.

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Appendix A: Assets and sectors

Table 6: List of assets selected in the model

N.	CODE	ASSET	SECTOR	WEIGHT	COUNTRY
1	ABN	ABN AMRO HOLDING	Banks	0.0227	Holland
2	AEG	AEGON	Insurance	0.0113	Holland
3	AHO	AHOLD KON.	Retail	0.0056	Holland
4	AIR	AIR LIQUIDE	Chemicals	0.0096	France
5	ALC	ALCATEL 'A'	Technology	0.0078	France
6	ALL	ALLIANZ (XET)	Insurance	0.0287	Germany
7	AIB	ALLIED IRISH BANKS	Banks	0.0095	Ireland
8	AXA	AXA	Insurance	0.0232	France
9	BSC	BANCO SANTANDER CENTRAL	Banks	0.0377	Portugal
10	BAS	BASF (XET)	Chemicals	0.0188	Germany
11	BAY	BAYER (XET)	Chemicals	0.0142	Germany
12	BBV	BBV ARGENTARIA	Banks	0.0281	Spain
13	BNP	BNP PARIBAS	Banks	0.0318	France
14	CAR	CARREFOUR	Retail	0.0125	France
15	CAG	CRÉDIT AGRICOLE	Banks	0.0105	France
16	DAI	DAIMLERCHRYSLER	Automobiles & parts	0.0213	Germany
17	DAN	DANONE	Food & beverage	0.0122	France
18	DEB	DEUTSCHE BANK (XET)	Banks	0.0247	Germany
19	DTE	DEUTSCHE TELEKOM (XET)	Telecommunications	0.0209	Germany
20	END	ENDESA	Utilities	0.0118	Spain
21	ENE	ENEL	Utilities	0.0160	Italy
22	ENI	ENI	Oil & Gas	0.0342	Italy
23	EON	E ON (XET)	Utilities	0.0321	Germany
24	FOR	FORTIS (AMS)	Banks	0.0190	Holland
25	FTE	FRANCE TÉLÉCOM	Telecommunications	0.0210	France
26	GEN	GENERALI	Insurance	0.0169	Italy
27	IBE	IBERDROLA	Utilities	0.0100	Spain
28	ING	ING GROEP CERTS.	Insurance	0.0323	Holland
29	LAF	LAFARGE	Construction & materials	0.0073	France
30	LOR	L'ORÉAL	Personal & household goods	0.0104	France
31	LVM	LVMH	Personal & household goods	0.0109	France
32	NOK	NOKIA	Technology	0.0372	Finland
33	PHI	PHILIPS ELTN.KON	Personal & household goods	0.0183	Holland
34	REN	RENAULT	Automobiles & parts	0.0076	France
35	REP	REPSOL YPF	Oil & gas	0.0142	Spain
36	RWE	RWE (XET)	Utilities	0.0158	Germany
37	SGO	SAINT GOBAIN	Construction & materials	0.0097	France

Continued on next page

Table 6 — continued from previous page

N.	CODE	ASSET	SECTOR	WEIGHT	COUNTRY
38	SAV	SANOFI-AVENTIS	Healthcare	0.0394	France
39	SPA	SAN PAOLO IMI	Banks	0.0072	Italy
40	SAP	SAP (XET)	Technology	0.0186	Germany
41	SIE	SIEMENS (XET)	Industrial goods & services	0.0311	France
42	SOG	SOCIÉTÉ GÉNERALE	Banks	0.0256	France
43	SUE	SUEZ	Utilities	0.0161	France
44	TIT	TELECOM ITALIA	Telecommunications	0.0150	Italy
45	TEL	TELEFONICA	Telecommunications	0.0310	Spain
46	TOT	TOTAL	Oil & gas	0.0718	France
47	UNC	UNICREDITO ITALIANO	Banks	0.0221	Italy
48	UNL	UNILEVER CERTS.	Food & beverage	0.0186	Holland
49	VIV	VIVENDI UNIVERSAL	Media	0.0152	UK
50	BMK	DJ EUROSTOXX 50	*Benchmark portfolio*		

Table 7: List of macro-sectors

N.	SECTOR	DIM.	ASSETS INCLUDED
1	Automobiles	2	DAI, REN
2	Banks	11	ABN, AIB, BSC, BBV, BNP, CAG, DEB, FOR, SPA, SOG, UNC
3	Chemicals	3	AIR, BAS, BAY
4	Constructions	2	LAF, SGO
5	Industrial	7	DAN, LOR, LVM, PHI, SAV, SIE, UNL
6	Insurance	5	AEG, ALL, AXA, GEN, ING
7	Oil & gas	3	ENI, REP, TOT
8	Retail	2	AHO, CAR
9	Technology	4	ALC, NOK, SAP, VIV
10	Telecommunications	4	DTE, FTE, TIT, TEL
11	Utilities	6	END, ENE, EON, IBE, RWE, SUE
12	Benchmark	1	BMK

Appendix B: Estimation results

Table 8 reports the FDCC first step estimation of the univariate GARCH(1,1) provided by equation (15); coefficients θ, α and β which have 5% statistical signifiance are shown in bold.

Table 8: Univariate GARCH models (1^{st} step estimation)

Asset	μ	θ	ω	α	β
ABN	0.0678	-0.0812	0.0468	**0.0736**	**0.8923**
	(0.0370)	(0.0439)	(0.0190)	(0.0203)	(0.0295)
AEG	0.0551	0.0705	0.0479	**0.1125**	**0.8751**
	(0.0477)	(0.0632)	(0.0205)	(0.0245)	(0.0248)
AHO	0.0329	-0.0410	0.0329	**0.0248**	**0.9662**
	(0.0658)	(0.0762)	(0.0132)	(0.0061)	(0.0076)
AIR	0.0654	-0.0608	0.0600	**0.0894**	**0.8649**
	(0.0351)	(0.0428)	(0.0239)	(0.0232)	(0.0348)
ALC	0.0926	-0.0124	0.0444	**0.1230**	**0.8527**
	(0.0335)	(0.0369)	(0.0273)	(0.0390)	(0.0516)
ALL	-0.0177	-0.0481	0.5161	**0.0851**	**0.8167**
	(0.0744)	(0.0795)	(0.1612)	(0.0260)	(0.0468)
AIB	0.0885	-0.0306	0.0478	**0.0930**	**0.8904**
	(0.0471)	(0.0597)	(0.0196)	(0.0214)	(0.0234)
AXA	0.0970	0.0447	0.1092	**0.0991**	**0.8587**
	(0.0482)	(0.0599)	(0.0400)	(0.0230)	(0.0316)
BSC	0.0486	**0.0888**	0.1083	**0.1138**	**0.8172**
	(0.0375)	(0.0447)	(0.0434)	(0.0275)	(0.0480)
BAS	0.0918	**-0.1172**	0.0585	**0.0905**	**0.8760**
	(0.0398)	(0.0482)	(0.0221)	(0.0198)	(0.0264)
BAY	0.1510	**0.1345**	0.1474	**0.1708**	**0.8050**
	(0.0502)	(0.0645)	(0.0601)	(0.0323)	(0.0403)
BBV	0.0842	-0.0375	0.0342	**0.0850**	**0.8909**
	(0.0349)	(0.0435)	(0.0141)	(0.0196)	(0.0247)
BNP	0.0681	-0.0380	0.0444	**0.0799**	**0.8954**
	(0.0397)	(0.0480)	(0.0196)	(0.0184)	(0.0243)
CAR	0.0261	-0.0784	0.0377	**0.0507**	**0.9252**
	(0.0399)	(0.0454)	(0.0175)	(0.0140)	(0.0214)
CAG	0.0905	-0.0550	0.0950	**0.0615**	**0.8917**
	(0.0451)	(0.0516)	(0.0409)	(0.0190)	(0.0341)
DAI	0.0321	-0.0847	0.0959	**0.0684**	**0.8914**
	(0.0489)	(0.0555)	(0.0343)	(0.0194)	(0.0293)
DAN	0.0685	**-0.1079**	0.0674	**0.1017**	**0.8476**

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Table 8 — continued from previous page

Asset	μ	θ	ω	α	β
	(0.0339)	(0.0372)	(0.0204)	(0.0206)	(0.0290)
DEB	0.0871	-0.0987	0.0439	**0.0565**	**0.9221**
	(0.0451)	(0.0528)	(0.0224)	(0.0174)	(0.0246)
DTE	0.0112	**-0.1427**	0.0557	**0.0753**	**0.8873**
	(0.0381)	(0.0450)	(0.0191)	(0.0212)	(0.0287)
END	0.0801	**-0.0814**	0.1673	**0.2442**	**0.6460**
	(0.0331)	(0.0404)	(0.0564)	(0.0486)	(0.0741)
ENE	0.0398	**-0.1017**	0.1517	0.0247	**0.8265**
	(0.0342)	(0.0343)	(0.1109)	(0.0214)	(0.1230)
ENI	0.0838	**-0.0788**	0.1841	**0.1073**	**0.7498**
	(0.0361)	(0.0393)	(0.0651)	(0.0320)	(0.0711)
EON	0.1071	**-0.1351**	0.0776	**0.0797**	**0.8712**
	(0.0388)	(0.0444)	(0.0274)	(0.0209)	(0.0312)
FOR	0.1028	-0.0793	0.0622	**0.1308**	**0.8358**
	(0.0360)	(0.0461)	(0.0178)	(0.0247)	(0.0276)
FTE	-0.0157	-0.1036	0.0889	**0.0402**	**0.9215**
	(0.0503)	(0.0565)	(0.0584)	(0.0188)	(0.0412)
GEN	0.0532	0.0127	0.0116	**0.0678**	**0.9233**
	(0.0301)	(0.0359)	(0.0072)	(0.0201)	(0.0232)
IBE	0.0757	**-0.1410**	0.2565	**0.2318**	**0.4206**
	(0.0267)	(0.0282)	(0.0788)	(0.0606)	(0.1430)
ING	0.1171	0.0336	0.0678	**0.1262**	**0.8445**
	(0.0413)	(0.0543)	(0.0210)	(0.0257)	(0.0281)
LAF	0.0679	-0.1098	0.0660	**0.0975**	**0.8806**
	(0.0450)	(0.0564)	(0.0210)	(0.0219)	(0.0230)
LOR	0.0397	**-0.1514**	0.1538	**0.1320**	**0.7712**
	(0.0390)	(0.0456)	(0.0497)	(0.0302)	(0.0507)
LVM	0.1018	0.0069	0.0973	**0.0847**	**0.8595**
	(0.0414)	(0.0472)	(0.0479)	(0.0256)	(0.0484)
NOK	0.0373	0.0239	0.0122	**0.0106**	**0.9858**
	(0.0649)	(0.0713)	(0.0073)	(0.0038)	(0.0046)
PHI	0.0526	-0.0499	0.0325	**0.0504**	**0.9386**
	(0.0522)	(0.0625)	(0.0161)	(0.0136)	(0.0159)
REN	0.0955	-0.0218	0.0564	**0.0475**	**0.9279**
	(0.0498)	(0.0562)	(0.0405)	(0.0206)	(0.0357)
REP	0.0875	**-0.0929**	0.0658	**0.0760**	**0.8728**
	(0.0357)	(0.0370)	(0.0254)	(0.0192)	(0.0318)
RWE	0.1415	**-0.0975**	0.1829	**0.1250**	**0.7833**
	(0.0443)	(0.0497)	(0.0650)	(0.0333)	(0.0562)
SGO	0.0899	-0.0805	0.0591	**0.1180**	**0.8560**

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Table 8 — continued from previous page

Asset	μ	θ	ω	α	β
	(0.0400)	(0.0506)	(0.0217)	(0.0266)	(0.0301)
SAV	0.0600	**-0.1522**	0.3079	**0.1165**	**0.7361**
	(0.0448)	(0.0503)	(0.1185)	(0.0366)	(0.0836)
SPA	0.1034	-0.1095	0.0608	**0.1126**	**0.8730**
	(0.0463)	(0.0575)	(0.0202)	(0.0233)	(0.0220)
SAP	0.0833	-0.0608	0.0266	**0.0538**	**0.9321**
	(0.0418)	(0.0487)	(0.0171)	(0.0221)	(0.0278)
SIE	0.0771	-0.0866	0.0328	**0.0551**	**0.9303**
	(0.0444)	(0.0520)	(0.0169)	(0.0156)	(0.0198)
SOG	0.0960	-0.0304	0.0526	**0.0914**	**0.8841**
	(0.0406)	(0.0501)	(0.0231)	(0.0250)	(0.0309)
SUE	0.1333	-0.0951	0.2249	**0.1873**	**0.7414**
	(0.0469)	(0.0590)	(0.0599)	(0.0458)	(0.0525)
TIT	0.0305	**-0.1102**	0.7105	**0.0870**	**0.4129**
	(0.0406)	(0.0388)	(0.3415)	(0.0432)	(0.2596)
TEL	0.0672	**-0.1032**	0.1031	**0.1297**	**0.7850**
	(0.0339)	(0.0392)	(0.0334)	(0.0309)	(0.0493)
TOT	0.0783	**-0.1030**	0.0451	**0.0734**	**0.8939**
	(0.0366)	(0.0427)	(0.0216)	(0.0204)	(0.0314)
UNC	0.0404	**-0.0026**	0.0160	**0.0640**	**0.9255**
	(0.0326)	(0.0369)	(0.0083)	(0.0154)	(0.0180)
UNL	0.0042	-0.0631	0.0156	**0.0446**	**0.9454**
	(0.0350)	(0.0447)	(0.0091)	(0.0196)	(0.0220)
VIV	0.0775	0.0638	0.0193	**0.0392**	**0.9518**
	(0.0472)	(0.0547)	(0.0108)	(0.0100)	(0.0119)
BMK	0.0773	**-0.0814**	0.0236	**0.0860**	**0.8887**
	(0.0281)	(0.0356)	(0.0091)	(0.0184)	(0.0231)

Table 9 shows the second step estimation of the FDCC model according to equation (19); note that the equation (21) is not violated for any of the estimated values of parameters a and b.

Table 9: Flexible DCC parameters (2^{nd} step estimation)

"a" parameters				
Sector	Coeff.	S.E.	t-stat	p-value
Automobiles	0.0478	0.0185	2.5744	0.0100
Banks	0.0769	0.0123	6.2330	0.0000
Chemicals	0.0679	0.0267	2.5450	0.0109
Constructions	0.0647	0.0197	3.2788	0.0010
Industrial	0.0526	0.0114	4.6248	3.7e-6
Insurance	0.0437	0.0159	2.7566	0.0058
Oil & gas	0.1047	0.0227	4.6196	3.8e-6
Retail	0.0770	0.0230	3.3502	0.0008
Technology	0.0678	0.0152	4.4644	8.0e-6
Telecommunications	0.0898	0.0212	4.2368	2.3e-5
Utilities	0.1104	0.0249	4.4419	8.9e-6
BMK	0.0915	0.0090	10.1340	0.0000
"b" parameters				
Sector	Coeff.	S.E.	t-stat	p-value
Automobiles	0.9607	0.0387	24.8270	0.0000
Banks	0.8662	0.0479	18.0950	0.0000
Chemicals	0.7921	0.1889	4.1929	2.8e-5
Constructions	0.9442	0.0314	30.0490	0.0000
Industrial	0.9153	0.0310	29.5480	0.0000
Insurance	0.8945	0.0629	14.2200	0.0000
Oil & gas	0.9155	0.0390	23.5030	0.0000
Retail	0.9082	0.0500	18.1680	0.0000
Technology	0.9274	0.0287	32.3600	0.0000
Telecommunications	0.8129	0.1020	7.9671	0.0000
Utilities	0.8386	0.0801	10.4630	0.0000
BMK	0.7559	0.0459	16.4530	0.0000

For each time series Table 10 provides:

- R_{T+5}: sum of realized out of sample returns (5 observations);
- $\hat{R}_{T+5}$: estimated expected returns vector from the FDCC model, according to equation (22);
- $\hat{R}_{BL}$: estimated Black and Litterman (1991) returns vector, according to equation (26);
- $\bar{R}$: 5 times sample means;
- $h^{1/2}$: realized risk from out of sample observations;
- $\hat{h}^{1/2}_{T+5}$: estimated risk vector from the FDCC model, according to equation (23);
- σ_y: variance of forecasts computed on sample variances (s^2) evaluated as $\sigma_y = \sqrt{5s^2}$;

Mean squared errors (MSE) for different estimated returns are also evaluated.

Table 10: Expected returns and risks

Ticker	R_{T+5}	$\hat{R}_{T+5}$	$\hat{R}_{BL}$	$\bar{R}$	$h^{1/2}$	$\hat{h}^{1/2}_{T+5}$	σ_y
ABN	1.6974	0.3097	1.3505	0.2050	2.5372	2.2793	3.0775
AEG	4.2787	0.3012	2.1992	0.1686	1.8464	4.0434	5.1395
AHO	0.7391	0.1496	1.9613	0.4756	4.2253	3.3916	5.5979
AIR	2.4594	0.3050	1.6622	0.2381	1.8645	2.1197	2.7094
ALC	3.3056	0.4587	1.9146	0.2464	1.3390	4.2966	2.6610
ALL	-0.7032	-0.1061	1.4341	0.2318	2.2339	4.0536	5.1290
AIB	2.8495	0.4313	1.9663	0.4083	2.3091	3.6438	4.4323
AXA	7.2311	0.5013	2.4628	0.4499	0.5680	3.6199	4.0885
BSC	1.0821	0.2753	1.5583	0.3617	0.9897	2.5182	2.9018
BAS	3.0239	0.4165	2.8810	0.3401	2.8245	2.1685	3.3061
BAY	7.6505	0.8041	3.0012	0.6111	5.7134	4.3825	4.4777
BBV	3.4803	0.4076	2.1658	0.3900	1.6435	2.4351	2.9222
BNP	3.0521	0.3269	1.8542	0.3727	2.3785	2.9784	3.1727
CAR	3.2143	0.1021	0.9193	0.1349	1.7048	2.3008	3.1425
CAG	4.6440	0.4326	1.5973	0.4197	4.3784	3.3017	3.2468
DAI	2.0927	0.1295	1.4482	0.1657	2.6890	2.7672	3.6044
DAN	4.0046	0.3033	1.1725	0.3197	1.6828	2.3974	2.6958

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Table 10 — continued from previous page

Ticker	R_{T+5}	$\hat{R}_{T+5}$	$\hat{R}_{BL}$	$\bar{R}$	$h^{1/2}$	$\hat{h}^{1/2}_{T+5}$	σ_y
DEB	3.5981	0.3994	1.7870	0.4692	2.7270	2.8877	3.4727
DTE	0.4781	0.0042	0.7409	0.0924	1.3564	1.9698	3.1634
END	0.8495	0.3707	1.7299	0.5130	1.0562	1.9719	2.6659
ENE	1.6455	0.1621	1.9644	0.1570	1.4546	2.1361	2.2902
ENI	2.9077	0.3901	2.9082	0.2926	0.5057	2.5319	2.5731
EON	5.9278	0.4865	2.0037	0.4425	2.5496	2.5989	2.9230
FOR	2.6212	0.4854	1.9331	0.4128	2.6434	2.8052	3.9349
FTE	1.0766	-0.1161	0.6917	-0.0373	1.7420	2.7966	3.6937
GEN	2.6687	0.2706	2.9295	0.0969	2.3218	2.9275	2.6256
IBE	2.0256	0.3271	1.8311	0.3574	1.5652	1.5728	1.9105
ING	2.8720	0.5978	2.2691	0.5010	3.0793	3.1250	4.3182
LAF	3.3673	0.2998	2.9974	0.3550	2.6304	4.5464	3.8733
LOR	3.6543	0.1435	1.1936	0.0999	1.7491	2.3150	3.0274
LVM	4.6152	0.5114	2.3977	0.3812	1.9089	3.4588	3.0893
NOK	1.4516	0.1953	0.8678	0.1375	5.6542	3.5582	4.6797
PHI	2.6547	0.2449	2.1412	0.2484	6.5318	4.0331	4.3218
REN	0.5372	0.4698	1.7043	0.4371	4.6360	3.2206	3.7236
REP	4.7563	0.4036	1.7853	0.2868	0.8066	2.4268	2.5490
RWE	3.7563	0.6720	2.0135	0.6420	2.6582	2.3995	3.3102
SGO	3.9221	0.4203	2.1149	0.3763	3.9059	3.4834	3.4815
SPA	1.9187	0.2445	1.1504	0.2376	3.9018	2.6648	3.3564
SAV	0.7299	0.4774	1.4987	0.4321	1.8788	2.8126	4.0423
SAP	2.7861	0.3946	1.5679	0.4203	1.0836	2.3708	3.2256
SIE	0.1913	0.3538	2.4157	0.3544	2.3851	3.8621	3.3889
SOG	3.9917	0.4689	2.4278	0.4607	1.8513	3.5202	3.2974
SUE	5.1455	0.6321	2.0077	0.5481	3.6155	3.0365	4.4912
TIT	-0.1377	0.1124	0.9655	0.0807	1.7739	2.2556	2.6822
TEL	0.8484	0.2984	1.2783	0.2470	1.0158	1.7767	2.5689
TOT	2.9388	0.3542	2.1805	0.2900	2.1825	2.9656	2.7325
UNC	0.7380	0.2009	1.7900	0.2835	3.4780	3.4702	2.6904
UNL	2.9187	-0.0020	0.8503	-0.0103	1.9032	2.2446	2.7393
VIV	1.5983	0.4108	1.4876	0.4225	0.9221	2.9824	3.7489
BMK	2.7440	0.3571	1.9944	0.2924	1.8357	2.1717	2.3837
MSE:							
FDCC			20.4494				
Black Litterman			12.7031				
Sample mean			20.7464				

Appendix C: Proof of equation (23)

The starting point of this proof is the multivariate generalisation of the first equation of equation (12) provided by equation (14). Focussing the attention upon the forecasts of the expected returns vector, after some recursive substitutions, the model becomes:

$$\sum_{j=1}^{\tau} \hat{y}_{T+\tau} = \mu + \Pi \sum_{j=0}^{\tau} (\tau - j)\Pi^j + \sum_{j=1}^{\tau} \Pi^j y_T + \sum_{j=1}^{\tau} \sum_{r=0}^{\tau-j} \Pi^r \varepsilon_{T+j}$$

Conditioning to the information set $\mathcal{F}_T$, the first and the second term of the above summation are constant, therefore the covariance matrix forecasts depend only upon the third one. Given the result

$$\sum_{j=0}^{\tau} \Pi^j = (I - \Pi)^{-1}(I - \Pi^{\tau+1}),$$

the forecast for the vector $\hat{y}_{T+\tau}$ conditional to the information set $\mathcal{F}_T$ is given by the following expression:

$$\sum_{j=1}^{\tau} \hat{y}_{T+j} \mid \mathcal{F}_T = (I - \Pi)^{-1} \sum_{j=1}^{\tau} (I - \Pi^{\tau-j+1})\varepsilon_{T+j}$$

For the standard hyphotheses of the VAR(1) model, $E(\varepsilon_{t+j}\varepsilon'_{t+r}) = 0$ for all $j \neq r$, hence, the conditional covariance matrix is

$$\hat{\Omega}_{T+\tau} = (I - \hat{\Pi})^{-1} \left[\sum_{j=1}^{\tau} (I - \hat{\Pi}^{\tau-j+1})\hat{\Omega}_{T+j}(I - \hat{\Pi}^{\tau-j+1})' \right] (I - \hat{\Pi}')^{-1}$$

equal to equation (23).