Università degli Studi di Ancona

DIPARTIMENTO DI ECONOMIA

Short-run bargaining, factors shares and growth

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QUADERNI DI RICERCA n. 188

Luglio 2003

Sintesi

In this paper we assume that firms and unions bargain efficiently on wages and employment, whereas work effort is optimally chosen by workers. In the short run, the bargaining process leads to the contract curve. Instead of solving the model and leaving the equilibrium dependent on an exogenous social partners bargaining power, we prefer to leave the wage rate undetermined. Using an endogenous growth model based on human capital, and on the hypothesis that firms invest profits in physical capital while workers optimally allocate their earnings between consumption and investment in human capital, we determine the wage rate that maximizes individual expected utility. Finally, we investigate the relationship between short run behaviour and long run optimality.

JEL Class.: D33, J24, O40
Keywords: Endogenous growth, Distribution, Labour share, Unemployment

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1 Introduction

During the 1950s and 60s, the economic literature studied the complex relationship between income distribution and economic growth. This issue was debated, amongst others, by Kaldor (1956), Pasinetti (1962, 1969) and Samuelson - Modigliani (1966). Attention focused mainly on the different propensities to save of the social classes comprising workers and capitalists, and on the change in the average value of the rate of saving brought about by variation in the proportions of total income accruing to one or other class.

A branch of growth theory focuses on the distributive conflict that takes place in non-competitive labour markets. It has long been recognized that trade unions are able to influence capital accumulation through their involvement in the fixing of wage and employment levels. In a competitive firm, higher wages lead to the substitution of labour by capital and to a fall-off in production. The overall effect on the capital stock is therefore ambiguous.

In a situation in which companies and unions bargain over both wages and employment, in the absence of binding agreements between the parties it is likely that the incentive to invest will diminish (Grout (1984), Van del Ploeg (1987)): in fact, a larger capital stock and greater labour productivity will induce the unions to demand higher wages, thereby eroding the expected return on capital. When firms see the shortfall in their expected return, they will have less incentive to invest.

Empirical estimates based on neoclassical growth theory gave rise to an unexplained component of the rate of growth, usually called the "Solow residual". The recent economic literature has highlighted the fundamental role of

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human capital in explaining economic growth. The “Solow residual” is now commonly considered as the human capital contribution to growth. There is little dispute that human capital accumulation plays an important role in the growth process because education, by producing a particular form of capital based on intellectual skills, increases output.

The traditional two-factor production function of earlier growth models has gradually been substituted by a three-factor production function based on two accumulable factors (human and physical capital) and a non accumulable one (labour, usually defined in efficiency units).

These approaches have been summarized in Lucas’s paper (1988) where he highlights that the inclusion of human capital in growth models gives rise to an unresolved question on the functional distribution of income: which factor earns the part of product due to human capital? By raising the efficiency of both workers and physical capital, human capital is a sort of “public good” (justifying the public financing of educational systems) whose earnings may be appropriated by both workers and capitalists.

Furthermore, other theories on endogenous growth strongly emphasize that labour market institutions may influence growth. Indeed, the conclusions of some studies may differ from others simply because an assumption is changed. Gilles (2000) and Irwen and Wigger (2001) assume that the marginal propensity to save on wages is higher than that on capital, while De Groot (2001) assumes the opposite. Their conflict of views derives directly from the assumption of no dynasty versus an infinitely long dynasty. This, ultimately, determines whether the rent bearing older generation will save a lot or will not save at all.

Lingen (2002) assumes that the ratio of labour employed in the productive sector to the labour employed in the R&D sector is sticky, and that therefore higher wages in the productive sector causes unemployment in both sectors. Lingens’ assumption eventually induces him to conclude that unions are growth reducing while Palokangas (2003), in contrast, concludes that real wage increases in the productive sector will increase labour supply to the R&D sector, causing growth.

Lastly, Parreño and Sánchez-Losado (1999) show that consideration of how unemployed resources are used is essential in determining effects of labour unions on growth. They show that if unemployed resources are used to educate, the unemployment caused by labour unions may be growth enhancing.

However, the results are diverse and the paths to results are also diverse, suggesting that there is no consensus on the effects of labour unions on endogenous growth. A conclusion to the argument will come when consensus is reached at each stage of building a model.

The aim of this article is to re-examine bargaining, functional distribution of income and endogenous growth based on human capital, following:

- the Lucas "indeterminacy" of factor shares when human capital enters the production function;
- the Kaldorian hypothesis of different propensities to save for the different social classes (or, more appropriately, earners of different types of revenue);
- the Mc-Donald, Solow (1981) idea of efficient bargaining between the social partners;
- the Shapiro, Stiglitz (1984) efficiency wage model, which gives rise to a positive relationship between effort supplied by workers and the unemployment rate.

Our main hypothesis is that, in the long-run, firms define the growth path of physical capital, basing their investment decisions on their profits; households determine the growth path of human capital by deciding the amount of income to invest in education.

We obtain the result that there is a labour share level (depending on preferences and technological parameters alone) which maximizes expected utility. To obtain precisely this labour share, the wage rate must be selected in such a way that the last unit invested in physical capital and in human capital generates the same increase in the current value of the utility deriving from consumption.

In section 2 we consider the effects on factor shares of different hypotheses on the division of the revenue accruing to human capital. Section 3 defines the behaviour of workers, trade unions and firms in the short run. Section 4 presents an endogenous growth model where firms invest profits in physical capital while workers optimally allocate their earnings between consumption and investment in human capital. In Section 4 we analyse the relationship between short run behaviour and long run optimality. Finally, we propose some concluding remarks.

2 The distributive conflict

Suppose the following constant return to scale production function:

$$Y = B(xL)^{\alpha} H^{(1-\alpha)\beta} K^{(1-\alpha)(1-\beta)} \tag{1}$$

where Y is production, B is an exogenous scale parameter, x is workers effort, L employment, H human capital and K physical capital.

With perfect competition in the product and the labour markets, it is usually assumed that the labour share equals $\alpha + (1 - \alpha)\beta$. Thus, human capital revenue completely accrues to workers.

Considering human capital as a skill which results from devoting time to its acquisition (like Lucas, 1988), it is impossible to distinguish "education" from the "educated" individual worker; in which case it seems plausible to accrue to workers the total increase in productivity due to education.

> Lucas supposes there to be a set of perfectly competitive risk neutral firms producing the single good for consumers. They maximize profits and pay a wage equal to labour's marginal product. Workers, by acquiring skills, forego some of their wage in favor of the higher wage their skill gain will command (F. Hahn, p. 9-10).

This of course seems very simplistic. In fact, higher skills and better education improve the productivity of machinery. Furthermore,

> ... intellectually skilled workers facilitate the transfer of technology...This suggests that a high level (rather than a high growth rate) of intellectual skills is associated with increase in output. If this alternative interpretation is correct, the conflict between the predicted and actual profit share may not be so easily resolved (K. Foley and R. Michl, 1999, page 173.)

We maintain that firms may be able to obtain a part of the product generated by human capital. By raising the efficiency of both workers and physical capital, human capital is a sort of "public good", whose earnings may be not completely appropriated by workers.

If this is the case, the labour share $q(w) = \frac{wL}{Y}$, where w is the wage rate per worker, depends on the bargaining power of the social partners. Because of the "indeterminacy" on the appropriation of human capital and given equation 1, the labour share is limited as follows:

$$\alpha \leq q(w) \leq \alpha + (1 - \alpha)\beta$$

For $q(w) = \alpha$ all the return on human capital goes to capitalists, whereas for $q(w) = \alpha + (1 - \alpha)\beta$ it goes completely to workers.

Hence, the competitive market is unable to split the revenue from human capital according to the criterion of the value of marginal productivity because of the public good nature of human capital. The distributive conflict arises when the trade unions act in order to increase the labour share. Furthermore, workers may react to the bargaining outcome in term of effort in the job.

The following sections will present a two stage model on the basis of the following hypotheses:

- in the short run, the social partners (trade unions and firms) bargain efficiently at a decentralized level over wages and employment; the contract curve set the employment level as a function of the wages. Workers choose their effort level considering the unemployment rate in the whole economy. Therefore, employment, effort, production and the factor shares are determined as a function of the bargained wage. According to the Nash bargaining model, determination of the equilibrium levels of employment and wages requires the definition of a given level of bargaining power for firms and trade unions. Instead of solving the model and leaving the equilibrium dependent on an exogenous bargaining power, we prefer to leave the wage rate undetermined in the short run. Therefore the short run equilibrium is "open" in the sense of the classical and the marxian theories of *conventional* wage models.

- in the long run, households maximize their expected utility. On the hypothesis that the capital share is completely reinvested, whereas the labour share is optimally allocated between consumption and investment in human capital[1], the optimal wage rate is determined. Therefore, in a long run perspective, households, as the owners of physical capital, decide to leave profits to firms in order to increase the physical capital stock and, as consumers and owners of human capital, decide how much to invest in human capital in order to maximize the expected utility.

3 The short run

3.1 Effort determination

We represent the expected utility of a worker (V_i^E) as follows:

$$V_i^E = p(x_i, u)U(w, x_i)$$

[1] Investment in human capital usually is measured by school enrolment, financed partly by the general taxation system and partly by households directly. Hence, the cost of schooling is mainly transferred to households.

where U represents workers' instantaneous utility, which depends positively on the wage rate w and negatively on the effort level on the job, x_i. The p function is the probability of being employed, which is assumed to depend[2] positively on workers' effort and negatively on the aggregate unemployment rate, u. In fact, if the worker loses his/her current job, s/he expects the probability of finding another job to be lower when the unemployment rate is high.

Our worker chooses the level of his/her effort (x_i), following the condition:

$$\frac{p'_{x_i}}{p(x_i, u)} = -\frac{U'_{x_i}}{U(w, x_i)} \tag{2}$$

Using a CRRA utility function[3] of the type:

$$U(w, x_i) = \frac{1}{1-\sigma}\left(\frac{w}{x_i^{\gamma}}\right)^{1-\sigma} \tag{3}$$

the right hand side of equation 2 becomes:

$$-\frac{U'_{x_i}}{U(w, x_i)} = \frac{\gamma(1-\sigma)}{x_i}$$

Let us suppose that the probability of being employed is given by:

$$p(x_i, u) = me^{(x_i-1)(1-u)}$$

where $p(1, u) = m$ and $p\left(1 + \frac{ln(1/m)}{1-u}, u\right) = 1$. Therefore, the left hand side of equation 2 becomes:

$$\frac{p'_{x_i}}{p(x_i, u)} = 1 - u$$

Hence, the equilibrium effort level chosen by worker i and, given identical workers, in the whole economy is:

$$x^*(u) = \frac{\gamma(1-\sigma)}{1-u} \tag{4}$$

[2] The p probability should also depend on the relative wage paid by the firm ($\frac{w}{E(w)}$); we suppose that workers, if fired, expect to find the same wage in every firm, so that $w = E(w)$

[3] An utility function separable in w and x would generate a time dependent difference $\frac{\dot{U}}{U} - \frac{\dot{w}}{w}$. See Bucci, Fiorillo and Staffolani, 2002 page 10.

3.2 Bargaining

We assume decentralized efficient bargaining, in the sense that each firm and each trade union bargains jointly on wage and employment (Mc Donald, Solow, 1984).

Let us suppose that the trade union at firm j maximizes the following union utility function:

$$W_j = L_j(w_j - v)^z$$

where v is some reference wage[4], z indicates the relative weight of the wage in trade unions utility. Risk neutral firms maximizes profits[5]:

$$\Pi_j = A(xL_j)^\alpha - w_j L_j$$

where, in the short run production function $Y(x, L) = A(xL)^\alpha$, we define $A = BH^{(1-\alpha)\beta}K^{(1-\alpha)(1-\beta)}$.

The contract curve is given by the set of tangency points between trade unions' iso-utility and firms' iso-profit curves [6] in the space w_j, L_j, so that $\frac{W_{L_j}}{W_{w_j}} = \frac{\Pi_{L_j}}{\Pi_{w_j}}$. Equating the two slopes we obtain the contract curve at firm level, which can be solved for the employment level and be aggregated across firms (with a mass equal to 1) in order to obtain the aggregate contract curve:

$$L(w, x) = \left(\frac{\alpha z A x^\alpha}{(z-1)w + v} \right)^{\frac{1}{1-\alpha}} \tag{5}$$

Along the contract curve, employment is decreasing in the wage rate if $z > 1$, which represents the case where trade unions care more about wages than employment.

3.3 The short run equilibrium

Substituting in equation 5 the optimal effort level obtained in equation 4, considering that $1 - u = \frac{L}{N}$, where N is the labour force, we obtain a final definition of the aggregate relationship between employment and wages:[7]

[4]This we assume to be invariant across firms. As we will see later, the reference wage is a crucial determinant of the long run equilibrium; for instance, it may represent the last bargained wage, the non-unionized sector wage, the wage of some foreign "reference" country, unemployment benefits and so on.

[5]Where x is exogenously given to individual firms

[6]From the definition of union utility function and firm profit function, we obtain the slope of the iso-utility curve, that is: $\frac{dw_j}{dL_j} = -\frac{w_j - v}{zL_j}$ and the slope of the iso-profits curve: $\frac{dw_j}{dL_j} = -\frac{Y'_{L_j} - w_j}{-L_j}$

[7]We can also write the effort as a function of the wage rate $x^*(w) = \frac{[\gamma(1-\sigma)]^{1-\alpha}}{\alpha} \frac{(z-1)w+v}{zA} \frac{1}{N}$ so that workers' effort is increasing in the wage rate.

$$L^*(w) = [\gamma(1-\sigma)N]^\alpha \frac{\alpha z}{(z-1)+\frac{v}{w}} \frac{A}{w} \tag{6}$$

Let us now consider the labour share, $q = \frac{wL(w)}{Y(x,L)} = \frac{wL(w)}{A(xL)^\alpha}$. First of all, consider that the product between effort and employment (xL) is independent of the wage rate; in fact, from equation 4 we obtain $xL = \gamma(1-\sigma)N$. Substituting this last equation and equation 6 in the definition of the labour share, we easily obtain:

$$q(w) = \alpha \frac{z}{(z-1)+\frac{v}{w}} \tag{7}$$

Hence, $q(w)$ is increasing in the wage rate.

If $w \geq v \geq 0$, so that the trade unions' objective is to increase the wage rate with respect to a given "reference" positive wage, we obtain that the labour share must lie between the "traditional"one, so that $q_{MIN} = \alpha$ when $w = v$ and the maximum possible level, given by $q_{MAX} = \alpha\frac{z}{z-1}$, when $v = 0$.[8]

Let us finally consider the unemployment rate, $u = 1 - \frac{L}{N}$. Equations 6, 7 and the definition of A allow us to write:

$$u(w) = 1 - \frac{q(w)}{w}[\gamma(1-\sigma)]^\alpha AN^{\alpha-1} \tag{8}$$

which is increasing in the wage rate if $z > 1$. Hence, for a given reference wage, v, and $z > 1$, the labour share and the unemployment rate increase with the wage rate.

4 Endogenous growth and wage determination

This section studies the optimal growth path of the economy in long run general equilibrium. In the steady state solution it is reasonable to suppose that worker's effort remains constant over time, that employment grows at

[8]It is worth noting that in this last case trade unions are able to accrue all the remuneration of human capital to workers. In fact, as we said above, given the production function of equation 1 workers are able to obtain all the remuneration of human capital when $q = \alpha + \beta(1-\alpha)$. Equating the two last equations, and considering that $z = 2$ because otherwise $q(w) > q_{MAX}$, we obtain $\beta = \frac{\alpha}{1-\alpha}$; given this value for β, the production function may be written: $Y = B(xLH)^\alpha K^{(1-\alpha)}$, where human capital is completely indistinguishable from labour measured in efficiency units. For given x and L, this last production function has the same form as the one used in many models of endogenous growth (see Barro and Sala-I-Martin, 1995, page 176).

the same constant rate as the population (n, exogenously given, so that the unemployment rate remains constant), and that the labour share remains constant (A more rigorous explanation of these hypotheses is given in section 5).

Let us consider production, human capital and physical capital in efficiency units, defining:

$$y(t) = \frac{Y(t)}{x(t)L(t)} \qquad h(t) = \left(\frac{H(t)}{x(t)L(t)}\right)^{1-\alpha} \qquad k(t) = \left(\frac{K(t)}{x(t)L(t)}\right)^{1-\alpha}$$

so that:

$$y(h,k) = Bh(t)^{\beta}k(t)^{1-\beta} \tag{9}$$

In the long run, the utility of each household depends on consumption level ($C(t)$) and effort. We suppose that households maximize their expected utility considering that effort must be constant over time. In order to keep the notation simple, we do not write it in the CRRA utility function of equation 3.

It is convenient to write consumption in terms of unity of efficiency as: $c(t) = \frac{C(t)}{xL(t)}$, where $C(t) = xL(t)c(t) = xL_0e^{nt}c(t)$, where n is the rate of growth of the population.[9]

Therefore the CRRA utility function becomes:

$$U(t) = \frac{1}{1-\sigma}c(t)^{1-\sigma}e^{n(1-\sigma)t}$$

Profits are always invested in physical capital ($\dot{k}(t)$), whereas labour income is optimally allocated between consumption ($c(t)$) and investment in human capital ($\dot{h}(t)$). Therefore:

$$\dot{k}(t) = [1 - q(w)]y(h(t), k(t)) - nk(t) \tag{10}$$

$$\dot{h}(t) = q(w)y(h(t), k(t)) - c(t) - nh(t) \tag{11}$$

Households must choose two variables: the wage rate, which defines the capital share and hence the accumulation of physical capital, and the consumption level, which determines the accumulation of human capital.

The Hamiltonian for the problem is:[10]

$$\aleph(c, w, h, k, \lambda, \mu) =$$
$$e^{-\theta t}\tfrac{1}{1-\sigma}c^{1-\sigma} + \mu[q(w)y(h,k) - c - nh] + \lambda[(1 - q(w)y(h,k) - nk]$$

[9] For simplicity we assume $xL_0 = 1$

[10] In what follows, we do not write the time index unless it is necessary.

where $\theta = \rho - n(1-\sigma)$; the first order conditions are:

$$\aleph'_c = e^{-\theta t} c^{-\sigma} - \lambda = 0 \tag{12}$$

$$\aleph'_w = y(h,k)(\mu - \lambda)\frac{dq}{dw} = 0 \tag{13}$$

$$-\aleph'_h = \dot{\lambda} = -[\lambda(1-q(w)) + \mu q(w)]\beta\frac{y(h,k)}{h} + \lambda n \tag{14}$$

$$-\aleph'_k = \dot{\mu} = -[\lambda(1-q(w)) + \mu q(w)](1-\beta)\frac{y(h,k)}{k} + \mu n \tag{15}$$

and transversality conditions are:[11]

$$\lim_{t\to\infty} \lambda(t)h(t) = 0 \qquad \lim_{t\to\infty} \mu(t)k(t) = 0$$

Given that $\frac{dq}{dw} \neq 0$, equation 13 implies $\lambda(t) = \mu(t) \forall t$, which, in turn, implies $\frac{\dot{\lambda}}{\lambda} = \frac{\dot{\mu}}{\mu} \forall t$.

The dynamic laws of equations 14 and 15 become, respectively:

$$\frac{\dot{\lambda}}{\lambda} = -\beta\frac{y}{h} + n \tag{16}$$

$$\frac{\dot{\mu}}{\mu} = -(1-\beta)\frac{y}{k} + n$$

So that:

$$k = \frac{1-\beta}{\beta}h \tag{17}$$

Therefore, along the optimal growth path, physical and human capital must grow at the same rate, so that the production function of equation 9 can be written as:

$$y(k) = \Phi k \tag{18}$$

for $\Phi = B\left(\frac{\beta}{1-\beta}\right)^{\beta}$ and:

$$y(h) = B\left(\frac{1-\beta}{\beta}\right)^{1-\beta} h \equiv \frac{1-\beta}{\beta}\Phi h \tag{19}$$

Substituting equation 18 in equation 10, we obtain:

$$\frac{\dot{k}}{k} \equiv g_k = [1-q(w)]\Phi - n \tag{20}$$

[11]The second one is respected if $\rho > (1-\beta)(1-\sigma)B\left(\frac{\beta}{1-\beta}\right)^{\beta}$.

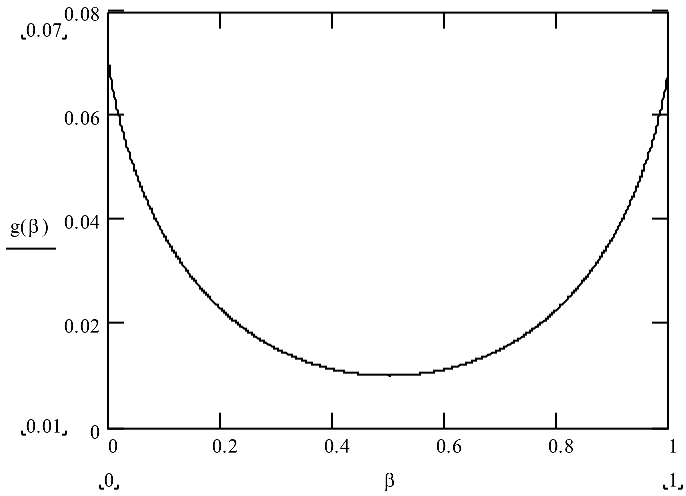

Figure 1: The relationship between the growth rate and the parameter β

parameters: $B = 0.06$, $\sigma = 0.5$, $n = 0.01$, $\rho = 0.02$

Equation 16 becomes:

$$\frac{\dot{\lambda}}{\lambda} = (1-\beta)\Phi + n \tag{21}$$

Following the same steps, we obtain the human capital growth rate:

$$\frac{\dot{h}}{h} \equiv g_h = \frac{1-\beta}{\beta}\Phi q(w) - \frac{c}{h}$$

Differentiating equation 12 with respect to time, we obtain:

$$\frac{\dot{\lambda}}{\lambda} = -\theta - \sigma\frac{\dot{c}}{c}$$

and substituting in equation 21 we obtain the consumption growth rate:

$$\frac{\dot{c}}{c} \equiv g_c = \frac{1}{\sigma}\left[(1-\beta)\Phi - n - \theta\right] \tag{22}$$

where $n + \theta = \rho + n\sigma$.

Therefore, subtituting Φ, the economy's optimal growth rate ($g = g_C$) is:

$$g = \frac{B\beta^{\beta}(1-\beta)^{1-\beta} - (\rho + n\sigma)}{\sigma}$$

The relationship between the growth rate and the parameter β is displayed in figure 1 which shows that the growth rate decreases with β for

$\beta < 0.50$ and increases for $\beta > 0.5$. Therefore, the economic growth rate is higher when the elasticity of output to human capital is very high or when it is very low (the same is the case when the elasticity of output to physical capital is very low or very high).[12]

In steady state, factor shares and the unemployment rate must be constant:

$$\frac{\dot{q}(w)}{q(w)} = \frac{\dot{u}(w)}{u(w)} = 0 \tag{23}$$

This means that, given equations 10 and 11, and given the steady state solutions for the per capita product 19 and 18, we can write:

$$\frac{c}{h} = \left(\frac{\beta}{1-\beta}\right)^{\beta} \left(\frac{q(w)}{\beta} - 1\right)$$

Using equations 22 and 20, we obtain the following implicit definition for the equilibrium labour share:

$$1 - q(w) = \frac{1}{\sigma}\left[(1-\beta) - \frac{\rho}{\Phi}\right] \equiv \frac{\kappa}{\sigma}$$

where $\kappa = (1-\beta) - \frac{\rho}{\Phi} > 0$ if $\rho < (1-\beta)\Phi$[13].

Therefore, the labour share that maximizes expected utility is obviously given by:

$$q^* = 1 - \frac{\kappa}{\sigma} \tag{24}$$

which depends on the "fundamentals", i.e. technology and preferences. However, we have no guarantee that bargaining between the social partners, as described in equation 7, leads the economy to the optimal equilibrium of 24. To reach the optimal growth path, the wage level should be such that the two equations mentioned are equal, so that:

$$w^* = \Omega v \tag{25}$$

where $\Omega = \frac{1-\frac{\kappa}{\sigma}}{\alpha z - (z-1)(1-\frac{\kappa}{\sigma})} = \frac{q^*}{q_{MAX} - q^*}\frac{1}{z-1}$ where q^* is defined in equation 24 and $q_{MAX} = \alpha\frac{z}{z-1}$ is the maximum acceptable value for the labour share.[14]

[12] Given that the growth rate aggregates different sectors, it seems that an economic system specialized in "physical capital intensive' or "human capital intensive" sectors should grow more than a less specialized system.

[13] Therefore, the intertemporal discount rate ρ must be greater than $(1-\beta)(1-\sigma)\Phi$ because of the convergency conditions, and smaller than $(1-\beta)\Phi$ because of the non negativity of capital shares; hence: $(1-\beta)(1-\sigma)\Phi < \rho < (1-\beta)\Phi$.

[14] The wage rate is always higher than the reference wage, because $\Omega \geq 1$ if $q^* \geq \alpha$; furthermore it is increasing in z and decreasing in α.

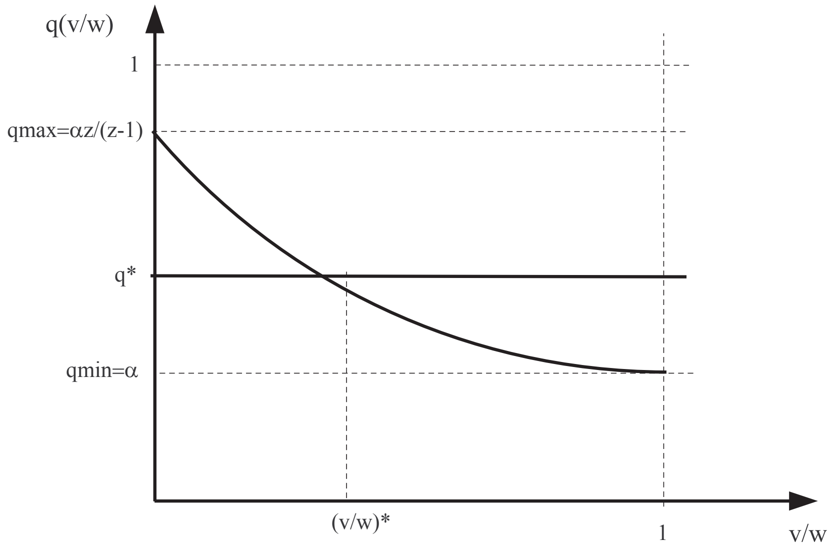

Figure 2: The long run equilibrium

5 Short run behaviour and optimality

The optimal long run equilibrium is described in figure 2, where the labour share is drawn with respect to the ratio between the reference wage and the effective wage ($\frac{v}{w}$). The long run equilibrium requires a labour share equal to q^* (see equation 24), which implies an optimal ratio $(\frac{v}{w})^*$

To obtain these results, we supposed that the labour share and the unemployment rate were constant over time (see equation 23).

Obviously, in order to keep the labour share constant, we must have:

$$\frac{\dot{w}}{w} = \frac{\dot{v}}{v} \tag{26}$$

From equation 5 we can write the dynamic of the unemployment rate:

$$\frac{\dot{u}}{1-u} = n - \frac{\dot{L}}{L} = \frac{\dot{w}}{w}\left[1 - \frac{v}{v+(z-1)w}\right] + \frac{v}{v+(z-1)w}\frac{\dot{v}}{v} - g = 0 \tag{27}$$

Hence, the unemployment rate is constant over time if:

$$\frac{\dot{w}}{w} - g = \frac{v}{v+(z-1)w}\left(\frac{\dot{v}}{v} - \frac{\dot{w}}{w}\right) \tag{28}$$

This equation implies that, if the labour share is constant, so that equation 26 is respected, the unemployment rate is constant too (equation 8), and the wage rate grows at the same rate as the whole economy.[15]

Are there factors which lead the economy to the optimal level of the labour share?

Let us think of the economy described as a sequence of short-run equilibria, where the trade unions determine the wage rate:

1. considering the existence of a wage rate which maximizes the growth rate (rational behaviour)

2. according to their goals, completely ignoring the existence of an optimal wage rate (myopic behaviour);

5.1 Rational trade unions

In the first hypothesis (rational trade unions) we analyse the dynamic behaviour of the wage rate when unions consider the optimal wage rate defined in equation 25.

The dynamic evolution of the wage rate over time is assumed to depend on the difference between the optimal wage rate and the current one, according to the following equation:

$$\dot{w}(t) = \chi[w^*(t) - w(t)] = \chi[\Omega v(t) - w(t)] \tag{29}$$

for $\chi > 0$ which represents the adjustment speed. Let us suppose that the reference wage ($v(t)$) grows at a constant rate. The trade unions' reference wage should be strictly linked to the optimal growth rate of the economy, g. If the trade unions act rationally, and if they know the long run determinant of optimal equilibrium, there are many logical mechanisms with which to obtain $\frac{\dot{v}}{v} = g$. Even if these mechanisms usually act together, and in some sense can reinforce themselves by operating simultaneously in moving the economic system along the optimal growth path, let us consider some of them, and precisely:

- the trade unions' *reference* wage is linked to per-capita income; hence, they grow at the same rate. This mechanism acts completely internally to the labour market.[16]

[15]In fact, the unemployment rate can be constant even if the labour share is not. The condition referring to the unemployment rate is less restrictive: from equation 27, we obtain that the weighted sum of the wage growth rate and of the reference wage growth rate must be equal to the aggregate growth rate.

[16]Let us write condition 28 in discrete time:

$$w_{t+1} = (1+g)w_t + \frac{1+g}{z-1}v_t - \frac{1}{z-1}v_{t+1}$$

- the role played by the state, mainly through unemployment benefit which should influence the *reference* wage, v. It is very hard to imagine a benefit completely independent of the evolution of the economic system over time; in the extreme case, the benefit could be a constant ratio of the per-capita income, so that $\frac{\dot{v}}{v} = g$.

- the existence of an *irregular* sector which indirectly determines the *reference* wage; this *irregular* sector moderates wage growth rate in the regular economy by dint of the same mechanism described as the marxian "industrial reserve army".

The explicit solution of equation 29 gives:

$$w(t) = \left(w(0) - \frac{\chi \Omega v(0)}{g + \chi} \right) e^{-\chi t} + \frac{\chi \Omega v(0)}{g + \chi} e^{gt}$$

remembering that $w(0)$ is the first optimal control and can be chosen in order to verify the limit condition, i.e., $w(0) = \frac{\chi \Omega v(0)}{g+\chi}$, the wage $(w(t))$, the exogenous reference wage $(v(t))$ and the economic system grow at the same rate; the labour share and the unemployment rate remain constant over time at their optimal value.

5.2 Myopic trade unions and Nash bargaining

Let us consider the second hypothesis (myopic trade unions), where trade unions are indifferent to households' well-being and pursue their own goals.

The standard procedure to solve for the wage in bargaining is based on the Nash bargaining model, which maximizes, with respect to employment and the wage rate, the weighted product of expected gains from bargaining obtained by trade unions and firms. The solution of the model gives the contract curve (equation 5) and the bargained wage rate (w_{SR}), which takes the form:

$$w_{SR} = \frac{\eta + \alpha(1-\eta)}{\eta[1 - z(1-\alpha)] + \alpha(1-\eta)} v$$

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and let us suppose that the *reference* wage is such that $v_t = E(w_t + \epsilon_t) = w_t + E(\epsilon_t) = w_t$; the same for time $t+1$, so that $v_{t+1} = w_{t+1}$. Substituting in the previous equation, we obtain $w_{t+1} = (1+g)w_t$, as expected. If, instead of rational expectations, we suppose myopic expectations, so that $E(v_{t+1}) = w_t$, we obtaine the following dynamic law for the wage rate:

$$w_t = \frac{w_0}{z + g(z-1)} \left[z(1+g)^t + g(z-1) \left(-\frac{1}{z-1} \right)^t \right]$$

which implies that the wage growth rate can be decomposed into two parts: a trend of dimension g, and oscillations, which, according to the value of z, can be both increasing or decreasing over time.

where $0 \leq \eta \leq 1$ is the trade unions' bargaining power (and $1-\eta$ is the firms' bargaining power). With efficient bargaining, the labour share of equation 7 becomes:

$$q(\eta) = \alpha + \eta(1-\alpha)$$

The short run labour share as defined in the previous equation and the optimal labour share (q^*) as defined in equation 24 are equal if :

$$\eta = \frac{q^* - \alpha}{1-\alpha} = 1 - \frac{\kappa}{(1-\alpha)\sigma} \tag{30}$$

Therefore, there exists a level of trade-union bargaining power that is "optimal" for the economic system; this is because human capital is financed by labour income alone and because, by raising the wage rate, trade unions are able to increase worker effort[17].

Hence, the "optimal" trade-union bargaining power must be higher when the long run optimal labour share (the one that allows the economy to grow at the optimal rate) is high and when parameter α is low.

Obviously, on a priori grounds there is no reasons to believe that trade union power is exactly the one described in equation 30. Suppose that bargaining power, and hence wages and the labour share, are higher than optimality: in this case, unemployment and effort are higher. A smaller number of people are working too hard. Even if this situation is a stable steady state, because equation 26 is respected, so that the labour share and the unemployment rate are constants, the labour share is higher than the optimal one, so the growth rate of physical capital is lower than optimality. This is a situation of capital shortage. Households consume more output, but at a reduced growth rate.

Are there endogenous mechanisms able to lead the bargaining power to the optimal level? It is obvious that trade-unions bargaining power is influenced by various factors, like the unemployment level, the general public's perception that the trade unions are doing the "right thing" in bargaining, or the wage rate that the unions are able to negotiate. But, at least in our model, there is no clear factor which leads the parameter η to the optimal value of equation 30.

Hence, even if unions are indeed useful for economic growth (by raising the wage rate over the competitive one and by inducing greater effort), they should incorporate household optimal behaviour in their objective function.

[17]This depends on the fact that, in our model, effort is a function of the aggregate unemployment rate, whereas wages are set at a decentralized level; therefore each individual firm does not internalize the effects on effort of a higher wage (as is normally assumed in efficiency wage models). Trade unions, by raising the wage and raising unemployment, push workers to produce a greater effort.

6 Conclusion

In this paper we have revisited, from a modern perspective, the relationship between the functional distribution of income and growth envisaged by the Ricardian tradition. In an endogenous growth model based on human capital, we have assumed that the revenue accruing to human capital must be split between the social partners according to some bargaining rule, and that the functional distribution of income influences investment in the accumulable factors.

In the short run, workers supply effort considering the aggregate unemployment rate. Efficient bargaining between the social partners determines the employment level as a function of the wage rate.

In the long run, the capital share accruing to firms defines the growth path of physical capital, whereas the labour share accruing to households is optimally split between human capital investment and consumption. Hence, households decide the path of human capital accumulation by choosing the amount of income to invest in education. There exists a given labour share, which depends on preferences and technology alone, which maximizes the expected household utility.

We have thus obtained analytical results for short run equilibrium (as the outcome of bargaining between firms and unions) and for long run optimal growth (as the outcome of households' intertemporal maximization).

We speculated whether the effective behavior of the social partners leads to the households' desired labour share, the one which maximizes their expected utility. We have shown that the wage rate must be above the competitive rate, so that unions are required to permit optimal growth. If the unions set the wage rate by considering the long run determinants of capital accumulation, the optimal growth path is reached, whereas if the unions myopically pursue their goals, there exists a given level of union strength in the bargaining process which allows the economy to reach the optimal growth path.

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