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Innovation Diffusion and the Evolution of Regional Disparities

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Abstract

This article investigates the effect that the interaction between the creation and the spatial diffusion of technology brings on regional disparities. We will show that an increase in the pace of innovation, as it has happened with the "technological revolution" can engender regional income disparities; we will also show that if, afterwards, the speed of diffusion also increases enough, these disparities can fade out. The paper will not, at this stage, address the problem of which effect will eventually prevail in the real World, since the process of ICTs diffusion is both increasing the easiness of transfer of blueprinted knowledge across places and having effects on the spatial concentration of innovation.

To analyse the problem, we will first build a simple model with technological disparities as the source of income disparities and a set up apt to represent all the knowledge flows. The first result is that wide enough technological differences can be the source of income disparities. The basic model will then be used in two different ways for the study of innovation and diffusion mechanisms. We will show that the most important variable to determine if income disparities exist is the ratio between the speed of diffusion and the speed of innovation. In particular, when this ratio is low, the most likely prediction is an equilibrium with both technological and income disparities. For intermediate values, technological disparities will exist, but will not be large enough to generate income disparities; for higher values, there will not be technological disparities anymore and, consequently, no income disparities.

The paper also supports an important policy conclusion: when trying to reduce regional disparities, targeting innovation diffusion and the acquisition of external knowledge by the less developed region will be more effective than favoring own innovation if the knowledge base of the lagging region is not wide enough. This is due to the fact that innovation is a process cumulative on previously possessed knowledge, which can be a scarce resource in most lagging regions.

Keywords: Spatial innovation diffusion, regional disparities, technological spillovers, ICTs

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Innovation Diffusion and the Evolution of Regional Disparities*

Ugo Fratesi

1 Introduction

According to a number of authors, the pace of technology has increased in the XXth century; in particular Freeman and Soete (1997) note that innovation changed and, from being the outcome of initiatives of inventors or single Shumpeterian entrepreneurs, became mostly the outcome of specifically designed R&D departments of the firms.

In such a context, the comparative advantage in terms of material resources is not anymore the main factor explaining the differentials of income among territories (Dollar, 1993), and the main cause of competitive advantage, for both firms and regions, has become the ability to produce new technical knowledge (Porter, 1998).

For this reason, the attention of scholars has been increasingly caught by the context in which innovative activities take place, sometimes defined as the Systems of Innovation (Lundvall, 1992, Edquist, 1997, Cooke et al. 1998). And also more orthodox economic theorists have successfully made of innovation the growth engine of their models (e.g. Romer, 1990; Grossman and Helpman, 1991; Martin and Ottaviano, 2001).

Innovation is a difficult process to export from one place to the other, due to its characteristics, for this reason, once technology has become the most important factor in the competition among countries and regions interest has grown in the development policies targeting R&D (Rodriguez-Pose, 2001).
Among all the characteristics of technology, the one that appears to play the largest role in inducing regional disparities is, as this paper will show, its cumulativeness, since new technology can only be built upon previously existing one.

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The focus on knowledge creation, however, should not obscure that, differently from an ordinary physical factor of production, the same knowledge can in theory be used in many different places and many productions at the same time. For this reason innovation, for our purposed defined as the creation of new knowledge, is only a part (even if the basic) of the mechanism and imitation (the acquisition of external knowledge through devoted efforts) and diffusion (the non costly acquisition of external knowledge) also play an essential role.

If, for example, the diffusion of technology were instantaneous (and not protected by some form of patenting), it would be irrelevant the physical place in which innovation takes place. Since knowledge is 'sticky', the location is relevant, but the extent of this relevance depends on the speed of spatial diffusion. This has been affected, especially in the last decade, by the expansion of the information and communication technologies, (ICTs) has made easier, faster and much less expensive the transfer of blueprinted knowledge from one place to the other. The limit consists in the fact that, in order to use this knowledge, it is always necessary to contextualize it: there is in fact the need for someone able to interpret and apply it.

In this paper we will investigate the interaction between the creation and the diffusion of technology in order to detect the effects in brings on regional disparities. We will also show that an increase in the pace of innovation, as the one that took place in the XXth century, can engender regional income disparities but if, successively, the speed of diffusion also increases enough, these disparities can fade out. We will not enter, however, in open the debate if the advancements in the ICTs and the 'New Economy' will reduce disparities or lead to the 'dead of distance' since, as Gillespie et al. (2001, p.110) noted, *"communication technologies should not be seen as simply pulling the balance of centrifugal and centripetal forces in one direction at the expense of the other, but rather at simultaneously strengthening both"*.

To address the research question, we will first build a base model that concentrates on location and technology as the causes of regional income disparities, without modeling growth and physical capita accumulation. For this reason the model is more a supplement to existing theories than a substitute, in that it identifies an important channel and strongly focus on it.

However, the model presents many interesting features: first of all it is able to rigorously and separately represent bi-directional processes of spatial technological diffusion.

The base model will then be used in two ways for the study of innovation and diffusion mechanisms. We will show that the most important factor determining the existence of income disparities is the ratio between the speed of spatial diffusion of knowledge and the speed of innovation. In particular, when the ratio is low, the model predicts an equilibrium with technology and income disparities. For intermediate values there will be technology disparities but not wide enough to generate income disparities. For higher values technological disparities will fade out and, consequently, income disparities will no longer exist.

The paper is organized as follows: section 2 features the base static model, which extends the north-south models by allowing symmetry and eliminating a-priori differentiation between the regions; section 3 introduces transport costs in the model and shows their effects on disparities and aggregate income; section 4 will present the first type of dynamics implemented, with probabilistic consequences for income disparities; in section 5 the representation of a more complex and realistic dynamics will allow to study, now deterministically, the effects of the parameters on the multiplicity of equilibria. The last section concludes with the policy prescriptions.

2 Base model

In this section a small static model is introduced. This model illustrates that different knowledge endowments can generate income disparities between regions when there is no or low mobility of workers.
At the same time the model shows that, in order to generate those disparities, the existence of technological differences is not enough but their level must lie above a certain level.

Positive transport costs, in this model, have the effect of making the incomes of the regions more similar, even if the effect on aggregate welfare is negative, as we will show in the next section. The base model is defined by 8 hypotheses:

Hypothesis 1 *The economy is composed of two regions A and B, with respective fixed endowments of workers L_a and L_b.*

In this two-region model, therefore, we don't need to assume anything on the relative or absolute size of the regions.

Hypothesis 2 *The technology is composed of a given number of varieties of goods whose production technique is known. Not all the production techniques are common knowledge of both regions.*

This hypothesis is due to the nature of technology: the production of certain goods require specific abilities that are not easily transferable because, for example, can be only acquired through learning by doing. This mechanism

can also be the outcome of some patenting preocess, which gives the firms exclusive rights on the production of certain goods; however, the patenting reason needs some additional mechanism to prevent firms located in one region from moving to the other.
We will indicate with N_a the number of varieties that it is only possible to produce in A (the "eclusive" to A), with N_b the number of varieties that it is only possible to produce in B (the exclusive to B) and with N_c the number of varieties that can be produced in both A and B (that we will call "common").

Hypothesis 3 *The production technique involves constant returns to scale and the use of labor and knowledge only. At the same time, in order to produce a variety, the technique on how to produce it must be known in the region. For this reason, if q_i is the amount produced of variety i,*

$$q_i = \xi l_i \mathbb{I}_{ji} \tag{1}$$

Where l_i is the labour employed and ξ is a constant positive and equal for all the varieties whose production technique is known in the region. $\mathbb{I}_{ji}$ is an indicator which assumes value 1 when the production technique of variety i is known in region j, but becomes equal to 0 when, on the contrary, the production technique of i is not known in the region.

Hypothesis 4 *The workers are not mobile across regions.*

This strong hypothesis is necessary for the results of the model but is justified by the evidence available. In fact labor is very sticky, especially within Europe (where only a small percentage of the workforce is born in another region) but not only since the mobility between the least developed countries and the advanced world, even if growing, still affects only a minor part of the population.

Hypothesis 5 *Only a quota of workers $1 - r_j$ is involved in production in region j.*

This is a technical hypothesis which allows to use the rest of the workers for other activities (e.g. for innovation).

Hypothesis 6 *The utility function is the same for all the consumers in both regions, all the varieties consumed enter symmetrically and there exists some degree of love for variety:*

$$U = [\int_0^N {q_i}^\rho]^{\frac{1}{\rho}} \text{ with } 0 < \rho < 1 \tag{2}$$

Where $N = N_a + N_b + N_c$ is the total number of varieties available and q_i is the quantity consumed of each good. The love for variety hypothesis is necessary for the results of the model, but even a small degree is enough, i.e. ρ can be close to 1.

Hypothesis 7 *There is perfect competition in the market, so that price discrimination is not allowed and all the revenue goes to the workers through wages.*

Hypothesis 8 *In this first over-simplified framework, we also assume there are no transport costs, so that all the consumers of both regions will be able to buy the same products.*

This simplifying hypothesis will be released in section 3, where we will also be able to observe the changes the release of this hypothesis entails for the results of the model.
When hypothesis 8 holds it bears the conclusion that each consumer, in either A or B, will have the same utility function and will maximize it by choosing the same varieties in the same proportion. Moreover, since there is love for variety, the consumers would demand the same quantity of all the varieties produced in the economy if the prices were equal. The prices, however, will not always be the same for all the varieties due to the technological constraints on the production side.

On the offer side in fact, the joint effect of concurrence and constant returns to scale would make profitable to produce in each region the same amount of each variety if all the production techniques were known.

By hypothesis 2, the varieties N_a can be exclusively produced in A, the varieties N_b can be exclusively produced in B. The N_c common varieties can be produced in both regions, and a market mechanism allocates their production as illustrated below.

When there is not a large difference in technology levels from one region to the other, the market will allocate the labour factor among the different varieties which is possible to produce inside the regions so that each variety existing in the economy (belonging to either N_a N_b N_c) will be produced in the same amount and sold at the same price. If we indicate by N_{ca} and N_{cb} the common varieties respectively produced in A and B ($N_{ca} + N_{cb} = N_c$), these values will be determined by the equation:

$$\frac{(1-r_a)L_a}{N_a + N_{ca}} = \frac{(1-r_b)L_b}{N_b + N_{cb}} \tag{3}$$

Equation 3 means that, if the two regions were of equal size in terms of active population and the differences of technology endowments were small

enough, A and B would produce the same total number of varieties and all the varieties in the same quantity.
Notice that, since the production function we use is linear, it is not important if some of the common varieties are produced in one region and some in the other or if, alternatively, each one is produced part in A and part in B; it is sufficient that the aggregate ratio is respected.

When the number of varieties which can be exclusively produced in B (N_b) is high enough with respect to N_a and N_c, i.e. when B is enough more technologically advanced, it will be profitable for its workers (or for B firms hiring B workers) to exit from the production of the common varieties, that will continue to be produced only in A, and produce only the N_b exclusive varieties. This happens when:

$$\frac{L_b(1-r_b)}{N_b} < \frac{L_a(1-r_a)}{N_a+N_c} \tag{4}$$

This condition implies complete specialization of B in the production of B varieties leaving to A the production of its (fewer if the regions are of equal population) N_a varieties and of the common varieties, which are, presumably, the less advanced since they can be produced everywhere and no patent exist on them.

When condition 4 is satisfied, due to the rationing of quantities the price is determined by the consumer maximization, which will be the same in the two regions:

$$MaxU = [\int_0^{Na+Nc} q_a{}^\rho + \int_0^{Nb} q_b{}^\rho]^{\frac{1}{\rho}} \tag{5}$$

under the constraint

$$q_a(N_a+N_c) + Pq_bN_b = M \tag{6}$$

Using the price normalisation $P_a \equiv 1$, $P_b \equiv P \equiv \frac{P_b}{P_a}$ and indicating with M the amount of money possessed by the individuals.
Since all N_b varieties are produced in the same amount and all the N_a and N_c varieties are also produced in the same amount, the Lagrangean will be:

$$L = [(N_a+N_c)q_a{}^\rho + N_bq_b{}^\rho]^{\frac{1}{\rho}} - \lambda[(N_a+N_c)q_a + PN_bq_b - M]$$

Giving these first order conditions:

$$\begin{aligned}
\frac{\partial L}{\partial q_a} &= 0 = \frac{1}{\rho}[(N_a+N_c)q_a{}^\rho + N_bq_b{}^\rho]^{\frac{1-\rho}{\rho}}(N_a+N_c)\rho q_a{}^{\rho-1} - \lambda(N_a+N_c) \quad (7)\\
\frac{\partial L}{\partial q_b} &= 0 = \frac{1}{\rho}[(N_a+N_c)q_a{}^\rho + N_bq_b{}^\rho]^{\frac{1-\rho}{\rho}}N_b\rho q_b{}^{\rho-1} - \lambda PN_b \quad (8)\\
&+ \quad \text{constraint}
\end{aligned}$$

By solving the system above we can find the unique equilibrium price:

$$P = (\frac{L_a}{L_b})^{1-\rho}(\frac{1-r_a}{1-r_b})^{1-\rho}(\frac{N_b}{N_a+N_c})^{1-\rho} \tag{9}$$

Once obtained the relative prices it is easy to calculate the aggregate nominal welfare of the two regions:

$$W_a = (N_a + N_c)q_a = (1-r_a)L_a \tag{10}$$

$$W_b = PN_bq_b = (\frac{L_a}{L_b})^{1-\rho}(\frac{1-r_a}{1-r_b})^{1-\rho}(\frac{N_b}{N_a+N_c})^{1-\rho}(1-r_b)L_b \tag{11}$$

Which gives the following formula for the ratio between the GDPs:

$$\frac{W_B}{W_A} = (\frac{L_b}{L_a})^{\rho}(\frac{1-r_b}{1-r_a})^{\rho}(\frac{N_b}{N_a+N_c})^{1-\rho} \tag{12}$$

The economic size of a region is relatively larger when it has larger population (but this effect decreases as the love for variety effect increases) and it is also relatively larger when it is more technologically advanced; differently from the previous one, this effect increases as the love for variety increases.

Through the price mechanism, the labor of the region more advanced region, even if not more productive in terms of quantity, will be better paid in nominal and real terms. This because the labour of A and B workers are only substitutable for the common varieties and it is not possible to move elsewhere the production of exclusive varieties. The ratio between the per capita incomes will in fact be:

$$\frac{y_b}{y_a} = \frac{W_b}{L_b}\frac{L_a}{W_a} = (\frac{1-r_b}{1-r_a})^{\rho}(\frac{\frac{N_b}{L_b}}{\frac{N_a+N_c}{L_a}})^{1-\rho} \tag{13}$$

If we jointly take into account the disperse case of equation 3, the agglomerate/specialised case of equation 4 and its symmetric, the relative wage of the two regions will be:

$$\begin{aligned} \frac{y_b}{y_a} &= (\frac{1-r_b}{1-r_a})^{\rho}(\frac{\frac{N_b}{L_b}}{\frac{N_a+N_c}{L_a}})^{1-\rho} \text{ if } \frac{L_b(1-r_b)}{N_b} < \frac{L_a(1-r_a)}{N_a+N_c} \\ &= (\frac{1-r_b}{1-r_a})^{\rho}(\frac{\frac{N_b+N_c}{L_b}}{\frac{N_a}{L_a}})^{1-\rho} \text{ if } \frac{L_a(1-r_a)}{N_a} < \frac{L_b(1-r_b)}{N_b+N_c} \\ &= 1 \text{ otherwise} \end{aligned} \tag{14}$$

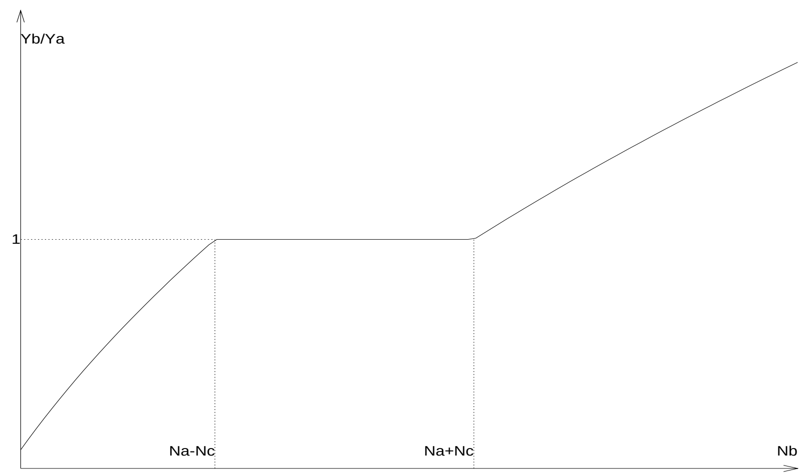

Figure 1: Relation between the number of varieties possessed in B and the relative incomes for given N_a and N_c.

The relative income of the two regions will therefore positively depend on their endowment of varieties per capita, and, negatively, on the proportion of workers that are not involved in production. In particular the effect of the number of varieties is depicted in Fig. 1. The relation between regional technological disparities and per capita income disparities, although not discontinous, will not be increasing, but just not decreasing. There will exist in fact a certain range of technological disparities which will not, due to the existence of shared knowledge, entail income disparities. Externally to this interval, the relation is increasing and the more different the number of varieties per capita, the larger the sage difference.

The effect of love for variety in eq. 14 has th be investigated. The ρ has no effect on the width of the flat trait of the curve of fig. 1; outside of this, it changes the curvature and we can prove that the larger the love for variety (the smaller the ρ) the larger the income disparities. In fact, assuming $r_a = r_b$ for simplicity, we find that the derivative of the disparity with respect to ρ is always negative:

$$\frac{\partial \frac{y_b}{y_a}}{\partial \rho} = -\left(\frac{\frac{N_b}{L_b}}{\frac{N_a+N_c}{L_a}}\right)^{1-\rho} \ln \frac{\frac{N_b}{L_b}}{\frac{N_a+N_c}{L_a}} < 0 \tag{15}$$

The outcome model is consistent to that of Krugman (1979), but the result is extended so that it takes into account 2 structurally identical regions, so that this model can be used to study inter-regional disparities inside countries. In fact this model is symmetric and avoids the north-south dichotomy by allowing each region to have its own exclusive varieties. In addition to this we have the market equilibrium dependent on a number of parameters, for example it is possible to allow regions of different size. It remains to study the effect of a transport cost parameter, which will be done in the next section.

3 Introduction of transport costs

When the hypothesis of null transport costs is removed, the utility functions in the two regions become different and it is no longer true that all the varieties are demanded and consumed in equal proportion in both regions. The easiest, and most diffused in the literature, way of introducing transport costs is by assuming iceberg transport costs, i.e to represent with a $t < 1$ the amount of good shipped from one region to the other that reaches its destination.

Since the hypothesis of perfect concurrence is still valid in the model, there will again exists a relative price for the goods produced in B with respect to those produced in A.

Since, however, the goods involved in the utility maximization will have different proportions depending on the region, we will use the following notation:

$$
\begin{aligned}
q_{aa} &\equiv \text{the amount of a good produced in } A \text{ and consumed in } A \\
q_{ab} &\equiv \text{the amount of a good produced in } A \text{ and consumed in } B \\
q_{ba} &\equiv \text{the amount of a good produced in } B \text{ and consumed in } A \\
q_{bb} &\equiv \text{the amount of a good produced in } B \text{ and consumed in } B
\end{aligned}
$$

There will be complete specialization with A producing all the common varieties, due to the concurrential markets hypothesis, when the consumers of both regions will find more convenient to buy C varieties from A, i.e. when the two following conditions apply simultaneously (note the similarity with equation 4):

$$
\begin{aligned}
t\frac{L_b(1-r_b)}{N_b} &< \frac{L_a(1-r_a)}{N_a+N_c} \text{ for } A \\
\frac{L_b(1-r_b)}{N_b} &< t\frac{L_a(1-r_a)}{N_a+N_c} \text{ for } B
\end{aligned}
$$

Because $t < 1$, the conditions above are simultaneously satisfied when

$$
\frac{L_b(1-r_b)}{N_b} < t\frac{L_a(1-r_a)}{N_a+N_c} \tag{16}
$$

From this equation we can see that the case of complete specialisation becomes less probable when the transport cost increases, and in fact, when $t = 0$, each region will produce its quota of common varieties no matter of any other parameter.

In case of complete specialization with B more technologically advanced, consumers of region A will maximize their utility as follows:

$$MaxU_A = [\int_0^{Na+Nc} {q_{aa}}^{\rho} + \int_0^{Nb} {q_{ba}}^{\rho} t^{\rho}]^{\frac{1}{\rho}} \tag{17}$$

under the constraint:

$$q_{aa}(N_a + N_c) + Pq_{ba}N_b = q_{aa}(N_a + N_c) + q_{ab}(N_a + N_c) = (1 - r_a)L_a \tag{18}$$

The lagrangean will be

$$L_A = [(N_a + N_c){q_{aa}}^{\rho} + N_b {q_{ba}}^{\rho} t^{\rho}]^{\frac{1}{\rho}} - \lambda[q_{aa}(N_a + N_c) + Pq_{ba}N_b - (1 - r_a)L_a]$$

And the first order conditions:

$$\begin{aligned} \frac{\partial L}{\partial q_{aa}} &= 0 = [...]^{\frac{1}{\rho}-1} N_a + N_c q_{aa}^{\rho-1} \rho - \lambda(N_a + N_c) && (19) \\ \frac{\partial L}{\partial q_{ba}} &= 0 = [...]^{\frac{1}{\rho}-1} N_b q_{ba}^{\rho-1} \rho t^{\rho} - \lambda N_b P && (20) \\ &+ \text{ constraint} \end{aligned}$$

From the equations 19 and 20 the price needed for equilibrium in A can be found:

$$P = (\frac{q_{ba}}{q_{aa}})^{\rho-1} t^{\rho} \tag{21}$$

In B a similar utility maximization will take place:

$$MaxU_B = [\int_0^{Na+Nc} {q_{ab}}^{\rho} t^{\rho} + \int_0^{Nb} {q_{bb}}^{\rho}]^{\frac{1}{\rho}} \tag{22}$$

under the constraint:

$$q_{ab}(N_a + N_c) + Pq_{bb}N_b = q_{ba}(N_b) + q_{bb}(N_b) = (1 - r_b)L_b \tag{23}$$

By solving the maximization, similarly of what done for A, we get the price needed for market equilibrium in B:

$$P = (\frac{q_{bb}}{q_{ab}})^{\rho-1} t^{-\rho} \tag{24}$$

The global equilibrium is the solution of a system of 6 equations (of which only 5 independent) in 5 unknown (the 4 quantities plus the equilibrium price). This system can become simpler adopting the following change of notation:

$$\begin{aligned}
q_{aa} &\equiv \alpha q_a = \alpha \frac{(1-r_a)L_a}{N_a+N_c} \\
q_{ab} &\equiv (1-\alpha)q_a = (1-\alpha)\frac{(1-r_a)L_a}{N_a+N_c} \\
q_{ba} &\equiv \beta q_b = \beta \frac{(1-r_b)L_b}{N_b} \\
q_{bb} &\equiv (1-\beta)q_b = (1-\beta)\frac{(1-r_b)L_b}{N_b}
\end{aligned}$$

With this new notation eq. 21 becomes:

$$P = \left(\frac{\beta}{\alpha}\frac{N_a+N_c}{N_b}\frac{(1-r_b)L_b}{(1-r_a)L_a}\right)^{\rho-1} t^{\rho} \tag{25}$$

And eq. 24:

$$P = \left(\frac{1-\beta}{1-\alpha}\frac{N_a+N_c}{N_b}\frac{(1-r_b)L_b}{(1-r_a)L_a}\right)^{\rho-1} t^{-\rho} \tag{26}$$

The two constraints 18 and 23 become identical and equal to:

$$(1-\alpha)(1-r_a)L_a - \beta(1-r_b)L_b P = 0 \tag{27}$$

The equations 25 26 and 27 now form a system of three equations in three unknown α, β and P. Unfortunately, the analytical solution to this non-linear system does not exist; nevertheless, something can be said about the effect of transport costs on consumption in the two regions.
As a first step, we have to remind that, when transport costs are null (i.e. $t = 1$), the consumers in the two regions will consume the goods in the same proportion, that is $\alpha = \beta$. Joining eq. 25 and eq. 26 it is possible to obtain the function that links α and β:

$$\frac{1-\alpha}{\alpha} t^{\frac{2\rho}{\rho-1}} = \frac{1-\beta}{\beta} \tag{28}$$

Eq. 28 verifies the condition $\alpha = \beta$ when $t = 1$ and also says that (as one could intuitively imagine), as t decreases each region will shift its consumption towards its domestic products. Moreover, an increase in love for variety (a decrease in ρ) reduces the effect of an increase in transport costs on the consumption shift.

As a second step, using a linear approximation around $t = 1$, one can also study the effect of the introduction of transport cost on prices. When, in fact, t shifts from 1 to $1 - \epsilon$ the variation of price will be:

$$\delta P = \epsilon P_0 \rho[-1 + 2\rho(1 - \alpha_0)(2\alpha_0 - 1)] \tag{29}$$

When, for example, B is the richest region without transport costs, $P_0 > 1$ and $\alpha_0 < 1/2$. In this case δP is negative, implying that, with the introduction of positive transport costs, regional disparities decrease. In fact, the difference of price is due to the effect of love for variety. In this framework, positive transport costs, have the effect (opposite to the previous) of introducing a bias in favor of home products that makes the demand functions of the two regions become different so that it is less convenient for the less advanced region to demand and consume the products of the more advanced. And this makes the (relative) price of the products of the most advanced region decrease.

The boundary case of no love for variety ($\rho = 1$) is also interesting since, in this case, for $t = 1$ in each region an amount whatever of varieties produced in either region is consumed; when an even very small transport cost exists, however, each region will immediately shift all its consumption on the domestic goods (which will be consumed in undefined proportions) and no trade at all will take place.

4 Innovation Dynamics

Since, as shown in the previous sections, the disparities of income may depend on technology, as represented by the number of varieties that it is possible to produce in each region, the dynamics of innovation (i.e. the creation of new varieties) and diffusion (i.e. the flows of knowledge on some varieties from one region to the other) will determine the long run equilibrium of a multiregional system in terms of relative income.

When innovation is not modeled as a separate sector, there are basically two relevant approaches in the literature, both coming from north-south models where innovation is assumed to be a process cumulative on previously possessed knowledge, with no decreasing returns to scale; the two approaches differ in what new knowledge is built on.

The first one is due to Krugman (1979) who assumes that:

$$\dot{N}_n = \dot{N} = \alpha N \tag{30}$$

In this approach, also adopted by Grossman and Helpman (1990), the quantity of new varieties invented in the north in a period is proportional to the varieties known in the north, that is, in a north-south model, to all the varieties existing in the economy.

The second approach is due to Dollar (1986) which instead assumes that:

$$\dot{N}_n = \dot{N} = \alpha N_n \tag{31}$$

In this second approach north innovation still builds on previous knowledge but now only on those varieties (N_n) which are currently produced by the north and not on all the varieties (N) that it would be able to produce.

The specificity of this literature is that the north and the south are assumed to be different: the north is, due to various plausible justifications, able to innovate, the south, instead, is not able to innovate (i.e. it can't produce any new variety unknown to the north) but only to imitate, i.e. acquire from the north the technology used to produce other varieties. This difference is also basically the definition of north and south. These models are in fact asymmetric by construction, but this does not constitute a relevant limit when they are applied to international trade and growth at world level.

Since the interest of this paper is instead on regional disparities or national disparities among similar countries, the asymmetry is indeed a limit.
For this reason, in sections 2 and 3 we developed a symmetric model which is consistent in the conclusions to the one of Krugman (1979), but with an important extension: the possibility for each region to be the most advanced and, therefore, no a-priori assumed or structural difference among the regions.

In this section we will investigate the dynamics and the extent of innovation and diffusion and analyse the results with respect to the static model developed in the previous sections. Without any assumption on exogenous differences among the regions, we will be able to evidence when regional disparities arise. We will use two different dynamic mechanisms, each inspired by one of the equations 31 and 30. The use of both is interesting, in addition to speculative interesting, because they lead to about the same conclusions but in different ways, the one in terms of probability, and the other with the presence of deterministic multiple equilibria.

In this section we will build a dymanic model that, used jointly with the static model of section 2, will be able to explain the evolution of regional disparities in relation to the magnitude of innovation and diffusion, without any assumption on exogenous differences.

4.1 First model of innovation dynamics

The first and simplest type of dynamics that we will implement is related to equation 31: we will here assume that each region will be able to innovate cumulatively creating new varieties on the base of varieties that it has

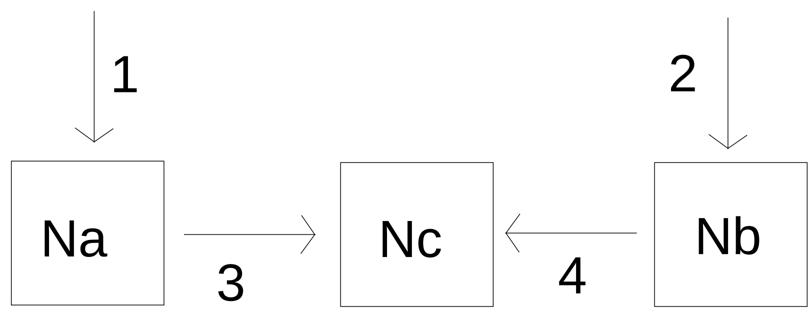

Figure 2: Type 1 representation of the flows of knowledge

exclusive ability to produce, i.e. that the common varieties are sufficiently old that no new product can be invented building upon their knowledge. The dynamics of innovation and diffusion will be defined by the following differential equations:

$$\dot{N}_a = \alpha N_a - \gamma N_a \tag{32}$$

$$\dot{N}_b = \beta N_b - \gamma N_b \tag{33}$$

$$\dot{N}_c = \gamma N_a + \gamma N_b \tag{34}$$

As a consequence:

$$\dot{N} = \alpha N_a + \beta N_b \text{ since } N = N_a + N_b + N_c \tag{35}$$

Where γ is the parameter of technological diffusion and α and β are now the regional parameters of innovation, which will depend on a number of factors including, for example, the quota of researchers r_a and r_b.

Graphically we can portray the process as in Fig. 2: arrows 1 and 2 represent innovation (the creation of new varieties) as flows into the sets of exclusive varieties N_a and N_b, regulated by the respective parameters α and β. Arrow 3 is the first diffusion flow of knowledge, which becomes available to B after having been available exclusively to A; this flow is regulated by the diffusion parameter γ. Arrow 4 is the diffusion of knowledge that becomes available to A after being available exclusively to B, also regulated by γ.

The dynamics of this system can be easily transformed in a set of two differential equations by defining:

$$\frac{N_a}{N} = x$$

$$\frac{N_b}{N} = y$$

After easy calculations, the dynamics will be entirely defined by the following two equations:

$$\dot{x} = x(\alpha - \gamma - \alpha x - \beta y) \tag{36}$$

$$\dot{y} = y(\beta - \gamma - \alpha x - \beta y) \tag{37}$$

$\dot{x}$ will be equal to 0 when:

$$x = 0 \text{ or } y = \frac{\alpha - \gamma}{\beta} - \frac{\alpha}{\beta}x$$

$\dot{y}$ will be equal to 0 when:

$$y = 0 \text{ or } y = \frac{\beta - \gamma}{\beta} - \frac{\alpha}{\beta}x$$

From the conditions above it is possible to obtain the phase diagram, represented in Fig 3. In the diagram (represented for the case in which $\alpha > \beta$) there are three equilibria of which one (unstable) (O) with all the varieties being common, one (A, stable) in which region A has a quota $\frac{\alpha-\gamma}{\alpha}$ of exclusive varieties and the rest are common and one (B, saddle) in which region B has a quota $\frac{\beta-\gamma}{\beta}$ of exclusive varieties and the rest are common.

According to the model of section 2, in case in which the regions are of equal size in terms of active workers, there exist income equality in an x,y, diagram inside the box shaded in Fig. 4. This means that if $\alpha > \beta > 2\gamma$, both A and B will be equilibria with regional disparities; when $\alpha > 2\gamma > \beta$ there will exist regional disparities only in A, which is however the only stable equilibrium. For $2\gamma > \alpha > \beta$, finally, in both the equilibria the technological differences will not be sufficient to generate income disparities among the regions. For this reason, we can conclude that the size of the diffusion of knowledge is essential in the determination of the existence of regional income disparities.

Since in this paper we are analysing the evolution of regional disparities among structurally identical regions, we are much more interested to case in which we don't assume any difference of innovative effort between the regions ($\alpha = \beta$); in this case the phase diagram is the one of Fig. 5, where any point in the segment AB is a feasible stable equilibrium outcome for the economy, but only the part included in the square OPQR is composed of equilibria without income disparities.

In particular, if $\alpha < 2\gamma$, in all the possible equilibria there will be equal income. If $\gamma = 0$ in all the equilibria differences of income will exist.

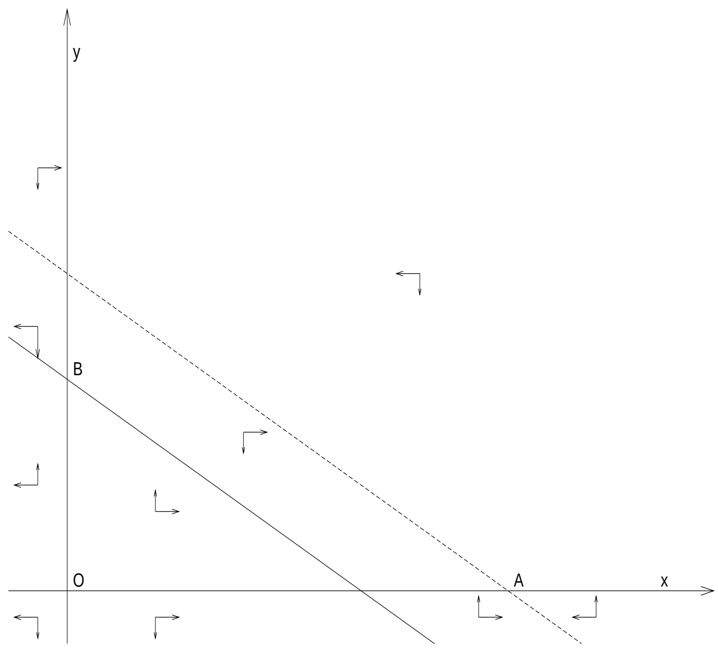

Figure 3: Phase diagram of type 1 dynamics with $\alpha > \beta$

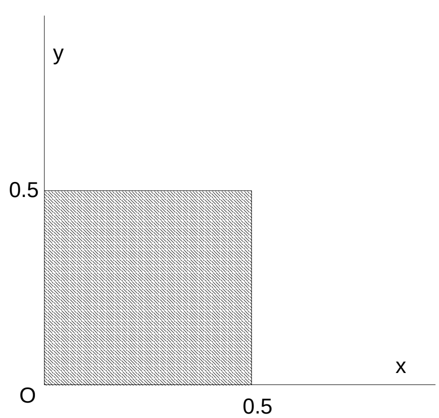

Figure 4: Values of x and y for which there is no income disparity

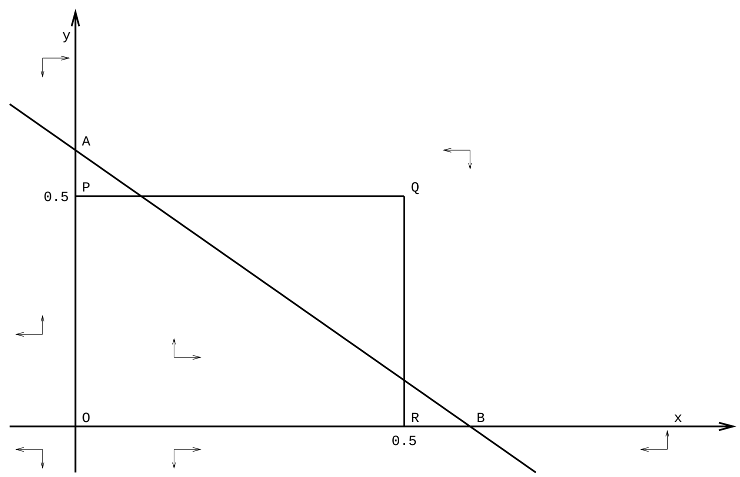

Figure 5: Phase diagram of type 1 dynamics with $\alpha = \beta$

An increase in α $(= \beta)$ shifts the segment AB upwards, whilst an increase in γ shifts it downwards. For this reason, when in an economy the pace of innovation increased, it becomes more probable to find equilibria with differences of income (since a smaller part of the segment AB will be inside OPQR). If, instead (or in a successive period), something makes easier the diffusion of knowledge from one place to the other, income disparities will become less probable. Again the effect on regional disparities of the innovation concurrence between regions depends on the interaction between the parameters regulating the speed of innovation and the speed of diffusion.

5 Second model of innovation dynamics

In section 4 we presented a dynamic model which has as outcome that regional disparities become more probable if innovation runs faster and less probable if technological diffusion become easier. However, we are not able to know which equilibrium will be the outcome of the model and, as a consequence, if regional disparities will indeed exist.

If, differently from the previous section, we build a more comprehensive model, now based on equation 30, we can get definite equilibria. Due to the higher complexity of this case, it will be useful to normalize the total number of varieties to 1 and, through appropriate processes of obsolescence, to make this number remain constant. For our purposes, in fact, it is the ratio of varieties possessed by the regions that matters: on the one hand, we

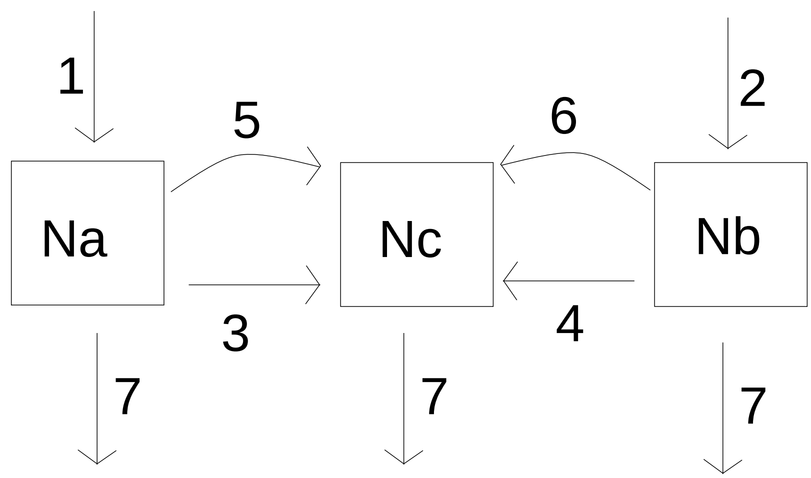

Figure 6: Type 2 representation of the flows of knowledge

are focusing on regional disparities and not on growth, therefore we are only interested in the ratios and the dynamics of N_a/N and N_b/N is not affected by the normalization we impose to N; on the other hand it is not completely realistic to allow the people to consume a whatever large number of goods in any small proportions.

There is another assumption to make, in favor of realism: we have to allow a region to invent products that the other region is already able to produce, and not only to imitate them. A clarifying historical example is the "run to space" between the Americans and the Russians: a part of the technology independently developed by both of them was the same even without espionage from the other.
How big would be the part of new region A technology already possessed by region B? The best assumption is to make it proportional to the size of region B technology compared to all the technology existing; in fact, this assumption verifies that all A innovations are known to B if B knows everything and all A innovations are new to B if B has no exclusive knowledge.

For the above reason, the innovation of A is now represented by two flows of knowledge: one is completely new innovation (arrow 1 in Fig. 6) and the other one is made of innovations that make some varieties previously exclusive to B become also available to A (arrow 6 in Fig. 6), this one adds to spontaneous diffusion, still represented by arrow 4.

Obsolescence, justified by the normalization but also by the fact that new products often substitute the old ones in the utility function, is represented by the three flows of arrows 7. At this stage, it is useful to add to the model the possibility to represent speedier obsolescence for the elder varieties, that is those common to both regions (since they were first invented and then imitated). We will introduce for this reason a parameter m, representing the ration between the speed of obsolescence of old varieties and the speed of obsolescence of new varieties.

We also allow the model to represent decreasing returns to scale in the creation of knowledge, by the addition of a parameter σ. Finally, in the calculations of the model a virtual parameter π is introduced as a means to more easily calculate the dynamic equations which satisfy all the hypotheses; the π obviously cancels itself out when solving the system.
The equations of the model, as described above, will be the following:

$$\dot{N}_a = \alpha\frac{N_a + N_c}{N}(N_a + N_c)^{\sigma} - \gamma N_a - \beta\frac{N_a}{N}(N_b + N_c)^{\sigma} - \pi N_a \tag{38}$$

$$\dot{N}_b = \alpha\frac{N_b + N_c}{N}(N_b + N_c)^{\sigma} - \gamma N_b - \beta\frac{N_b}{N}(N_a + N_c)^{\sigma} - \pi N_b \tag{39}$$

$$\pi = [\alpha(N_a + N_c)^{1+\sigma} + \beta(N_b + N_c)^{1+\sigma}]/[N_a + N_b + mN_c] \tag{40}$$

$$\dot{N}_c = \gamma(N_a + N_b) + \alpha N_b(N_a + N_c)^{\sigma} + \beta N_a(N_b + N_c)^{\sigma} - m\pi N_c \tag{41}$$

With these equations, it is always verified that:

$$\frac{N_a}{N} = N_a = x \ , \ \frac{N_b}{N} = N_b = y \ , \ N = 1 \text{ and } \dot{N} = 0$$

After easy computations we obtain two dynamic differential equations describing all the dynamics of the system, since $\frac{N_c}{N} = 1-x-y$ by construction:

$$\dot{x} = \alpha(1-y)^{1+\sigma}\frac{m + (1-m)(x+y) - x}{m + (1-m)(x+y)} - \gamma x - \beta x(1-x)^{\sigma}\frac{m + (1-m)(x+y) + (1-x)}{m + (1-m)(x+y)} \tag{42}$$

$$\dot{y} = \beta(1-x)^{1+\sigma}\frac{m + (1-m)(x+y) - y}{m + (1-m)(x+y)} - \gamma y - \alpha y(1-y)^{\sigma}\frac{m + (1-m)(x+y) + (1-y)}{m + (1-m)(x+y)} \tag{43}$$

5.1 Dynamic behaviour

We are now able to study the long run equilibria of the model, depending on the parameters.
We begin with the easiest case and assume no decreasing returns to scale (as it is most common in the literature) in innovation ($\sigma = 1$), obsolescence

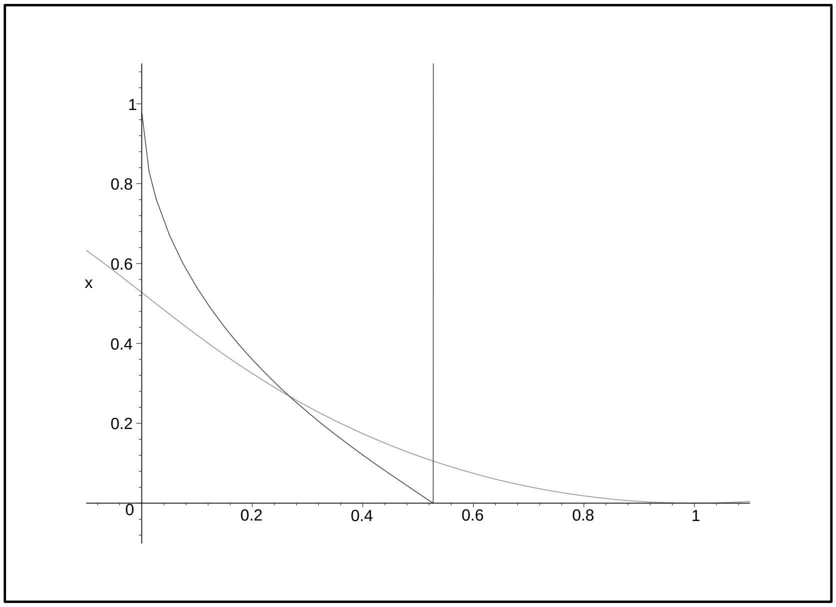

Figure 7: Phase diagram of type 2 dynamics with m=1

affecting old products in the same measure as the new products ($m = 1$) and regions equally investing in innovation ($\alpha = \beta$), since we are always investigating disparities between identical regions.

In this case the system only has one equilibrium which is stable, shown in the phase diagram in fig. 7. The modification of the parameter affects the equilibrium only quantitatively, since the curves still cross the axes at (1,0) and (0,1) but are slightly rotated. When α increases, the equilibrium shifts upwards; when γ increases, the equilibrium shifts downwards; these shifts leave unchanged the fact that in equilibrium $x = y$; the only thing that changes is the quota of common varieties existing in the economy, this quota is higher the faster the spatial diffusion of knowledge.

If instead, while maintaining $\alpha = \beta$ and $\sigma = 1$, we allow, realistically, to have obsolescence affecting faster elder varieties ($m > 1$), the phase diagram gives more interesting results: in fact, for relatively low values of α and relatively high values of γ (Fig. 8), the phase diagram is similar to the one shown above, with only one (stable) symmetrical equilibrium. However, if α becomes relatively high compared to γ (Fig. 9), the symmetric equilibrium of the phase diagram becomes a saddle point and two asymmetric equilibria arise, one with A owning a large part of the technology, and one with B in the same situation.

Since, once assumed $\alpha = \beta$ and $\sigma = 1$, the equilibria depend only on m

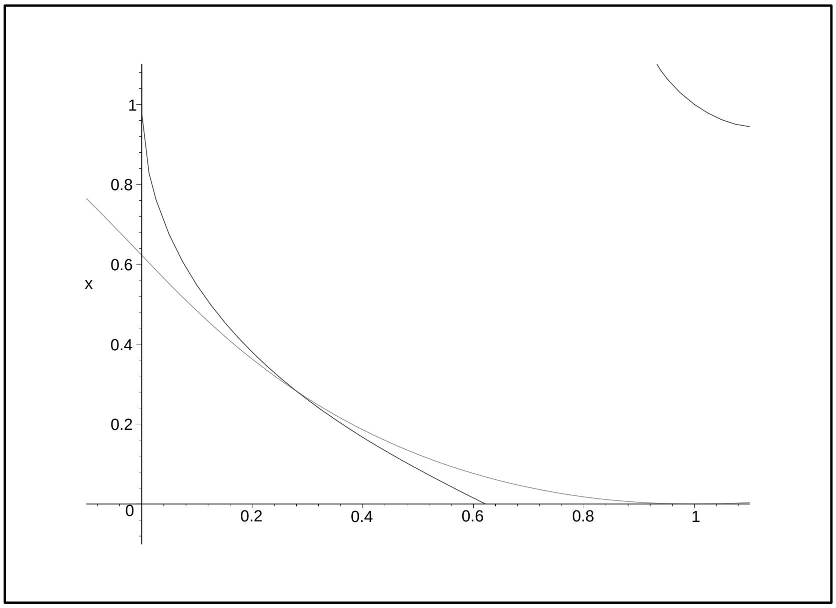

Figure 8: Phase diagram of type 2 dynamics with m=2, α=0.5 and γ=0.2

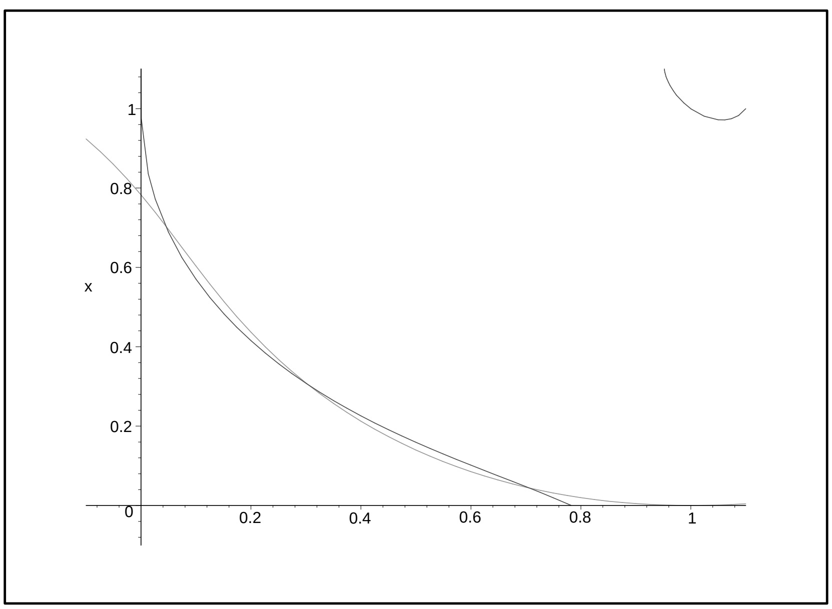

Figure 9: Phase diagram of type 2 dynamics with m=2, α=0.5 and γ=0.1

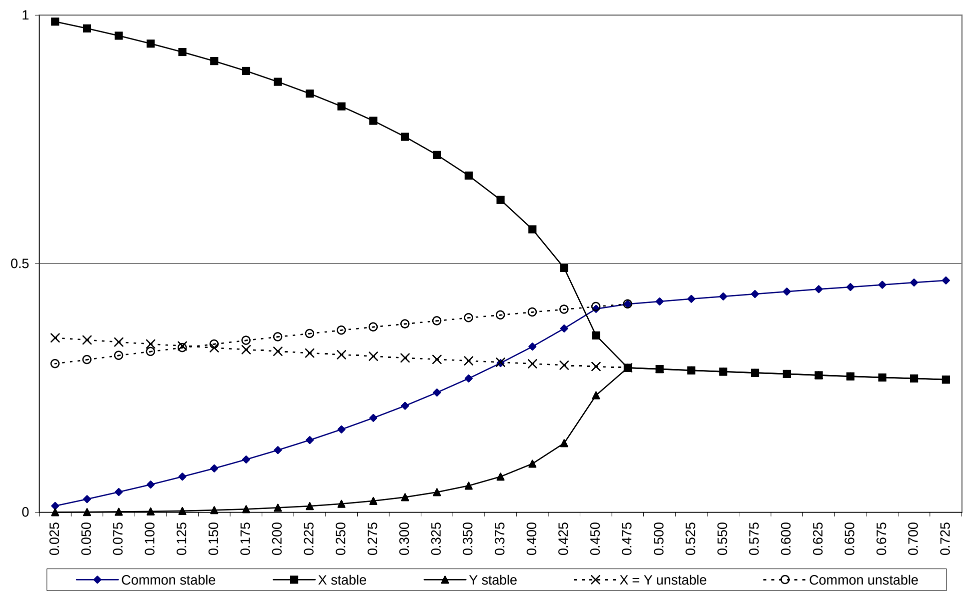

Figure 10: Representation of the equilibria of the model, depending on the size of $S = \gamma/\alpha$, case of $m = 3$.

and on the ratio of the speeds:

$$S = \frac{\gamma}{\beta} \tag{44}$$

It is possible to solve numerically the system for any m and S and obtain, as in Fig. 10, a diagram that, for given m, plots the equilibrium values as a function of S.

In figure 10 we observe that, for low values of S, i.e. for innovative forces relatively high with respect to diffusion, there exist one unstable equilibrium with $N_a = N_b$ and two symmetric stable equilibria (of which Fig. 10 plots only the one with $x > y$) with one of the two regions owning a large part of the varieties existing and only a few varieties owned by the other region or common.

As far as S grows, in the stable equilibria the most advanced regions becomes relatively less endowed of technology and the number of common and other region's varieties grow.

There exists a point S^* in which the proportion of varieties owned by the most advanced region reaches 0.5[1]. For values of $S \geq S^*$, even if one region is

[1]This is the critical value in this easier case in which $L_a = L_b$, $r_a = r_b$ and the number

more technologically endowed, the difference is not, according to the model of section 2, large enough to generate income disparities.

If S increases further, it reaches a point S^{**} where the stable and the unstable equilibria begin to coincide. For $S = S^{**}$, $N_a = N_b$ and there are neither income disparities nor technological disparities. If S grows above S^{**}, the model does not change qualitatively but only the quota of common varieties becomes larger with respect to those that are exclusive of one of the two regions.

What is the effect of m, the parameter which represents the ratio between the speed of obsolescence of old varieties with respect to new ones? In the most plausible case in which $m > 1$, a change of m affects the predictions of the model only qualitatively, by moving S^* and S^{**}; in particular an increase in m increases both S^* and S^{**}, i.e. it makes easier the emergence of regional technological and income disparities.

The essence of this reasoning is that for low values of S (i.e. with innovation being fast compared to technological diffusion) the economy will have regional technological disparities generating income disparities. For intermediate values of S the equilibrium of the economy will present regional technological disparities but less wide, so that there will not be income disparities. For High values of S, the model predicts stable symmetric equilibrium with all the regions being equally endowed technologically and, therefore, with no income disparities.

The model has therefore this very important prediction: from a symmetric situation, the acceleration of the technological pace will drive to an equilibrium with regional disparities. Even if a decrease of innovative effort is not possible[2], however, there still is a way to make the disparities decrease, by increasing the speed of the processes of diffusion of knowledge.

6 Conclusions and policy consequences

This article, despite using a simplified framework[3] has achieved a step towards the modeling of innovation diffusion and evidenced the consequences this process entails for regional disparities. The base model of section 2 evidenced that regional disparities can be caused by different technological endowments, when these differences are wide enough. The model features

of total varieties is standardized to 1. Otherwise, according to the model of section 2 we would have to write the condition $\frac{x}{L_a(1-r_a)} = \frac{1-x}{L_b(1-r_b)}$.

[2] And not desirable for general growth and aggregate welfare reasons, which are left out of this article.

[3] It does't have capital and growth, and the innovation sector is not modeled directly.

the possibility of identifying who will produce the varieties whose knowledge is common between the regions, in this way it becomes symmetric and apt to represent bi-regional systems in which a region is similar to the other; in this sense, it is an advancement on the traditional north-south models in which the two regions are assumed to be different. In particular this feature is useful for the representation of flows of knowledge from one region to the other and vice versa.
The introduction of transport costs in this model, in section 3, gives results which are common in the recent economic geography literature, since positive transport costs have negative aggregate welfare effects but diminish the disparities between the regions.

By using the technology endowment layout of the basic model, and its consequences for income disparities, in section 4 we have been able to study the dynamics of the flows of knowledge and evidence that, for higher values of the speed of innovation, it becomes more probable to have income disparities, whilst, if diffusion becomes faster, income disparities become less probable.

In section 4, we analysed the same problem in a more realistic and complex framework and obtained a similar but more precise result. The key variable is the ratio between the speeds of diffusion and innovation. When it is low, the system present multiple equilibria in which there are regional technological and income disparities. For intermediate values, even there exist multiple equilibria in which a region is more technologically advanced, the spatial diffusion mechanism allows the other region to have the same income. For higher values of the ratio, there exists only one equilibrium in which the two regions are equally endowed of technology and, as an obvious consequence in this model, have the same income.

The conclusion of this model is that a process of 'technological revolution', like the one that took place in the XXth century, could have contributed to generate regional disparities. However, even if a reduction in the speed of innovation is not envisageable, the model also predicts that, if the diffusion of technology becomes fast enough, regional disparities can decrease. A huge number of studies point out that the transfer of blueprinted knowledge is not by itself enough to make other people able to use that information; however, the ICTs revolution which is taking place is making more smooth the diffusion of innovations from one region to the other. At the same time, the speed of innovation might increase further.
For this reason we are not able to identify the final outcome of these World processes at this stage but, with the results of the model, it is possible to draw an important policy prescription that comes from the very important role of the spatial diffusion of knowledge after invention has taken place: due to the importance of knowledge for competitiveness, the policy makers

are in fact often "obsessed" by the desire to create the next Silicon Valley (Audretsch, 2002), but this strategy, in addition to being overwhelmingly difficult, can also be wrong for the development of least advanced regions.

Since innovation is a cumulative process, in fact, there must be a technological base wide enough to build upon it; if a region is lagging behind and does not possess front-line technology, even implementing strong innovative efforts it will not be able to create a large quantity of new knowledge, and, in addition to that, a large part of the 'new' discoveries, could even be already 'old' for the most advanced regions.

It can be therefore an easier and more effective strategy, when trying to make an under-developed region catch up with the richer, to target, at least in a first phase, the spatial diffusion of knowledge and to allow in this way the lagging region to enter rapidly in competition with the foremost regions in the production of goods which remain invented by the most advanced regions.

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