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Public Expenditure and Economic Growth

A critical extension of Barro's (1990) model

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PUBLIC EXPENDITURE AND ECONOMIC GROWTH

A critical extension of Barro's (1990) model

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Abstract

I intend to verify whether the results obtained by Barro (1990) in relation to the effects of both productive investments and public consumption on economic growth are also confirmed in a more general context. As is well-known, public expenditure may exert an effect on the economic growth rate through the positive externality in the productivity of the capital stock. When public expenditure in the households' utility function is considered, a further effect operates to modify the saving and investment decisions of households, depending on the relative weight of public consumption. In particular, if households consider public expenditure to be useful, I shall show that – whatever the exogenous fiscal policy may be – the growth rate is always higher than it is in the case of productive investments alone. Moreover, if households are able to choose the optimal income tax rate, an optimal growth rate greater than the maximum one may be obtained.

JEL:H5-O4
KEYWORDS: public expenditure, economic growth

Indice

1. Introduction

The basic hypothesis of Barro's model (1990)[1] is that the government purchases a constant share of private output: *$g(t) = \tau y(t))$* and uses it to provide free public services to private producers. Barro considers all public expenditures that produce externalities generalised to the firms' system, such as the defence of property rights, spending on justice, national defence, education,[2] and so on. This public spending affects the constant returns to scale[3] production function in two productive factors, *$k(t)$* and *$g(t)$*. On maximising households' utility, one obtains a steady-state growth rate which is influenced by public spending on production services. An increase in the tax rate τ reduces the income available for consumption and private investment, but it increases public services *$g(t)$* to firms. Which of the two effects will prevail depends on the form of the production function. In the case of a Cobb-Douglas production function, an increase in the tax rate boosts the growth rate until $\tau < \alpha$, and it reaches the maximum when $\tau = \alpha$..[4]

A natural extension of this important result has been described by Barro (1990) himself.: "*We could also allow for public consumption services as an influence on households' utility.*" ...: "*The growth rate lies uniformly below the value γ, ..., that would have been chosen if $\tau_h = 0$*".[5]

However, the growth rate obtained by Barro (1990) (equation 25, p.117) does not

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[1] R. J. Barro (1990), *op. cit.* See also Barro and Sala-I-Martin (1992), (1995) . Public investments are non rival and non excludable, like public goods.

[2] Lucas (1988) argued that investment in education increases the stock of human capital. Therefore, the public provision of general education modifies the optimal accumulation of human capital and the long run pattern of economic growth. Similarly, government expenditures on research and development (Romer, 1990), on health (Bloom *et al*, 2001) and on public infrastructures may influence the optimal rate of economic growth by introducing an externality in private decisions. Turnovsky (1996, 2000) notes that distorsive taxation may internalise the effect of these externalities, inducing an efficient intertemporal allocation. For an interesting survey of the literature on fiscal policy and economic growth , see Zagler and Durneker (2003). In a recent paper, Peretto (2003) shows that in a model of endogenous growth that does not exhibit the scale effect, the level and composition of public expenditure have no effect on steady-state growth, but only on per capita income. See also Greiner and Hanush (1998), Bajo-Rubio (1998).

[3] A similar hypothesis is present in all contributions to the New Growth Theory . For a different hypothesis, see Jones and Manueli (1990). For an analysis of the policy implications of endogenous growth theory, see Scott (1992) and Shaw (1992).

[4] See Barro (1990), *op.cit.,* p. 109.

[5] See Barro (1990), p.117.

consider the weight β that the households give to public consumption as an alternative to private consumption. Then β close to zero or β close to unity would be the same! But, if $\beta=0$ (why it would be excluded ? , as in Barro (1990), p.117), which would mean that public consumption services are entirely wasted resources, a lower rate of growth due to such a squandering of resources would be justified. If instead $\beta=1$, then households want public services only and save and invest all their disposable income, with obvious positive effects on growth.

Equation (25) in Barro (1990) shows no evidence of this, in that it considers only the effects of public spending on the productivity of the global (public and private) capital stock, net of income tax. Nor does it consider that the presence of public consumption in the households' utility function modifies the intertemporal elasticity of substitution. This latter cannot be the elasticity of substitution relative to private consumption alone; it must also evaluate the elasticity of substitution relative to public consumption. Consequently, households' consumption and saving choices are modified, with important effects on private investments.

The classification of public expenditures in Barro (1990) and Barro and Sala-i-Martin (1995) distinguishes between all public expenditures that produce externalities generalised to the firms' system and all public consumption services, excluding expenditures on national defence and on public education.

This unusual classification of public expenditures between productive externalities and consumption services calls to mind the analogous distinction between productive and unproductive works drawn by Adam Smith in *Wealth of the Nations.* If applied literally, it would produce some paradoxical situations. For example, the services of a university library (considered as a public expenditure on education) would generate productive externalities and would increase the growth rate, while the services of a municipal library (considered as an expenditure on culture) would be public consumption services that slow down economic growth. Analogously, the services of a swimming-pool used by the army (public expenditure on national defence) would favour economic growth, while the services of a municipal swimming-pool (public expenditure on sport) would slow down it. As a matter of fact, the defence of property rights, national security , education and health are values useful not only for production but also for consumers in general, and for workers in particular.

In my opinion this unusual distinction is unacceptable because it gives rise to confusion and uncertainty in the attribution of public expenditures, both in relation to their use and in relation to whom they favour.

The correct way to reproduce Barro's (1990) analysis is to consider all public expenditures, net of transfers, as useful to households; the exponent β in the utility function defines the relative weight. If public expenditure is useful, it has positive effects on citizens and workers. For example, a higher level of education or of health care enables citizens-workers both to more appreciative of private consumption and to be more productive. Thus useful public expenditure would produce positive externalities on production through greater labour productivity. The exponent α in the production function defines the relative weight of this externality, obviously as an average effect.

This paper examines whether the results obtained by Barro (1990) are confirmed in this different context as well. It assumes that all public expenditure enters both the household utility function and the production function, and it studies the effect on the growth rate of an exogenous income tax rate. It then assumes that the government adopts the economic policies most desired by households and analyses which is the optimal income tax rate and which the optimal growth rate.

2. *A model of endogenous growth with public expenditure.*

I consider a simple model of endogenous growth assuming that the government imposes a proportional income tax rate, $0<\tau<1$ and uses the public budget $g(t)=\tau y(t)$ to furnish both households with public consumption and firms with productive investments.

The production function, in terms of constant labour units,[6] is a Cobb-Douglas with constant returns to scale in private capital and public investment. If the economy knows the exogenous public policy: $g(t) = \tau y(t)$, we can obtain a production function at constant returns to scale in private capital $k(t)$ only:

[6] The units of population are normalised to 1 and the rate of growth of the population is assumed to be nil

1 $$y(t) = ak(t)^{1-\alpha} g(t)^{\alpha} = ak(t)^{1-\alpha} (\tau y(t))^{\alpha} = A(\tau)k(t)$$

where : $A(\tau) = a^{\frac{1}{1-\alpha}} \tau^{\frac{\alpha}{1-\alpha}}$, $A_{\tau} = A(\tau)\frac{\alpha}{(1-\alpha)\tau}$ is the derivative of A(τ) with respect to τ.

Disposable household income is spent on private consumption and investment:

2 $$\dot{k}(t) = (1-\tau)A(\tau)k(t) - c(t)$$

The utility function is hypothesised as being of CRRA type, its arguments being per capita private consumption *c(t)* , with weight (1-β), and per capita public consumption *g(*t)=*τy(t)*, with weight $0 \le \beta \le 1$ [7]:

$$u(t) = \frac{\left[c(t)^{1-\beta} g(t)^{\beta}\right]^{1-\sigma} - 1}{1-\sigma} = \frac{\left[(\tau A(\tau))^{\beta(1-\sigma)} c(t)^{(1-\beta)(1-\sigma)} k(t)^{\beta(1-\sigma)}\right] - 1}{1-\sigma}$$

$$u_c(t) = (\tau A(\tau))^{(1-\sigma)\beta} c(t)^{(1-\beta)(1-\sigma)} k(t)^{\beta(1-\sigma)} \frac{1-\beta}{c(t)}$$

$$u_k(t) = (\tau A(\tau))^{(1-\sigma)\beta} c(t)^{(1-\beta)(1-\sigma)} k(t)^{\beta(1-\sigma)} \frac{\beta}{k(t)}$$

The intertemporal optimum problem of households requires maximisation of the Hamiltonian function:

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[7] In a recent article, Flessing and Rossana (2003) find consistent evidence that private consumption (non durable goods, services, and the stock of durable goods) and government expenditures (federal defence expenditures, federal non-defence expenditures, state and local expenditures) are net substitutes, even if the elasticity of substitution is not high.

$$H[k(t), c(t), \mu(t)] = e^{-\rho t}u(t) + \mu(t)\left[(1-\tau)A(\tau)k(t) - c(t)\right]$$

choosing the optimal consumption path *c(t)* (*control variable*) in respect of the dynamic constraint (equation 2) and the usual non-negativity conditions. We also assume that the economy considers the income tax rate τ to be given and constant; in other words, *fiscal policy is exogenous*.

I introduce the first-order maximum conditions obtained by deriving the Hamiltonian function with respect to the control variable *c(t)* and the state variable *k(t)*:

3 $\quad e^{-\rho t}u_c(t) - \mu(t) = 0$

4 $\quad -\dot{\mu}(t) = e^{-\rho t}u_k(t) + \mu(t)(1-\tau)A(\tau)$

where $\rho > 0$ represents the intertemporal discount rate and $\mu(t)$ is the costate variable.

It is also necessary to impose the limit condition: $\lim_{t \to \infty} \mu(t)k(t) = 0$.

Substituting equation (3) in (4), I obtain the rate of growth of the costate variable:

4a $\quad -\gamma_\mu(t) = \dfrac{\beta}{1-\beta}\dfrac{c(t)}{k(t)} + (1-\tau)A(\tau)$

where $\gamma_x(t)$ represents the rate of growth of a variable x(t).

From the economy's budget constraint, I can obtain the growth rate of capital stock:

2a $\quad \gamma_k(t) = -\dfrac{c(t)}{k(t)} + (1-\tau)A(\tau)$

In steady state, the rate of growth of the capital stock must be constant; by differentiating (2a) with respect to time, one can demonstrate that it must be: $\gamma_c(t) = \gamma_k(t)$.

I take the logarithmic of the condition (3) and derive it with respect to time. Taking account of the above equality between rates of growth, the rate of growth of consumption is derived:

$$\gamma_c(t) = \frac{1}{\sigma}\left[-\gamma_\mu(t) - \rho\right]$$

and substituting (4a) , we obtain:

5 $$\gamma_c(t) = \frac{1}{\sigma}\left[\frac{\beta}{1-\beta}\frac{c(t)}{k(t)} + (1-\tau)A(\tau) - \rho\right]$$

The value of the ratio between consumption and capital stock can be obtained by equalising equations (5) and (2a):

6 $$\frac{c(t)}{k(t)} = \frac{1-\beta}{\sigma + \beta(1-\sigma)}\left[\rho - (1-\sigma)(1-\tau)A(\tau)\right]$$

Note that in order to have *positive* private consumption c(t)/k(t), the following condition must be satisfied *for all values of σ,* also for σ less than one:

7 $$\frac{\rho}{1-\sigma} > (1-\tau)A(\tau) > \rho$$

where (1-τ)A(τ) represents the share of productivity of the capital stock which increases the net income of households.

Finally, substituting (6) in (5) yields the rate of growth of private consumption, which in steady-state is constant and equal to the rates of growth of the per capita capital stock and output :

8 $$\gamma = \frac{1}{\sigma + \beta(1-\sigma)}\left[(1-\tau)A(\tau) - (1-\beta)\rho\right]$$

$$sign\frac{\partial \gamma}{\partial \beta} = sign\left[\frac{\rho}{1-\sigma} - (1-\tau)A(\tau)\right] > 0$$ for condition (7)

$$sign\frac{\partial \gamma}{\partial \tau} = signB_{\tau} = sign(\alpha - \tau) >< 0$$ iff $\tau <> \alpha$

Examination of equation (8) reveals the two channels through which public expenditure influences the economy's rate of growth. It positively affects the productivity of composite capital (private and public), and it modifies the saving decisions of households. If households consider public expenditure to be useful $\beta>0$, then public consumption substitutes private consumption, thus freeing resources to increase private saving and investment (*substitution effect*).

It is interesting to consider the growth rates in the two extreme cases $\beta=1$ and $\beta=0$.

(a) When households regard only public consumption to be useful $\beta=1$, the corner solution foresees null private consumption, and all disposable income is invested. The rate of growth will be:

8a $$\gamma(\tau, \beta = 1) = (1-\tau)A(\tau)$$

(b) When households do not consider public expenditure to be useful, $\beta=0$, the corner solution foresees null public consumption. The growth rate will be:

8b $$\gamma(\tau, \beta = 0) = \frac{1}{\sigma}\left[(1-\tau)A(\tau) - \rho\right]$$

The *case* $\beta=0$ allows comparison to be made with the results obtained by Barro (1990). It follows from (8b) that the economic growth rate is a concave function of the tax rate which reaches its maximum for $\tau^\circ = \alpha$, as in Barro (1990) p.109 and 117:

8c $$\gamma^\circ(\tau^\circ = \alpha, \beta = 0) = \frac{1}{\sigma}\left[a^{\frac{1}{1-\alpha}} (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} - \rho \right]$$

In general, the following proposition holds:

Proposition 1: $\gamma[\tau, \beta > 0] - \gamma[\tau, \beta = 0] > 0$ *for condition (7)*

Whatever the exogenous income tax rate τ, *if households evaluate public expenditure positively, the growth rate* $\gamma(\tau, \beta>0)$ *is always higher than it would be in the case of public productive investments alone* $\gamma(\tau,\beta=0)$ **.**

In conclusion, all public expenditure may be a valid means to promote growth. The reason for this is that the availability of public expenditure in the utility function partially substitutes private consumption and enables households to save and to invest more. This substitution effect will be the more robust, the greater the weight of public consumption β in the utility function.

3. The optimal endogenous fiscal policy.

The foregoing solution of the optimum problem has been obtained under the hypothesis that the income tax rate τ is exogenous and constant. Let us now drop this hypothesis and imagine that the electoral choices of the economy – at least in the long run – may induce governments to establish an optimal income tax rate τ^*. In other words, I consider the government to be a benevolent planner.

For this purpose we must zero-set the prime derivative of the Hamiltonian function with respect to the policy variable τ:

9 $$e^{-\rho t}u_{\tau} + \mu(t)k(t)\left[-A(\tau) + (1-\tau)A_{\tau}\right] = 0$$

where: $u_{\tau}(t) = (\tau A(\tau))^{(1-\sigma)\beta} c(t)^{(1-\beta)(1-\sigma)} k(t)^{\beta(1-\sigma)} \dfrac{\beta}{(1-\alpha)\tau}$

Solving equations (9) and (3), we obtain the ratio between consumption and capital:

10 $$\frac{c(\tau)}{k(\tau)} = \frac{1-\beta}{\beta}(\tau-\alpha)A(\tau) > 0 \qquad \text{for } \tau > \alpha$$

Finally, equalising equations (10) and (6), we obtain the following implicit relation which defines the optimal income tax rate:

11 $$V(\tau^*) = A(\tau^*)\left[\beta(1-\alpha)(1-\sigma) + \sigma(\tau^* - \alpha)\right] - \beta\rho = 0$$

where $V(\tau)$ is an increasing convex function of τ and $V(\tau = \alpha) < 0$. There will therefore be an optimal $\tau^* > \alpha$ such the $V(\tau^*) = 0$.

Moreover, the optimal tax rate depends positively on both β (because of condition 7) and ρ. Hence, as households' impatience increases, the economy prefers greater intervention by the government and is willing to accept a tax rate τ^* higher than $\tau^{\circ} = \alpha$.

In the admissible range of the optimal income tax rate, the optimum growth rate can be rewritten thus:

12 $\alpha < \tau^* < 1$

$$\gamma^*(\tau^*) = \frac{1}{\sigma + \beta(1-\sigma)}\left[(1-\tau^*)A(\tau^*) - (1-\beta)\rho\right]$$

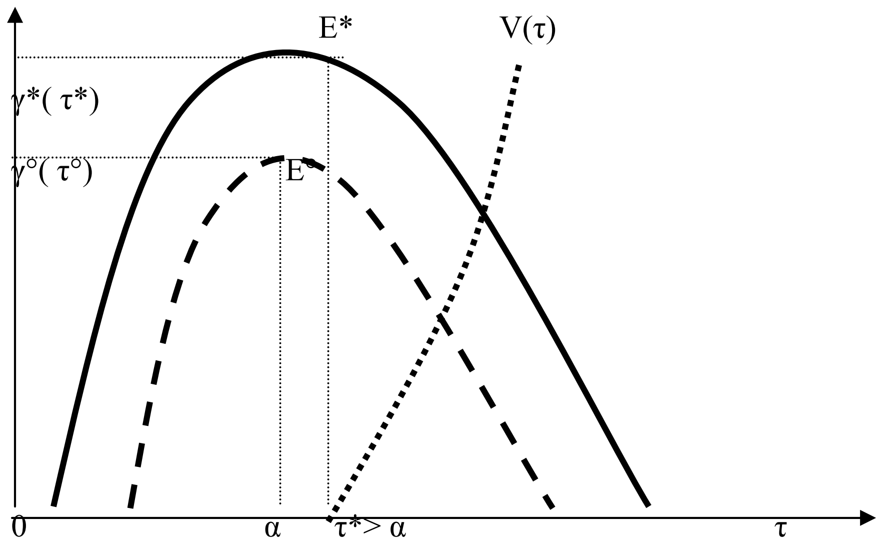
Note that this result contradicts Barro (1990) when he writes: *"The growth rate lies uniformly below the value γ,..., that would have been chosen if τ_h =0"(p.117).*

In general, the following proposition holds:

Proposition 2: $\gamma^*(\tau^* > \alpha) \geq \gamma°(\tau° = \alpha)$

When households choice the optimal tax rate, τ is greater than α and the optimal growth rate γ*(τ*>α) is higher than the maximum growth rate γ°(τ°= α) obtained in the case of public investment only*

Figure 1 *The optimal tax rate V(τ) (dotted line), the optimal rate of growth γ*(τ) (continuous line) and Barro's growth rate γ °(τ) (dashed line) in relation to the income tax rate. E* and E° are respectively the optimal and the maximum rate of growth.*



4. *Conclusion*

The foregoing analysis has enabled me to verify the robustness of Barro's (1990) result that the most powerful policy instrument is public spending on productive investments. When we consider that all public expenditure interacts with private consumption to increase utility, a further channel for influence on the growth rate opens up. Public consumption modifies the economy's saving and investment decisions. Therefore, by applying the same exogenous tax rate, it is possible to obtain a higher growth rate than the one obtained with public investment only. The difference between the two rates of growth is due to a different intertemporal elasticity of substitution which depends on the weight of public consumption β in the utility function. Finally, if we assume that in the long run the government adopts the policies most desired by households, we obtain an optimal income tax rate $\tau^* > \tau^\circ = \alpha$ positively related to β, and the optimal growth rate $\gamma^*(\tau^*)$ may be greater than the maximum one $\gamma^\circ(\tau^\circ)$ deriving from public investment only.

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