THE STUDY OF COMPETITION AMONG LOCAL AREAS: A FUNCTIONAL ANALYSIS APPROACH

FRANCESCO CHELLI

§1. INTRODUCTION

The long and ramified debate in Italy prompted by the rise of metropolitan areas has failed to resolve the issue of the balance to be struck between centralism and polycentrism. Although it is true that the problems of urban living often extend across the administrative boundaries separating municipalities, and therefore require the co-ordinated management of territorially dispersed resources, it is equally true that the interest of a local area, even a small one, sometimes resides in fragmentation, rather than in economic and social integration with a larger area.

In fact, the greater articulation of local powers is only able to increase the economic competitiveness of an area if fair and efficient governance can prevent the triggering of a localist interests chain contrary to the common good.

The article studies interactions in a territorial system with a hierarchy of centres of attraction. It proposes an original methodology for the definition of areas dysfunctional to those centres of attraction – that is, areas which tend in the long run to hamper the evolutionary dynamic of one or more of the poles identified.

Thus laid is the basis for a post-functionalist reading of territorial space in which analysis of the inter-relations among the points of the system considered leads into the broader scenario of territorial competition.

The paper is organised as follows: the second section reviews the criteria used to identify functional regions and introduces the methodology for defining dysfunctional ones; the third section describes three cases of perturbation to the transition matrix; and the fourth proposes an application of the methodology to the matrix of the commuter flows among the municipalities of the Marches region of Italy – as surveyed by ISTAT.

§2. THE METHOD FOR IDENTIFYING FUNCTIONAL AND DYSFUNCTIONAL AREAS

Given a spatial system consisting of n area entities and a transition matrix P, whose generic element p_{ij} expresses the probability of moving from the origin i to the destination j in a given interval of time,
let:

$$\pi = \pi P$$
$$Z = (I - P + \pi)^{-1}$$

be respectively the limit vector and the basic matrix of P.

For the construction of functional areas Brown and Holmes (1971) propose a method based on analysis of the mean first passage time matrix, M, whose generic element:

$$m_{ij} = \sum_k p_{ik} m_{kj} - p_{ij} m_{jj} + 1$$

expresses the mean number of steps necessary to reach, for the first time, the j-th destination starting from i.

Having ranked the points of the system on the basis of their limit probability values, Brown and Holmes (BH) first identify the poles and then define the functional areas by means of a rule which assigns the hierarchically lower units to the latter.
Given any whatever pole, called d, the assignment rule considers the d-th column of the M matrix: $\underline{m}_d$, whose elements are standardised.
Using μ_d and σ_d to denote respectively the arithmetical mean and the standard deviation of the elements of $\underline{m}_d$, the rule can be stated as follows:
Point g is assigned to pole d if:

- g is hierarchically lower than d;
- $q_{gd} = (m_{gd} - \mu_d)/\sigma_d <= s$, (1)
  where the threshold value s, fixed a priori, is set generally equal to -1.

Assigning point g to pole d means considering g in the functional area of d – or in other words, in its area of attraction.
The mean first passage time m_{gd} is a measure of the functional distance between the origin g and the destination d. If this standardized functional distance is below a particular threshold, one may conclude that pole d exerts a certain force of attraction on g.
The pole's force of attraction on a particular point in the system is not a simple expression of the properties of the pole itself; rather, it is the result of comparison between the latter's properties and those of the origin that is attracted.
The nature of the link established between two points therefore concerns the properties that two points possess, and it's from these that so-called 'place utilities' derive.
By utility of place is meant the assessment made by an individual belonging to g of the attraction, or lack of attraction, of all the possible destinations reachable from g. This assessment is made on the basis of subjective perceptions which do not necessarily comply with a principle of economic and spatial rationality.

When individuals perceive, on average, a significant difference between the utility of destination d and that of origin g to which they belong, the propensity to move from g to d is likely to be high and the mean first passage time m_{gd} is likely to be short.
Consequently, the values of m_{gd} can be read as resulting from the decisions that individuals belonging to g take on the basis of their assessments of d.
If we denote the sum of individual utilities with respect to place d, perceived by persons belonging to g, with

$$U_d^{(g)}$$

the difference between the place utilities of destination d and the origin g can be written as:

$$\Delta_{gd} = U_d^{(g)} - U_g^{(g)}$$

It is therefore possible to specify a function h, monotonic and decreasing, which connects the utility differences to the mean first passage times:

$$\Delta_{gd} = h(m_{gd})$$

Rule (1) is therefore susceptible to a further interpretation: assigned to pole d are those origins of inferior rank whose utility differentials exceed a certain threshold as one moves towards the pole.
According to BH, these origins have a behaviour functional to pole d, while the connections between the pole and the other points in the system are considered to exert no influence.
In this way functional analysis neglects an important aspect of the phenomenon being studied. It can be shown in fact, that some connections between the pole and the origins not functional to it are equally important.
The point can be clarified by considering the matrix F:[f_{ij}] of the flows among the n states of a system comprising a pole d, an origin r different from d, and two destinations g and f. A unitary variation, respectively positive and negative, is applied to flows f_{rg} and f_{rf}, so that the sum of the r-th row of F remains constant.
In general, this will trigger a series of chain reactions in the transactions system which will extend to d as well. In order to assess the effects of the variation introduced using functional analysis tools, one must first verify the position of g and f with respect to d's area of influence.
One may therefore argue that if g pertains to pole d, the positive variation will have a favourable effect on the latter; vice versa, if f pertains to it, the effect will be unfavourable. If both g and f pertain to d, contrasting effects will be generated whose result on the pole is uncertain. Finally, when g or f pertain to the functional area of d, the effects of the variation are not significant.
This is not how matters stand, however, and evaluating the effect exerted on d by the alteration, in one direction or the other, of a connection is a rather complex undertaking beyond the scope of functional analysis.
This can be shown by citing a result obtained by Conlisk (1985), who examines the relative position of pole d corresponding to a variation in the interactions among certain points of the system, doing so on the basis of the derivative of the d-th element of the limit vector.

Let us consider the generic row r of a transition matrix P and apply to the elements p_{rg} and p_{rf} a variation (perturbation) equal to ε, ε>0, identical in module but of opposite sign:

$$p_{rg}(\varepsilon) = p_{rg} + \varepsilon$$
$$p_{rf}(\varepsilon) = p_{rf} - \varepsilon.$$

The transition matrix may therefore be written as P(ε)=P+εV where V:[v_{ij}] is such that:

$$\begin{aligned} v_{ij} &= 1, && i=r, j=g; \\ v_{ij} &= -1, && i=r, j=f; \\ v_{ij} &= 0, && \textit{otherwise.} \end{aligned}$$

This perturbation will trigger a series of chain reactions which will contribute to determining a different limit vector π(ε) associated with P(ε).
The variation of the d-th element of **π(ε)**:**π(ε)**$_d$ (in the case in which g≠d and f≠d) is equal to:

$$\begin{aligned} \frac{\partial \pi(\varepsilon)_d}{\partial \varepsilon} &= (m_{fd} - m_{gd})\pi_r \pi_d \\ &= (z_{gd} - z_{fd})\pi_r \end{aligned} \qquad (2)$$

where z_{ij} is the generic element of the basic matrix Z.
Let us seek to interpret this result. Let us consider the average number of time s that the process finds itself in j in the first v steps (including the initial position) starting in any state and denote it with:

$$M_i[\overline{y_j^{(v)}}]$$

This is, I believe, a good approximation of the place utility differential. In fact, the higher the sum of individual perceptions of utility in moving from the origin i to the pole j, the greater the average number of visits that j will receive from i.
On the other hand, it can be shown that if the Markov chain is regular, then:

$$M_g[\overline{y_d^{(v)}}] - M_f[\overline{y_d^{(v)}}] \to z_{gd} - z_{fd}, per v \to \infty$$

Hence (2) is directly proportional to the differential of average presences in d for the two different initial states g and f, with a proportionality factor equal to π_r.
The sign and the magnitude of the variation in **π(ε)**$_d$ consequent on a perturbation ε of the transition probabilities therefore depends, besides π_r, on the difference between the average of presences in d of individuals originating from g and f respectively.
In other words, the advantage to d deriving from the perturbation grows with an increase in the utility differential from the origin g and with an decrease in that from f.
The foregoing suffices to state that definition of a pole's advantage cannot be made in purely functional terms but should be framed in a broader context. Consequently, a method which restricts analysis to identification and study of only the points functional to the pole cannot be considered complete.
There exists, in fact, a second category of origins distant from d and therefore characterized by a low utility differential whose action contrasts, in the long period, with the pole's territorial dominance.
In particular, we may state the following:

- origins g with high utility differentials with respect to pole d (low mean first passage times) are those located in the latter's functional area (Afu), and they can therefore be identified by means of rule (1) of the BH method;

- origins f with low utility differentials with respect to pole d are not simply the origins different from g, or in other words those that do not belong to d's functional area; instead, they are only those whose mean first passage times are above a certain threshold.

The rule for identifying the origins f is straightforwardly derived from (1) and may be written as follows:

Point f is an origin with a low utility differential for pole d if:

- f is of lower hierarchical rank than d;
- $q_{fd} = (m_{fd}-\mu_d)/\sigma_d >= u$, (3)

where the value of threshold u can be established *a priori*, by symmetry with rule (1), at around unity.

Origins f are characterized by their antagonism against pole d. In other words, the benefits deriving from strengthening the links with these origins give rise to decidedly higher costs for the pole: hence the expression 'pole's *dysfunctional* area' that can be applied to the set of origins f identified by rule (3).

§3. THE PERTURBATIONS

Introduced in the previous section was perturbation as a variation in certain elements of the transition matrix P.
Theoretically, therefore, a very large number of perturbations can be applied to P even if its dimensions are reduced.
Consequently, in order to construct a theoretical reference scheme for the derivative of the limit vector, one most choose few and significant perturbations from the many that are possible.
Now illustrated are three cases which, I believe, are important from the point of view of variations. In these cases the variations concern:

a. only one row and two columns;

b. all the rows and two columns (column gain);

c. only one row and several columns.

For the sake of completeness, added to these is the more general case in which the variations are applied to:

d. all the rows and several columns.

Let us now define the admissibility conditions: a perturbation is admissible of for those values of ε such that P(ε) respects the conditions imposed on a transition matrix.
In the above-cited study, Conlisk considers two different types of perturbations, elementary and composite. In both cases the perturbed matrix P(ε) is a linear function of the variable ε.
A perturbation is called 'elementary' if, taking a ε>0, all the variations are in module equal to ε. A perturbation is called 'composite' if, taking a ε>0, each variation is expressed by the product of ε and a non-null coefficient.
It is evident that the elementary perturbation is a particular case of the composite one with coefficients equal to 1.
In studying the above-listed cases, therefore, it was decided not to analyse the elementary perturbation in order to avoid unnecessary duplication.
In case (a), where the perturbation concerns only one row and two columns, we have:

$$p_{rg}(\varepsilon) = p_{rg} + \varepsilon w$$
$$p_{rf}(\varepsilon) = p_{rf} - \varepsilon w.$$

The transition matrix can therefore be written as P(ε)=P+εV where V:[v_{ij}] is such that:

$$\begin{aligned} v_{ij} &= w, & i = r, j = g; \\ v_{ij} &= -w, & i = r, j = f; \\ v_{ij} &= 0, & \textit{otherwise.} \end{aligned}$$

The derivative

$$\frac{\partial \pi(\varepsilon)}{\partial \varepsilon} = \pi V Z,$$

is a row vector whose d-th element is equal to:

$$\frac{\partial \pi(\varepsilon)_d}{\partial \varepsilon} = \pi_r w(z_{gd} - z_{fd}).$$

In case (b), where the composite perturbation concerns all the rows and only two columns of P, we have:

$$p_{rg}(\varepsilon) = p_{rg} + \varepsilon w_r, \qquad r = 1,\ldots,n;$$
$$p_{rf}(\varepsilon) = p_{rf} - \varepsilon w_r, \qquad r = 1,\ldots,n.$$

The transition matrix can therefore be written as P(ε)=P+εV where V:[v_{ij}] is such that:

$$v_{ij} = w_i, \qquad i = 1,\ldots,n, \quad j = g;$$
$$v_{ij} = -w_i, \qquad i = 1,\ldots,n, \quad j = f;$$
$$v_{ij} = 0, \qquad otherwise.$$

The derivative

$$\frac{\partial \pi(\varepsilon)}{\partial \varepsilon} = \pi V Z,$$

is a row vector whose d-th element is equal to:

$$\frac{\partial \pi(\varepsilon)_d}{\partial \varepsilon} = (z_{gd} - z_{fd}) \sum_{i=1}^{n} \pi_i w_i.$$

In case (c), with only one row and several columns, the composite perturbation affects P as follows:

$$p_{rg}(\varepsilon) = p_{rg} + \varepsilon w_{rg}, \qquad g = g_1,\ldots,g_k;$$
$$p_{rf}(\varepsilon) = p_{rf} - \varepsilon w_{rf}, \qquad f = f_1,\ldots,f_h.$$

The transition matrix can therefore be written as P(ε)=P+εV where V:[v_{ij}] is such that:

$$v_{ij} = w_{ij}, \qquad i = r,\ j = g_1,\ldots,g_k;$$
$$v_{ij} = -w_{ij}, \qquad i = r,\ j = f_1,\ldots,f_h;$$
$$v_{ij} = 0, \qquad otherwise.$$

Because of the admissibility condition:

$$\sum_j p(\varepsilon)_{rj} = 1$$

and therefore

$$\sum_{g=g_1,\ldots,g_k} w_{rg} - \sum_{f=f_1,\ldots,f_h} w_{rf} = 0.$$

The derivative

$$\frac{\partial \pi(\varepsilon)}{\partial \varepsilon} = \pi V Z,$$

is a row vector whose d-th element is equal to:

$$\frac{\partial \pi(\varepsilon)_d}{\partial \varepsilon} = \pi_r \Big(\sum_{g=g_1,\dots,g_k} w_{rg} z_{gd} - \sum_{f=f_1,\dots,f_h} w_{rf} z_{fd} \Big).$$

The composite perturbation in case (d), with all rows and several columns, is:

$$\begin{aligned} p_{rg}(\varepsilon) &= p_{rg} + \varepsilon w_{rg}, \qquad r = 1,\dots,n,\ \ g = g_1,\dots,g_k; \\ p_{rf}(\varepsilon) &= p_{rf} - \varepsilon w_{rf}, \qquad r = 1,\dots,n,\ \ f = f_1,\dots,f_h. \end{aligned}$$

The transition matrix can therefore be written as P(ε)=P+εV where V:[v_{ij}] is such that:

$$\begin{aligned} v_{ij} &= w_{ij}, \qquad i = 1,\dots,n,\ j = g_1,\dots,g_k; \\ v_{ij} &= -w_{ij}, \qquad i = 1,\dots,n,\ j = f_1,\dots,f_h; \\ v_{ij} &= 0, \qquad \textit{otherwise}. \end{aligned}$$

In this case too, due to the admissibility condition:

$$\sum_j p(\varepsilon)_{rj} = 1, \qquad r = 1,\dots,n$$

and therefore

$$\sum_{g=g_1,\dots,g_k} w_{rg} - \sum_{f=f_1,\dots,f_h} w_{rf} = 0, \qquad r = 1,\dots,n.$$

The derivative

$$\frac{\partial \pi(\varepsilon)}{\partial \varepsilon} = \pi V Z,$$

is a row vector whose d-th element is equal to:

$$\frac{\partial \pi(\varepsilon)_d}{\partial \varepsilon} = \Big(\sum_{g=g_1,\dots,g_k} z_{gd} \sum_{i=1}^{n} \pi_i w_{ig} - \sum_{f=f_1,\dots,f_h} z_{fd} \sum_{i=1}^{n} \pi_i w_{if} \Big).$$

§4. THE STUDY OF A TERRITORIAL CASE

We have seen that flanking Brown and Holmes' method with Conlisk's result on the derivative of the limit vector of a perturbed transition matrix P(ε) enables identification of areas which tend in the long period to hamper the evolutionary dynamic of the pole, and which may therefore be termed 'dysfunctional' to the latter.
To date, analysis of interactions in a particular territorial system has gone no further than dividing the space considered into functional areas, delimited by ideal boundaries and sometimes overlapping, each of them consisted of elementary entities aggregated together by the force of attraction exerted by a pole.
The functionalist literature on real space has therefore been trapped within its own confines, being unable to see any other significant inter-relations lying outside them.
Analysis of the derivative of the limit vector extends the boundaries of the pole's functional area, and it lays the basis for analysis of the relations among the points of the system within the broader scenario of territorial competition.
Gaining greatest advantage from this competition will be the party that, within the area of its influence, is best able to attract flows away from its competitors. In other words, the outcome of the competition depends on the pole's ability to perturb the transition matrix to its own advantage.
Described in the previous section was a class of perturbations εV which was linear in ε and admissible – that is, such that P(ε)=P + εV is still a transition matrix.
The next step consists in defining, in terms of ε, the problem of the concrete applicability of the perturbation to the matrix P: the constraints imposed by the admissibility are not in fact sufficient to guarantee respect by perturbation of the economic principles which regulate the dynamic of the phenomenon described by the transition matrix P.
It is therefore necessary to impose a further condition – called the 'plausibility' condition – which keeps the variations caused by the perturbation within the limits suggested by economic and social considerations. The poles, in fact, will not move in accordance with an abstract model of behaviour, but will rather do so along the lines that their political and economic function makes practicable.
This section applies the methodology with which to identify functional and dysfunctional areas to the matrix of commuter flows among the 246 municipalities of the Marches surveyed by the 1991 ISTAT Census of the Population. Also studied will be criteria on the basis of which the pole may orient its strategy in order to increase its limit probability. Having obtained the transition matrix P, the BH procedure can be used to rank the states on the basis of the limit vector π and to define the class of poles as that comprising the highest ranking municipalities (see Table 1).

Municipalities	π_i
ANCONA	0,1504
OSIMO	0,0449
FALCONARA MARITTIMA	0,0441
JESI	0,0427
MACERATA	0,0380
PESARO	0,0351
CAMERANO	0,0300
CIVITANOVA MARCHE	0,0262
CASTELFIDARDO	0,0253
FABRIANO	0,0227
RECANATI	0,0206

Table 1 – Highest-ranking municipalities – poles – with respect to the limit vector

The municipality of Ancona, which occupies first place in the regional ranking, was the most important pole in our application.
We applied rules (1) and (3) to it in order to identify its functional and dysfunctional areas, respectively.

Figure 4 – Ancona's functional (white) and dysfunctional (black) areas

To be noted first is that the pole in question has two clearly-defined dysfunctional areas, one to the south which we shall call the 'Fermo area' and one to the north, the 'Urbino' area. These are not contiguous to the functional area but are separated from it by municipal zones not classified as either of the two types examined.
Also to be noted is that the dysfunctional areas are not the ones most geographically distant from the pole: that is, there is not a perfect correspondence between functional distance and spatial distance.
As said, it is advisable to introduce a number of general criteria – and which are therefore also valid in different contexts – which orient the choice of the rows and columns to perturb while ensuring that the manoeuvre achieves the objectives pursued by the pole.
The rows to be perturbed should be selected from among those with direct links with both the pole's functional area and its dysfunctional area, for two reasons. First because the presence of links with the functional area means that positive variations increase already-existing flows, and does not create new ones, so that the plausibility of the manoeuvre is ensured. Second because the presence of links with the dysfunctional area is a necessary condition for the elements of the perturbed matrix to be admissible when a negative variation is applied.
Among the rows of the transition matrix with the necessary requirements, priority should be give to those:

- connected to municipalities in the functional area whose mean first passage times assume very low values, and to municipalities in the dysfunctional area whose mean first passage times to the pole are, vice versa, very high;
- geographically closer to the pole's functional area than to its dysfunctional area.

In this manner, in fact, the choice of the elements of P is conditioned to maximization of (i) the advantage that the pole derives from the perturbation, and (ii) the utility of the commuters who, following the perturbation, find that the total distance that they travel has been reduced.
For the Ancona pole, the elements of P to perturb must be selected separately for each dysfunctional area according to the criteria just outlined.
Tables 2 and 3 show the transition probabilities of some of the municipalities in the Fermo (perturbation's case c) and Urbino (perturbation's case a) dysfunctional areas affected by the manoeuvre and identified by the procedure described above .

.

		g_1	g_2	g_3	g_4	f_1	f_2	f_3	f_4
		CAMERANO	NUMANA	OSIMO	SIROLO	FERMO	M.GRANARO	P.TO S.ELPIDIO	S.ELPIDIO M.
r_1	PORTO RECANATI	0,0082	0,0148	0,0252	0,0193	0,0015	0,0045	0,0052	0,0074
r_2	POTENZA PICENA	0,0052	0,0007	0,0052	0,0059	0,0081	0,0214	0,0037	0,0251
r_3	RECANATI	0,0152	0,0020	0,0461	0,0112	0,0025	0,0025	0,0010	0,0025

Table 2 – Submatrix P for the Fermo dysfunctional area

		G_5	f_5
		FALCONARA	URBINO
r_4	JESI	0,0082	0,0015

Table 3 – Submatrix P for the Urbino dysfunctional area

The matrix of coefficients V thus assumes the following form:

$$
\begin{aligned}
v_{ij} &= w_{ij}, & i &= r_1,\ldots,r_3,\ j = g_1,\ldots,g_4;\\
v_{ij} &= -w_{ij}, & i &= r_1,\ldots,r_3,\ j = f_1,\ldots,f_4;\\
v_{ij} &= w, & i &= r_4,\ j = g_5;\\
v_{ij} &= -w, & i &= r_4,\ j = f_5;\\
v_{ij} &= 0, & & \textit{otherwise.}
\end{aligned}
$$

The criterion adopted to determine the 26 elements of the matrix of coefficients V required by the manoeuvre is that the admissible impact on the perturbed matrix P(ε) should be maximum for ε=1.
This is straightforwardly obtained by assigning to the 13 elements of V relative to the dysfunctional areas values which are the reverse of those of the corresponding transition probabilities (Tables 4 and 5).
Of the positive elements of V for the functional area, the one corresponding to the Jesi row could be measured univocally because of the row constraint (Table 4), while for the others it was necessary to introduce a plausible calculation criterion.

		G_5	f_5
		FALCONARA	URBINO
r_4	JESI	0,0015	-0,0015

Table 4 – Submatrix V for the Urbino dysfunctional area.

The criterion used, which respected the row constraint, was of proportional type, viz.:

$$w_{rj} = p_{rj} \frac{\sum_{f=f_1,\ldots,f_4} p_{rf}}{\sum_{g=g_1,\ldots,g_4} p_{rg}}, \quad r = r_1,\ldots,r_3, j = g_1,\ldots,g_4;$$

The values thus calculated are set out in Table 5.

Origin \ Destination		g_1	g_2	g_3	g_4	f_1	f_2	f_3	f_4
		CAMERANO	NUMANA	OSIMO	SIROLO	FERMO	M.GRANARO	P.TO S.ELPIDIO	S.ELPIDIO M.
r_1	PORTO RECANATI	0,0022	0,0041	0,0069	0,0053	-0,0015	-0,0045	-0,0052	-0,0074
r_2	POTENZA PICENA	0,0177	0,0025	0,0177	0,0202	-0,0081	-0,0214	-0,0037	-0,0251
r_3	RECANATI	0,0018	0,0002	0,0053	0,0013	-0,0025	-0,0025	-0,0010	-0,0025

Table 5 – Submatrix V for the Fermo dysfunctional area

The first result to be checked was the sign of the derivative of $\pi(\varepsilon)$, i.e. the change in the direction of the probability limit induced by the perturbation. Figure 5 shows the derivatives for the 246 municipalities of the Marches in aggregate form. Their range of variation has been divided into three intervals: the first for values less than -0.00001(white); the second for values between -0.00001 and 0.00001(grey); and the third for values of more than 0.00001(black). Simple indication of the sign of the derivative, in fact, seemed inadequate to express the direction of the change for values in a very small neighbourhood of zero.
The result is reassuring, not only for Ancona but also for almost all the municipalities in its functional area, whose positive derivatives go in the manoeuvre's expected direction. The exceptions are Serra San Quirico, which displays a negative sign, and Staffolo, Poggio San Marcello, Mergo, Castel Colonna, Poggio San Vicino and Genga, whose variations from the limit probability are not appreciably different from zero.

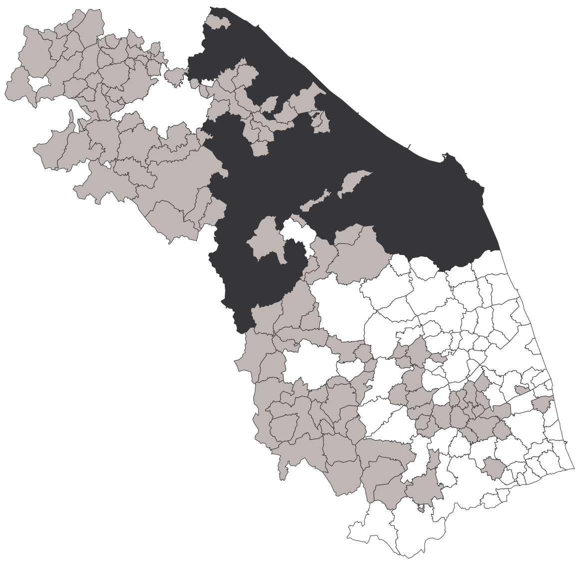

Figure 5 – Derivatives of the limit vector $\pi(\varepsilon)$ at the municipal level

Outside Ancona's functional area, the municipalities with positive variations are located north of the pole, and those with negative variations to its south, with the exception of Urbino.
We now move to estimation of what might be the long-period effects of a real perturbation.
As said, matrix V was constructed in order that it exerted the maximum admissible impact on P when the variable ε assumes value 1, so that for any positive value of ε no higher than 1, P(ε) was still a transition matrix.
The analysis of plausibility instead requires some sort of objective benchmark. To simplify the treatment, we may say that on the basis of the procedure for calculating the coefficients of V used thus far, the decreased amount of the flow from the origins considered ($r_1 \ldots r_4$), to the municipalities in the dysfunctional area is equal to the value of ε.
For example, if ε is set equal to 0.25, the flow decreases by 25%, which in absolute value is equivalent to 32 commuters.
In Figure 6, the ordinate represents the variables: number of commuters re-routed from the dysfunctional area to the functional area; absolute variation in commuters induced by the perturbation in the municipality of Ancona; and absolute variation in commuters induced by the perturbation in Ancona's functional area. The abscissa is the variable ε.
The relations are linear. Consequently, the proportionality relationships of the variables with respect to the values of ε are constant and respectively equal to 128, 364 and 1293.
In other words, every commuter re-routed by the perturbation is replaced, in the long period, by a further 3 in the municipality of Ancona, and a further 10 in its functional area.
The results therefore seems of a certain interest, especially if one considers that the initial flows to be re-routed were relatively modest (for any positive value of ε no greater than 1) and therefore did not obstruct the manoeuvre in question.

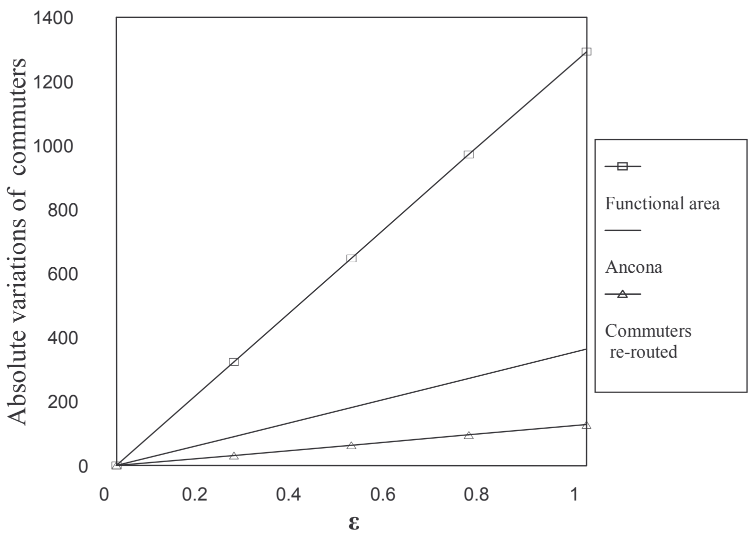

Figure 6 – Absolute variations in the number of commuters induced by the perturbation

Alternatively, the efficacy of the perturbation can be measured by the index of variation of the limit probability:

$$\Delta\pi(\varepsilon)_d = \frac{\pi(\varepsilon)_d - \pi_d}{\pi_d}$$

which is a linear function of ε.
The index in question, calculated both for the municipality of Ancona and for its entire functional area, increases with variation in ε at constant rates which are respectively equal to 1.58 and 1.59 percentage points.

§4. CONCLUSIONS

This article has laid the basis for superseding the functionalist approach to the study of interactions among territorial entities belonging to a particular geographical area. It has used Conlisk's result concerning the derivative of the limit vector of a perturbed transition matrix P (ε) to obtain a generalization of Brown and Holmes' method which enables identification, besides functional regions, also of regions that hamper the pole's evolutionary dynamic and which may be termed 'dysfunctional' to it.
First described was a class of linear perturbations, εV, able to ensure that the perturbed matrix P(ε)=P+εV is still a transition matrix (admissibility condition). Then introduced was the concept of effective applicability: it is reasonable, in fact, to require the perturbation's respect for the economic principles which regulate the dynamic of the phenomenon described by the transition matrix P.
The methodology described was applied to the transition matrix derived from the interactions for the purposes of commuting among the municipalities of the Marches region surveyed by ISTAT. Thus identified were two areas dysfunctional to the pole of Ancona. Interestingly, the areas identified were neither contiguous to the pole's functional region nor most geographically distant from it: that is, there was no perfect correspondence between functional distance and spatial distance.
Finally, it was estimated that every commuter re-routed by a perturbation from the pole's dysfunctional area to its functional one contributes to the growth of the latter to a tenfold greater extent.

Bibliografia

Brown L. A. , Odland J. e Golledge R. G. , "Migration, Functional Distance and the Urban Hierarchy", Economic Geography, 46, 1970.

Brown L. A. e Holmes J. , "The delimitation of functional regions, nodal regions, and hierarchies by functional distance approaches", Journal of Regional Science, vol. 11, n.1, 1971.

Chelli F. , Mattioli E. e Merlini A. , "La delimitazione di aree e regioni metropolitane: due metodi a confronto", Atti della IX Conferenza Italiana di Scienze Regionali, AISRe, vol II, Torino, 1988.

Chelli F. , Mattioli E. e Merlini A. , "Le regioni metropolitane dello Stretto e loro caratteristiche", in "La regione metropolitana dello stretto di Messina. Evoluzione e caratteristiche socio-economiche" a cura di O. Vitali, ESI, Napoli, 1991.

Chelli F. e Rosti L., "Age and Gender Differences in Italian Workers Mobility", International Journal Of Manpower, n. 5, Vol 23, Settembre 2002.

Conlisk J. , "Comparative statics for Markov chains", Journal of Economic Dynamics and Control, 9, 1985.

Conlisk J. , "Monotone Mobility Matrix", Journal of Mathematical Sociology, Vol. 15(3-4), 1990.

Dardanoni V. , "Measuring Social Mobility", Journal of Economic Theory, 61, 1993.

Ellison G. , "Basins of Attraction, Long-Run Stochastic Stability, and the Speed of Step-by-Step Evolution", Review of Economic Studies, 67, 2000.

Kemeny J. G. , "Generalization of a Fundamental Matrix", Linear Algebra and its Applications, 38, 1981.

Kemeny J. G. e Snell J. L. , "Finite Markov Chains", Van Nostrand, 1960.

Seneta E. , "Non-negative Matrices and Markov Chains", Springer-Verlag, Berlin, 1981.