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THE MEASUREMENT OF COHERENCE IN THE EVALUATION OF CRITERIA AND ITS EFFECT ON RANKING PROBLEMS ILLUSTRATED USING A MULTICRITERIA DECISION METHOD

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ABSTRACT

This paper aims to pursue two closely connected purposes. The first is to provide a theoretical framework, based on coherence constraints, for a technique of multicriteria analysis that allows to convert a partial pre-order into a total pre-order. The second it to show the possibility of measuring statistically the amount of modification implicitly made to the evaluations concretely expressed in the binary comparisons in order to make them coherent. This information may be useful both to the decision-maker who may wish to redefine certain evaluation criteria, and to the user of the ranking obtained with the multicriteria method, in order to determine its reliability.

Keywords : Multicriteria decision analysis, Ranking methods.

Contents

The measurement of coherence in the evaluation of criteria and its effect on ranking problems illustrated using a multicriteria decision method

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1. Introduction

The problem of ranking a set of entities – projects, territorial units, price indexes, economic policy measures, and the like – while simultaneously taking account of several assessment criteria associable with the individual units, and of the importance assigned to each of these criteria, has been subject to a large number of studies which adopt highly diverse theoretical positions and analytical approaches. In order to provide a reference scheme for the exposition in the rest of this paper, it is useful to distinguish among situations in which:

- information is available regarding both the evaluation of the criteria and their importance. In this regard, here I do no more that refer to the large body of knowledge on the construction of index numbers of prices and quantities;
- information on the evaluation of the criteria and their importance is implicitly expressed in the preferences between pairs of units. In this case, I refer to one of the broadest strands of analysis, which comprises the studies and applications of the Analytic Hierarchy Process (AHP);
- situations intermediate to the former are treated, and problems are addressed using multicriteria analysis methods.

My analysis is conducted within this last ambit, and it will have two purposes:

- the first is to furnish a theoretical justification to an empirical technique by which a partial pre-order, obtained using a multicriteria analysis method, is transformed into a total pre-order;[1]

[1] Note that the partial pre-order relation may comprise both non-comparable elements and comparable ones, whereas in the complete pre-order relation all the elements are comparable.

– the second, which is of greater practical interest, consists in the possibility of measuring statistically the amount of modification implicitly made to the evaluations concretely expressed in the binary comparisons in order to make them coherent.

This is a set of information useful both to the decision-maker who may wish to redefine certain evaluation criteria, and to the user of the ranking obtained with this multicriteria method, in order to determine its reliability.

In the next section I shall briefly outline the additive comparison scheme implicit in the multicriteria method analysed in the rest of paper, considering it in analogy with the better known and more widely used multiplicative method. The third section deals succinctly with those aspects of the PROMETHEE method necessary to define the notation and concepts used in the fourth and fifth sections, which set out the results of the paper.

2. The multiplicative and additive comparison schemes

Let $A = \{ a_1, \ldots, a_n \}$ be the set of entities or, in equivalent terms, the collective of units that are to be ranked, and let $c_m: A^2 :\rightarrow \hat{\mathbf{A}}^+$ be the preference function between two entities defined following the multiplicative scheme. In this case:

$$c_m(a_i, a_j) = r_{ij}; \quad i,j=1, \ldots, n, \tag{1}$$

measures *how many times* alternative a_i is preferred to alternative a_j. In this case:

$$r_{ij} > 1 \;\Rightarrow\; a_i \text{ is preferred to } a_j: \quad a_i \succ a_j ; \tag{2.1}$$

$$r_{ij} < 1 \;\Rightarrow\; a_j \text{ is preferred to } a_i: \quad a_i \prec a_j ; \tag{2.2}$$

$$r_{ij} = 1 \;\Rightarrow\; a_i \text{ is indifferent to } a_j: \quad a_i \sim a_j . \tag{2.3}$$

Coherence between the bilateral comparisons requires satisfaction, besides the identity property:

$$r_{ii} = 1 ; \quad i=1, \ldots,n , \qquad 3)$$

also of the reversibility property:

$$r_{ij} = 1/r_{ji} ; \qquad i, j=1, \ldots, n , \tag{4}$$

while the coherence between multiple comparison requires also the transitive propriety:

$$r_{ij} = r_{ik}r_{kj} ; \qquad i, j = 1, \ldots, n . \tag{5}$$

As is well known, in this case it is possible to rank the entities a_i by assigning to each of them a value w_i determined up to a multiplicative constant which depends on the choice of the unit of measurement, so that:

$$r_{ij} = w_i/w_j; \qquad i, j = 1, \dots, n. \tag{6}$$

If instead the additive scheme is followed, the assessment of the preferences between two entities is expressed by a function $c_a: A^2 :\rightarrow \hat{\mathbf{A}}$,

$$c_a(a_i,a_j) = s_{ij}; \qquad i, j = 1, \dots, n . \tag{7}$$

which measures *to what extent* alternative a_i is preferable a_j.

In this context:

$$s_{ij} > 0 \;\Rightarrow\; a_i \text{ is preferred to } a_j: \quad a_i \succ a_j ; \tag{8.1}$$

$$s_{ij} < 0 \;\Rightarrow\; a_j \text{ is preferred to } a_i: \quad a_i \prec a_j ; \tag{8.2}$$

$$s_{ij} = 0 \;\Rightarrow\; a_i \text{ is indifferent to } a_j: \; a_i \sim a_j . \tag{8.3}$$

The properties of identity and reversibility and the transitive property are respectively expressed by the relations:

$$s_{ii} = 0 , \qquad i = 1, \dots,n ; \tag{9}$$

$$s_{ij} = - s_{ji} , \qquad i, j = 1, \dots,n ; \tag{10}$$

$$s_{ij} = s_{ik} + s_{kj} , \qquad i, j, k = 1, \dots,n . \tag{11}$$

In this context too, if the preference function (7) satisfies the transitive property (11), it is possible to rank the entities a_i by associating with each of them a value v_i determined up to an additive constant, so that:

$$s_{ij} = v_i - v_j \; ; \qquad i, j = 1, \dots,n . \tag{12}$$

The formal connection between the multiplicative and additive schemes is easily stated: suffice it to consider the logarithmic transform of relations (1)–(6) to link them to the corresponding relations of the additive scheme. Vice versa, performing an exponential transform of the latter brings us back to the relations of the multiplicative scheme.

However, a distinction should be drawn between the formal analogies and the substantial aspects that induce a preference for one scheme rather than the other according to the problem to be addressed. Once this choice has been made, however, one must proceed with logical coherence in defining the indicators measured on the individual units. Particular account should be taken of the linkages between the functional specification of the indicators, according to whether they are associated positively or negatively with desirability, and the synthesis functions used to perform the comparisons and rank the units. This, I believe, is an aspect of fundamental importance which is too often neglected. For particularly wide-ranging and significant treatment of it, as regards theory and practical application, see Vitali and Merlini (2001), and for certain essential specifications developments the recent study by Merlini (2001).

In what follows I shall examine the PROMETHEE[2], one of the best known and most widely used methods of multicriteria analysis,[3] to show that it can been related to the coherent additive scheme. I shall also show that the passage from PROMETHEE I to PROMETHEE II, proposed as an empirical procedure with which to construct a total pre-order from a partial one, can be formalized by imposing constraints (9) to (11) – the coherence constraints for a total pre-order relation – on the preference functions (7).

3. Outline of the PROMETHEE method

Evaluation of preference between two generic units emerges from the joint consideration of a set of indicators or decision criteria:

$$f_k : A \to \hat{\mathbf{A}}, \qquad k=1, \ldots, K. \tag{13}$$

In order to make choices, however, it is necessary to be able to compare pairs of units and to decide which of the two is preferred. To this end, a preference function P_k is introduced which indicates the intensity of one alternative (a_i) over another (a_j) for each indicator f_k:

$$P_k : A^2 \to [0, 1]. \tag{14}$$

This preference function is normally considered as a function of the difference between the evaluations of a_i and a_j performed with the decision criterion f_k:

[2] Acronym for Preference Ranking Organisation METHod for Enrichment Evaluation.

[3] Note that this method is still considered a referent in the study and verification of other methods of multicriteria analysis: see on this Leyva-Lopez and Fernandez-Gonzalez (2003).

$$P_k(a_i, a_j) = \begin{cases} 0 & \text{se } f_k(a_i) \le f_k(a_j) \\ \psi_k(f_k(a_i) - f_k(a_j)) & \text{se } f_k(a_i) > f_k(a_j) \end{cases} \quad (15)$$

where Ψ is a non-decreasing function.[4]

The PROMETHEE method yields a first synthesis of the evaluations by means of multicriteria preference indices defined as:

$$s_{ij} = s(a_i,a_j) = \sum_{k=1}^{K} P_k(a_i, a_j)\pi_k = \sum_{k=1}^{K} p_{kij}\,\pi_k \quad (16)$$

where π_k, k=1, …, K are positive weights adding to one and reflecting the relative importance attached by a decision maker to the criteria P_k.

The indices (16) may be considered as elements in an n-order matrix **S:**

$$\mathbf{S} = [s_{ij}] \quad (17)$$

Each index s_{ij} measures the intensity of the preference for unit a_i over a_j considering all the criteria simultaneously.[5]

To illustrate the treatment so far, Table 1 shows the evaluation of the indices (17) exemplified in the work by Brans and Vincke (1985), henceforth BV85, and frequently cited by studies on multicriteria analysis.

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[4] The analytical specifications of these preference functions are such that any problem of concrete practical relevance can be handled. Besides those initially proposed by Brans and Vincke (1985), see the preference functions devised by Diakoulaki and Koumontsos (1991) and by Huylenbroeck (1995).

[5] Note that the additive nature of these indices entails as regards the choice of the functions fk, depending on whether these are associated positively or negatively with desirability, the coherence constraint stated by Merlini (2001, Theorem 1, p. 137) on the basis of the duality principle proposed by that author.

Table 1 – Multicriteria preference indices s_{ij}. BV85 data.							
	a_1	a_2	a_3	a_4	a_5	a_6	t_i
a_1	0	0.296	0.250	0.268	0.100	0.185	1.099
a_2	0.462	0	0.389	0.333	0.296	0.500	1.980
a_3	0.236	0.180	0	0.333	0.056	0.429	1.234
a_4	0.399	0.505	0.305	0	0.223	0.212	1.644
a_5	0.444	0.515	0.487	0.380	0	0.448	2.274
a_6	0.286	0.399	0.250	0.432	0.133	0	1.500
u_i	1.827	1.895	1.681	1.746	0.808	1.774	

Table 1 also shows the sums:

$$t_i = \sum_{j=1}^{n} s(a_i, a_j) = \sum_{j=1}^{n} s_{ij} \quad \text{e} \quad u_i = \sum_{j=1}^{n} s(a_j, a_i) = \sum_{j=1}^{n} s_{ji} \tag{18.1}$$

which respectively measure the intensity of the preferences for the generic i-th unit over all the others, and the preference of all the others with respect to the i-th. It is convenient to write (18.1) in the more compact form:

$$\mathbf{t} = \mathbf{Si} \quad \text{e} \quad \mathbf{u} = \mathbf{S'i}\ , \quad , \tag{18.2}$$

where **i** denotes a vector with all the unitary components.

As is evident from the exemplification in Table 1, and as normally happens in practice, the multicriteria preference indices s_{ij} satisfy neither the reversibility property (10) nor the transitive property (11), so that they are not coherent for ranking the units as in (12).

Recall that the PROMETHEE I method enables to construction of a partial preference pre-order using the following rules:

$[(t_i>t_j)\cap(u_i<u_j)]\cup[(t_i>t_j)\cap(u_i=u_j)]\cup[(t_i = t_j)\cap(u_i<u_j)] \Rightarrow a_i \succ a_j$ (a_i is preferred to a_j);

$[(t_i<t_j)\cap(u_i>u_j)]\cup[(t_i<t_j)\cap(u_i=u_j)]\cup[(t_i= t_j)\cap(u_i>u_j)] \Rightarrow a_i \prec a_j$ (a_j is preferred to a_i),

$(t_i = t_j) \cap (u_i = u_j) \Rightarrow a_i \sim a_j$ (a_i is indifferent to a_j);

$[(t_i > t_j)\cap(u_i > u_j)] \cup [(t_i < t_j)\cap(u_i < u_j)] \Rightarrow a_i \prec\succ a_j$ (a_i is non-comparable with a_j).

Observe that:

$a_i \succ a_j \Leftrightarrow a_j \prec a_i$ (asymmetric relation);

$a_i \sim a_j \Leftrightarrow a_j \sim a_i$ (symmetric relation);

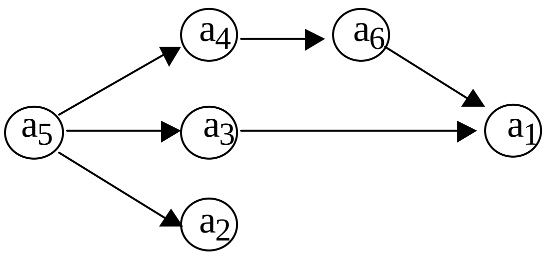
$a_i \prec\succ a_j \Leftrightarrow a_j \prec\succ a_i$ (symmetric relation).

The binary relations that represent the partial pre-order generated by PROMETHEE I on the basis of the information in Table 1 and shown in Table 2 and illustrated by Figure 1.

Table 2 – Multicriteria preference relations. BV85 data

	a_1	a_2	a_3	a_4	a_5	a_6
a_1	$a_1 \sim a_1$					
a_2	$a_2 \prec\succ a_1$	$a_2 \sim a_2$				
a_3	$a_3 \succ a_1$	$a_3 \prec\succ a_1$	$a_3 \sim a_3$			
a_4	$a_4 \succ a_1$	$a_4 \prec\succ a_2$	$a_4 \prec\succ a_3$	$a_4 \sim a_4$		
a_5	$a_5 \succ a_1$	$a_5 \succ a_2$	$a_5 \succ a_3$	$a_5 \succ a_6$	$a_5 \sim a_5$	
a_6	$a_6 \succ a_1$	$a_6 \prec\succ a_2$	$a_6 \prec\succ a_3$	$a_6 \prec\succ a_2$	$a_6 \prec a_5$	$a_6 \sim a_6$

Figure 1 – Partial ranking. BV85 data



Recall finally that, according to the PROMETHEE II methods, the passage from the partial pore-order to the total one takes place by calculating the scores to assign to the individual units as differences among the overall preference intensities:

$$\mathbf{h} = \mathbf{t} - \mathbf{u} \; . \qquad (19)$$

The ranking of the units is shown in Table 3 and illustrated in Figure 2.

Table 3 – Ranking of the units. BV85 data		
Unit	Range	Score
a_1	6	-0.728
a_2	2	0.085
a_3	5	-0.447
a_4	3	-0.102
a_5	1	1.466
a_6	4	-0.274

Figure 1 – Total ranking. BV85 data

$$a_5 \longrightarrow a_2 \longrightarrow a_4 \longrightarrow a_6 \longrightarrow a_3 \longrightarrow a_1$$

4. Consideration of coherence constraints in the PROMETHEE method

As pointed out with regard to Table 1, the multicriteria preference indices defined in an additive scheme satisfy neither the reversibility (10) nor the transitive property defined for that scheme.

However, it is of interest to find that the reversibility property is satisfied by multicriteria preference indices that measure the preference differential between the evaluations relative to symmetrical pairs of units. These are straightforwardly obtained from the indices (17) as follows:

$$d_{ij} = d(a_i,a_j) = s_{ij} - s_{ji}\ ; \qquad i,j = 1, \ldots, n, \tag{20}$$

or with matrix notation:

$$\mathbf{D} = \mathbf{S} - \mathbf{S'} \tag{21}$$

The values assumed by these indices in the numerical example considered are set out in Table 4.

Table 4 – Multicriteria preference differential indices d_{ij}. BV85 data						
	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	-0.166	0.014	-0.131	-0.344	-0.101
a_2	0.166	0	0.209	-0.172	-0.219	0.101
a_3	-0.014	-0.209	0	0.028	-0.431	0.179
a_4	0.131	0.172	-0.028	0	-0.157	-0.220
a_5	0.334	0.219	0.431	0.157	0	0.315
a_6	0.101	-0.101	-0.179	0.220	-0.315	0

From this, violation of the transitive property is easily ascertained. For example

$$d_{13} = 0.014 \neq d_{12}+d_{23} = -0.166+0.209 = 0.043.$$

These indices are therefore incoherent for ranking the units and hence for constructing a total pre-order for them. This thus raises the problem of modifying them in order that they satisfy this property. Moreover, because these indices result from empirical observations which take account of concrete needs both objective and subjective, the weights v_i to associate with the individual units in order to satisfy relations (10) must be such as to make the smallest modifications possible to these multicriteria preference indices: in other words, there must be the minimum possible difference between the observed indices and those that are theoretically coherent. Formalizing this choice in the least squares criterion[6] entails the problem of minimizing the function:

$$\varphi(\mathbf{v}) = \sum_{i=1}^{n}\sum_{j=1}^{n}(d_{ij} - v_i + v_j)^2 =$$

$$= \mathrm{tr}[(\mathbf{D} - \mathbf{v}\,\mathbf{i}' + \mathbf{i}\,\mathbf{v}')'(\mathbf{D} - \mathbf{v}\,\mathbf{i}' + \mathbf{i}\,\mathbf{v}')] \,. \qquad (22)$$

The first-order conditions require:

$$\frac{\partial}{\partial \mathbf{v}}\varphi(\mathbf{v}) = -4\mathbf{D}\,\mathbf{i} + 4n\mathbf{v} - 4\mathbf{i}\,\mathbf{i}'\,\mathbf{v} = \mathbf{0}\,, \qquad (23)$$

[6] It should be also pointed out that, according to the same criterion, in the domain of skew symmetric matrices, which satisfy the less stringent constraint of reversibility, matrix D is to be preferred in that it minimises the norm $\|S-D\|$. I am grateful to an anonymous referee for this suggestion.

whence:

$$(n\mathbf{I} - \mathbf{i}\,\mathbf{i}')\,\mathbf{v} = \mathbf{D}\,\mathbf{i}\,. \tag{24}$$

It is evident that the ($n\mathbf{I} - \mathbf{i}\;\mathbf{i}'$) matrix is singular,[7] which for that matter was to be expected given that the values v_i have been determined up to an additive constant. If this is chosen so that the sum of the weights is annulled, i.e. $\mathbf{i}'\;\mathbf{v} = 0$, one obtains the particular solution:

$$\begin{aligned}\mathbf{v}^* &= n^{-1}\mathbf{D}\,\mathbf{i} = \\ &= n^{-1}(\mathbf{S} - \mathbf{S}')\,\mathbf{i} = n^{-1}(\mathbf{t} - \mathbf{u}) = \\ &= n^{-1}\,\mathbf{h}\,. \end{aligned} \tag{25}$$

This last relation justifies, I believe, the procedure for determining the scores to assign to the individual units foreseen by the PROMETHEE II to pass from the partial pre-order to the total one. In fact, scores (19) differ from weights (25) only for the multiplicative constant n^{-1}, which is entirely without influence on the total pre-order of the units.

5. The alteration produced by the coherence constraints

The approach proposed appears to be of especial interest because it enables evaluation of the divergences between the multicriteria preference indices observed and those closest to them in the sense of (22) but coherent with a total pre-order of the units. The practical and operational need for a ranking of the units that enables identification of those that should be regarded as preferable induces consideration of the partial pre-order, and therefore of the cases of non-comparability shown by the PROMETHEE I method, only as a negative aspect. Instead, they yield useful information that can be profitably used by the decision-maker to reconsider some of the choices previously made. The ranking yielded by PROMETHEE II does not furnish information of this type, nor does it indicate the extent to which the total pre-order obtained is realistic with respect to the actual preference indices observed.[8] The procedure set out in the previous section enables one to address

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[7] Suffice it to consider that ($n\mathbf{I} - \mathbf{i}\,\mathbf{i}')\,\mathbf{i} = \mathbf{0}$.

[8] This is an aspect also emphasised by the authors of the multicriteria decision-making method cited here, who already noted in their work of 1985 (Brans and Vincke, p. 656) that “PROMETHEE II provides a

naturally and immediately the problem of evaluating the alteration introduced by the coherence constraints. In fact, for each pair of units, one need only consider the difference between the values of the preference differential indices d_{ij} and the corresponding values calculated using weights (25):

$$e_{ij} = d_{ij} - d^*_{ij} = d_{ij} - v^*_i + v^*_j, \quad i,j = 1, \ldots, n. \tag{26}$$

The values assumed by these (asymmetric) differences in the numerical example considered are shown in Table 5 below.

Table 5 – Differences between theoretical and observed values: $e_{ij} = d_{ij}{}^* - d_{ij}$. BV85 data.

	a_1	a_2	a_3	a_4	a_5	a_6
a_1	0	0.031	-0.061	0.027	-0.022	0.025
a_2	-0.031	0	-0.120	0.203	-0.011	-0.041
a_3	0.061	0.120	0	-0.086	0.112	-0.208
a_4	-0.027	-0.203	0.086	0	-0.104	0.249
a_5	0.022	0.011	-0.112	0.104	0	-0.025
a_6	-0.025	0.041	0.208	-0.249	0.025	0

For the purpose of succinct analysis of the results of the procedure, I calculated the following indices:

$$ç_i = \left(\frac{1}{n-1} \sum_{j=1}^{n} e^2_{ij} \right)^{1/2} \qquad i = 1, \ldots n, \tag{27}$$

$$ç = \left(\frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} e^2_{ij} \right)^{1/2} \tag{28}$$

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complete ranking which is ‘agreeable’ to the decision-maker, but some useful information about incomparabilities gets lost”. Brans, Vincke and Mareschal (1986, p. 234) provide a numerical example of the advantage of also considering and analysing a partial pre-order which highlights the misleading information that a complete pre-order may furnish, but without proposing measures of the diversities between the two rankings.

$$R^2 = 1 - \sum_{i=1}^{n}\sum_{j=1}^{n} e_{ij}^2 \Big/ \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}^2 \qquad (29)$$

which enable evaluation of the extent of the dissimilarity due to the individual units and the procedure as a whole, as shown in Table 6.

Table 6 – Dissimilarity indices. BV85 data							
ç$_1$	ç$_2$	ç$_3$	ç$_4$	ç$_5$	ç$_6$	ç	R^2
0.036	0.108	0.127	0.157	0.070	0.147	0.116	0.712

In order to further illustrate the usefulness of an objective measure of the difference between the partial and total pre-orders, I reconsidered the example provided in above-cited Brans, Vincke and Mareschal (1986, henceforth BVM86). It should be borne in mind that the diversity between the two orders is evaluated by these authors only by means of comparison between the two corresponding graphs shown in Figures 3 and 4.

Figure 3 – Partial ranking. BVM86

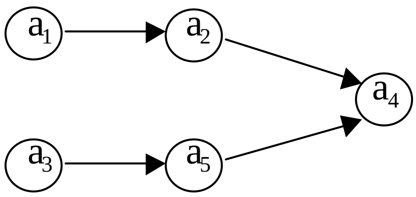
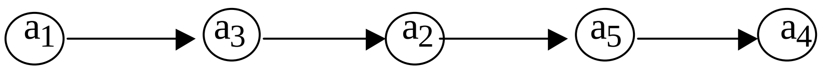


Figure 4 – Total ranking. BVM86



I used indices (27), (28) and (29) to measure the dissimilarity between the two pre-orders quantitatively. The results are given in Tables 7 and 8.

Table 7– Differences between theoretical and observed values: $e_{ij} = d_{ij}{}^* - d_{ij}$. BV86 data.					
	a_1	a_2	a_3	a_4	a_5
a_1	0	0.075	-0.167	0.160	-0.068
a_2	-0.075	0	0.071	-0.198	0.202
a_3	0.167	-0.071	0	0.085	-0.181
a_4	-0.160	0.198	-0.085	0	0.047
a_5	0.068	-0.202	0.181	-0.047	0

Table 8 – Dissimilarity indices. BV86 data						
$ç_1$	$ç_2$	$ç_3$	$ç_4$	$ç_5$	$ç$	R^2
0.128	0.151	0.135	0.136	0.142	0.138	0.669

The methodological approach proposed here also enables analysis to be made of the sensitivity of the complete final ranking to the choice of weights[9] π_k, k=1,…,K, for the criteria P_k. By substituting relation (16) in (21) and (25), from (26) one obtains:

$$e_{ij} = \sum_{k=1}^{K}(p_{kij} - p_{kji} - n^{-1}(\sum_{j=1}^{n}(p_{kij} - p_{kji}) - \sum_{i=1}^{n}(p_{kji} - p_{kij})))\pi_k \tag{30}$$

which makes it relatively simple to connect the measure of violation of the coherence constraints with evaluation of the importance assigned to the individual criteria.

6. Concluding comments

The paper has pursued two closely connected purposes. The first has been to provide a theoretical framework, based on coherence constraints, for a technique of multicriteria analysis with which to convert partial pre-order into a total pre-order. The second has been to use this framework to evaluate the alteration produced in the observed binary

[9] I am again grateful to an anonymous referee for this suggestion.

comparisons in order to eliminate incomparabilities. I believe that this latter aspect, which is of greater practical relevance, may of use to decision-makers for re-examination of both the data and the criteria used, and that it may also be utilised to evaluate the reliability of the total pre-order yielded by the multicriteria method.

I intend to illustrate this aspect in a subsequent study which analyses a concrete case. Here, however, it has seemed opportune to restrict the treatment to a numerical example with already-known and immediately verifiable data.

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