



UNIVERSITÀ DEGLI STUDI DI ANCONA
DIPARTIMENTO DI ECONOMIA

**COMPETITION-LED ENDOGENOUS
GROWTH WITH LOCALIZED
TECHNOLOGICAL CHANGE**

EDOARDO GAFFEO

QUADERNI DI RICERCA n. 91

QUADERNI DI RICERCA



UNIVERSITÀ DEGLI STUDI DI ANCONA
DIPARTIMENTO DI ECONOMIA

**COMPETITION-LED ENDOGENOUS
GROWTH WITH LOCALIZED
TECHNOLOGICAL CHANGE**

EDOARDO GAFFEO

QUADERNI DI RICERCA n. 91



giugno 1997

Comitato scientifico:

Antonio G. Calafati
Paolo Ercolani (coordinatore)
S. Manzocchi
A. Niccoli

Competition-Led Endogenous Growth with Localized Technological Change

*Edoardo Gaffeo**

Abstract

The paper aims to amend the standard monopolistic competition framework - and its endogenous growth extension - introducing both heterogeneity among producers and a form of 'Schumpeterian' competitive selection process. According to our view, the introduction of heterogeneity implies that the notion of a symmetric Nash long-run equilibrium solution has to be discarded. This amendment permits to appreciate the role played by preferences and technology in determining the degree of persistency of nominal shocks when the assumption of perfect foresight is abandoned. Furthermore, a trade-off between short run and long run neutrality is assessed.

The revised model is then employed in providing some contents to a couple of stylized facts about long-run aggregate growth. Instead of being modelled as a continuous increase of the commodity space, growth is described as succesful R&D investments in decreasing production-cost technologies. Innovation and imitation processes are modelled as activities requiring different amounts of investment, whose outcome is uncertain both as regards the time it takes to occur and the amount of effective increase in productivity it allows. Furthermore, technological change is governed by localized spillovers, so that it can be analytically treated by means of a Markovian random fields' machinery. Preliminary results obtained through a 'Mean Field' approximation allows us to appreciate the influence that alternative assumptions on the nature of localization of technological change bear on the rate of growth of average productivity.

Key words: *Endogenous Growth; Schumpeterian Hypothesis; Localized Technological Change; Local Interaction;*

JEL classification: *C60; L22; O33; O40.*

**Dipartimento di Economia
Università di Ancona
Via Birarelli, 11
60121 - Ancona, Italy.
tel. 071 - 2203900
fax 071 - 53621
e.mail: edoardog@deanovell.unian.it*

TABLE OF CONTENTS

1. Introduction	p. 1
2. Some stylized facts about markets, technological change and growth	3
3. Introducing heterogeneity in a standard monopolistic competition model	5
3.1 A simplified monopolistic competition model	6
3.2 Real effects of nominal shocks	7
3.3 Effective competition and market selection	11
4. Transitional dynamics towards a new long-run equilibrium	13
5. Growth driven by localized technological change	17
5.1 Technological change as a two-stage stochastic process	19
5.2 Simulation results	22
6. A mean field analysis	24
6.1 Approximating innovation	26
6.2 Approximating imitation	29
6.3 Mean field approximation of the productivity growth rate	30
7. Concluding remarks	31

1. Introduction*

In the last few years the monopolistic competition framework has gained the centre stage of macroeconomics, one of the main reasons being the embarrassing lack of empirical evidence supporting the predictions of standard perfect competition models. In fact, coupled with nominal and real rigidities both in product and labour markets, monopolistic competition has become pretty soon the cornerstone of the new-keynesian approach¹.

Since the works by Romer (1990) and Grossman and Helpman (1991), non-competitive (in a neoclassical sense) markets have been introduced in dynamic contexts as well, as a useful benchmark for the theory centred upon endogenous growth. In particular, Grossman and Helpman have associated growth with a continuous increase in the variety of commodities available for consumption. Furthermore, in such a setting the assumption of preferences of the Dixit-Stiglitz type implies that an increase in variety amounts to an improvement on welfare.

While particularly useful in introducing an endogenous channel in the implementation of R&D investments, our general impression is that these models fail in recognizing a striking empirical feature. The commodity space, i.e. the number of goods available for consumption, has historically not merely increased, its intertemporal movement being described also by a continuous rightward shift. In other words, while it is undeniable that completely new goods are introduced at a given rate, it must be stressed that at the same time a certain number of old goods disappear from our shops' shelves. The introduction of gasoline engines, of electric lighting and of kevlar tennis rackets has contributed to the growth of per-capita output and to the welfare of consumers all around the world (at least the industrialized one), but at the same time steam engines, oil lighting and wooden rackets do not continue to be available for consumption with a positive price: they simply went out of market. As a consequence, the methodological choice of approximating growth by the mean of a continuous increase in the number of available commodities seems to us at least questionable. On the contrary, we advance that monopolistic competition may be usefully employed in modelling growth due to process innovation as well.

* Previous drafts have highly benefited from comments and suggestions by Reiner Franke, Mauro Gallegati, Stefano Manzocchi, Stefano Staffolani, Alessandro Sterlacchini, and the participants at the 2nd workshop on Economies with Interacting Agents, Ancona, May 30-31 1997. The usual disclaimer applies.

¹ Among the copious literature nowadays available on macroeconomic models based on monopolistic competition, see Blanchard and Kiyotaki (1987), Galí (1994), Kiyotaki (1988), Krugman (1991), Murphy, Shleifer and Vishny (1989) and Startz (1989), as well as the surveys by Benassy (1991), Matsuyama (1995) and Silvestre (1993).

The arguments above are no more than examples, but they are useful in illustrating the basic point of '*creative destruction*', which relies both on process and on product innovations as the engine of growth. Introduced by Joseph A. Schumpeter in the first half of this century (Schumpeter (1934), (1942)), the idea of selection through competition among alternative technologies has been revived recently both by evolutionary associates and by neo-classical ones².

The main goal of this paper is to contribute in throwing a bridge between the nowadays prevailing approach of Chamberlinian inspiration and the more heterodox evolutionary stream of thought. This attempt is pursued extending the standard monopolistic competition framework in two directions: first, introducing heterogeneity among producers; second, investigating how and why suppliers of consumption goods should interact among them, respectively. In particular, heterogeneity allows us to shed some light on the Bertrand-type selective dynamic process which drives the economic system towards an asymmetric long-run equilibrium. In turn, dynamic considerations implies to explicitly take into account how R&D activities interact with the selective process led by the competitive pressure. In a few words, the competitive position held by a producer plays a double role: while efforts in R&D are a function of the competitive position reached so far (measured by profits), successfully R&D allows the firm to increase its advantage over competitors, in a process of microeconomic cumulative technological growth, from profits to innovation to growth to new profits due to increased competitiveness. Agents knows to be heterogeneous, and it is rational to take advantage of heterogeneity interacting. Hence, we assume that investments in R&D are characterized by some kind of positive spillovers.

The argument is definitely not new, and it has so far received a considerable attention by the profession. However, both the non-competitive endogenous growth theorists and the Schumpeterian ones have largely considered technological spillovers as phenomena generalized to the whole economy. According to this view, each firm can gain from the (embodied or disembodied) knowledge belonging to any other firm, independently of its technological or geographical distance. On the contrary, when distance - whatever form it takes - matters we talk of localized technological change. We explicitly analyze how localized spillovers in R&D shape the path of growth of our model economy. In doing it, we will follow the pioneering

²For evolutionary contributions, see Henkin and Polterovich (1991), Iway (1984), Nelson and Winter (1982), Silverberg (1984), Winter (1984) and the survey by Witt (1992); for neo-classical counterparts, see Aghion and Howitt (1992), Cheng and Dinopolos (1992) and Segerstrom, Anant and Dinopolos (1990).

work of Hans Föllmer (1974) on local interaction systems modelled as random fields.

The paper proceeds as follows. After a brief discussion, contained in section 2, of some stylized facts searching for a unified interpretative framework, section 3 discusses how heterogeneity modifies the long-run properties of the basic monopolistic competition model. The passage from comparative statics to transitional dynamics is analyzed in section 4, where we explicitly introduce learning-by-doing and learning-by-using phenomena in technological adoption. Section 5 extends the discussion dealing with the influence on growth of the two most important R&D activities: innovation and imitation. Both of them are supposed to be localized, in a sense that will be discussed in some detail. Section 6 focuses on a mean field analysis of the composite stochastic process we adopt to describe technological change. Finally, section 6 summarizes the paper.

2. Some stylized facts about markets, technological change and growth.

More or less like a lazy tourist who decides to transmit home a rough idea of the landscape he admired sending a postcard, economists have found particularly convenient to state what they want to explain through the theories they elaborate using simplifying images of how real economies work. Admittedly, this behaviour is particularly useful - and probably the only one we are allowed to use - whenever we have to describe a continuously changing and complex reality.

In this section we adopt the same strategy, proposing a selected list of stylized facts which in our opinion are at the core of the dynamics of growth and decline of firms in a competitive setting. Some of them are nowadays widely accepted by the profession, even if not always included in standard neo-classical models. On the contrary, a couple of facts with heavy implications for the story we want to tell, i.e. the presence of effective competition among heterogeneous agents and the localization of technological change, have been seldom enclosed in macroeconomic models. The facts we marshal are the following:

- i. Usually, markets are characterized by firms using heterogeneous technologies to produce non-perfectly substitutable commodities. As a consequence, at any point in time they experience differences in their relative competitiveness. This argument has two important implications: first, the empirical evidence is really at odds with the standard assumption heavily used by modern macroeconomics of a 'representative firm'; second, the most appropriate theoretical

framework to be used in studying economics is likely to be the monopolistic competition model, not the perfect competition one.

- ii. Each firm tries to improve its competitiveness through the adoption of more efficient technologies, often incorporated into capital equipments. Besides, adoption does not usually involve an immediate increase of productivity, because of a set of '*retardation factors*'. Among the others, retardation is a function of learning-by-doing and learning-by-using phenomena, of problems in adapting a new vintage of capital to old vintages, and of the need of retraining the employees.
- iii. Innovation and diffusion processes are characterized by localized spillovers. 'Local' is not here just considered in a geographic meaning, but also in the degree of substitutability among commodities and among the technologies used to produce them. In other words, it is easier (and provides higher incentives) to discover innovative and less expensive ways to produce whenever close (in some sense) to technologically up-to-date competitors. On the other side, it results easier to imitate an external innovation if this is somehow related to the technology we are presently using.
- iv. Innovation and diffusion are inherently uncertain activities. For any given level of investment in R&D, uncertainty matters at two levels: the time it takes to obtain some results, and the degree of innovativeness of the R&D outcome with respect to the present technology.
- v. Imitation is hardly instantaneous. We are not referring simply to the well-known fact that imitative processes follow logistic diffusion patterns among the firms' population, so that a new available technology is adopted by different firms at different times. We label as 'lack of contemporaneity' the presence in almost every country of a patent system or of other legal protection's devices preventing follower firms from adopting the most efficient technology. Furthermore, the very fact that a technology is incorporated in capital equipments implies that a firm has to face sunk costs in discarding old technologies. As a consequence, at any time there is a non-negligible number of firms using technologies not positioned on the efficient frontier.

The stylized facts from i) to v) refer exclusively to microeconomics' phenomena. Given that the aim of this paper is to explore how competitive struggles among firms cause the growth process of aggregate economies and

shape the form it takes, a couple of macroeconomics regularities on growth follows, which in our opinion are strictly related to the previous ones. A fascinating introduction to the topic of interrelations between technological change, its rate of growth and the development of industrialized economies is provided by Abramovitz (1989) and Landes (1969). Here we claim that:

- vi. Technological change represents one of the most important factor determining aggregate economic growth. The forms in which it has appeared during the last two centuries have been basically an increase in the number and the quality of commodities available for consumption, and an increase in the productivity to obtain them.
- vii. Per-capita output increases at a rough constant rate in the very long-run. However, if we analyze the process more closely, i.e. for shorter periods, we can observe that its path is not smooth at all, but it might be better described as an increasing process subject to continuous and irregular fluctuations around one (or more) hypothetical trend(s).

The remaining of the paper tries to compose into a unifying picture the several postcards here presented. In particular, we will try to explain how the macroeconomic stylized facts about long-run growth vi) and vii) can be obtained as the result of a model where the microeconomic regularities singled out above are explicitly taken into account.

3. Introducing heterogeneity in a standard monopolistic competition model.

As we already noticed, the class of growth models bearing on the Ramsey-Cass-Koopmans framework employs a notion of competition which has nothing to do with what is currently intended by the very notion. In turn, the increasing available evidence seriously threatening the correctness of its predictions has led a consistent part of the profession to look for more realistic ways of describing how economic systems really work.

The resurgence of attention for imperfect competition models recently occurred represents a first step in this route. Unfortunately, the assumptions of homogeneity among agents and of symmetry along the final long-run equilibrium have deeply permeated the literature at hand. Some useful insights could be obtained whenever these assumptions are dropped. In order to do this, let us focus on a simplified version of the standard monopolistic competition model developed by Blanchard and Kiyotaki (1987).

3.1. A simplified monopolistic competition model.

Consider an economy with K final goods, each one produced by a single firm operating as a monopolist. A representative consumer endowed with a constant elasticity of substitution (CES) additive utility function determines the following standard demand curve for the commodity j :

$$Y_j^d(t) = \left(\frac{p_j(t)}{Q(t)} \right)^{-\sigma} \frac{Y^d}{K} \quad (1)$$

where $p_j(t)$ represents the price charged by the monopolist j , Y^d is the real aggregate income available for consumption in the whole economy, σ is the constant elasticity of substitution between couples of good, with $\sigma > 1$; and finally $Q(t)$ is a CES index for the aggregate price level:

$$Q(t) = \left(\frac{1}{K} \sum_{j=1}^K p_j(t)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (2)$$

We assume that producers are heterogeneous. In our setting, heterogeneity is postulated to matter just at the cost level. Thus, a generic monopolist j can obtain its output only bearing the total cost:

$$C_j(t) = \frac{Y_j(t)^\alpha}{e_j(t)} + F \quad (3)$$

with $\alpha > 1$. Expression (3) states that for a given peculiar technology parameter $e_j(t)$, the monopolistic firm experiences increasing static marginal costs (for a convincing evidence for this assumption see Bilal, (1987)). Costs could nevertheless be dynamically decreasing, whenever the variable $e_j(t)$ follows a sufficiently fast increasing path. In turn, F captures the fact that each technology is embodied in some kind of capital equipment. In order to simplify the exposition, we assume that the capital depreciates in one period, so that firms must incur in a fixed cost each period; that fixed costs are homogeneous among producers; and finally that fixed costs remain constant as the technology embodied in the capital equipment varies, respectively. These assumptions allow us to avoid any discussion about the role played by overlapping vintages of capital stock and possible linkages between capital stock depreciation and productivity.

At the beginning of time t , $\forall t \in T$, the K monopolistic firms play a one-shot game adopting Bertrand-type strategies. Monopolist j determines the price

he decides to fix in selling the good he produces, given consumers preferences and the aggregate price index. Recall the latter measures the average behaviour of his competitors. Thus, the problem consists in maximizing the expression for period t profits with respect to the relative price $\frac{p_j(t)}{Q(t)}$:

$$\max \pi_j(t) = p_j(t) \left(\frac{p_j(t)}{Q(t)} \right)^{-\sigma} \frac{M(t)}{Q(t)} - \frac{1}{e_j(t)} \left[\left(\frac{p_j(t)}{Q(t)} \right)^{-\sigma} \frac{M(t)}{Q(t)} \right]^\alpha - F \quad (4)$$

where for ease of notation we substituted $\frac{M(t)}{Q(t)}$, the real amount of money,

for $\frac{Y^d}{K}$. This substitution may be considered a standard trick in this literature, and it follows from the assumption of money as a medium of exchange supplied by the mean of a quantitative rule. The 'speed' of currency is assumed fixed and equal to the inverse of the number of goods produced in the economy (see Rotemberg, (1987)).

From the first order condition we obtain the monopolist j 's Best Replay Function (BRF):

$$\frac{p_j(t)}{Q(t)} = \left[A \frac{1}{e_j(t)} \left(\frac{M(t)}{Q(t)} \right)^{\alpha-1} \right]^{\frac{1}{\delta}} \quad (5)$$

where $A = \frac{\sigma\alpha}{\sigma-1}$, and $\delta = 1+\sigma(\alpha-1)$.

3.2. Real effects of nominal shocks.

In the monopolistic competition literature the equilibrium solution is typically found as a symmetric Bertrand-Nash static equilibrium. This characterization proves to be particularly useful to overcome the problem of determining the aggregate price index. Indeed, when the one-shot game is played simultaneously by homogeneous players, the right value of $Q(t)$ can always be found simply because each firm has all the relevant information one needs to calculate the optimal price of other players. At each time t , each monopolist is allowed to choose its optimal price along its BRF, which in turn is a function of time t aggregate price index: at least implicitly, the

aggregate price index must be forecasted. Introducing heterogeneity raises general questions on the workability of such an equilibrium concept.

In a heterogeneous environment, what one needs to calculate both its optimal price and, at the very same moment, the aggregate price index is a perfect knowledge of the technology employed by each competitor. Whenever the economy is large enough, this requirement might prove to be too an unrealistic one. It follows that the current level of the aggregate price index can be just forecasted on the basis of the available information. According to what we stated above, the symmetric equilibrium solution might be associated with perfect foresight in forming expectations. Our point is that perfect foresight and heterogeneity are not mutually consistent assumptions: heterogeneity is likely to be associated with less than perfect foresight.

Given the assumption of a bounded capacity to forecast the future, the simplest mechanism one can postulate is a static expectation formation rule:

$$Q^e(t) = Q(t-1)$$

Substituting in the BRF of the generic monopolist j , and aggregating through expression (2), we have:

$$Q(t) = \left[\frac{1}{K} \sum_{j=1}^K \left(\left[A \left(\frac{M(t)}{Q(t)} \right)^{\alpha-1} \varepsilon_j \right]^{\frac{1}{\delta}} Q(t-1) \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad (6)$$

where for ease of notation we substituted ε_j for $(1/e_j)$. In other terms, (6) can be rewritten as:

$$Q(t) = A^{\frac{1}{\delta}} \left(\frac{M(t)}{Q(t)} \right)^{\frac{\alpha-1}{\delta}} Q(t-1) v(t) \quad (7)$$

where $v(t) = \left[\frac{1}{K} \sum_{i=1}^K \left(\varepsilon_i^{\frac{1}{\delta}} \right)^{\frac{1}{\sigma}} \right]^{\sigma}$ might be interpreted as an index of average

productivity. Linearizing by the mean of a logarithmic transformation we obtain:

$$q(t) = \gamma + \frac{\alpha-1}{\delta} [m(t) - q(t)] + q(t-1) + u(t) \quad (8)$$

where lower case letters represent the logarithm of nominal variables, $u(t)$ the logarithm of $v(t)$, and γ the logarithm of $A^{\frac{1}{\delta}}$. After some algebraic manipulations one obtains:

$$q(t) = \gamma + c_1 m(t) + c_2 q(t-1) + c_3 v(t) \quad (9)$$

$$\text{where } \gamma = \gamma^* \frac{\delta}{\delta + (\alpha - 1)}, \quad c_1 = \frac{\alpha - 1}{\alpha + \sigma(\alpha - 1)} \text{ and } c_2 = c_3 = \frac{1 + \sigma(\alpha - 1)}{\alpha + \sigma(\alpha - 1)}.$$

The simplest way to assess whether our framework allows real effects of nominal shocks is that of transforming it in order to obtain a relationship involving growth rates. Deriving with respect to time expression (9) one has:

$$q^{\wedge}(t) = c_1 m^{\wedge}(t) + c_2 q^{\wedge}(t-1) + c_3 v^{\wedge}(t) \quad (10)$$

where the superscript $\wedge$ represents growth rates. Calculating the value of parameters c_i , $i = 1, 2, 3$, through a simple visual inspection of equation (10) we are allowed to state the two following results:

Result 1: A nominal (i.e. monetary) shock influences the rate of growth of prices less than one to one.

Result 2: The nominal innovation creates persistency (or hysteresis), whose degree is measured by the magnitude of c_2 .

The main message here comprised is that in order to obtain stickiness of aggregate prices in a monopolistic competition setting it is not necessary to introduce nominal rigidities (e.g., see Ball and Romer (1991) and Caplin and Leahy (1992)), but it suffices to drop the assumption of perfect foresight, the last issue coming from the call for more realism when introducing heterogeneity³.

Notice that in the long run $q(t) = q(t-1) = q^*$, i.e.:

$$q^* = m^* + \frac{1 + \sigma(\alpha - 1)}{\alpha - 1} v^* \quad (11)$$

³Ball and Romer analyze the case of heterogeneity as well (pag. 547), where heterogeneity regards menu costs. In their model agents must know the cumulative distribution function $H(z)$ of menu costs in order to be able to calculate precisely the aggregate price P and the gain from adjusting their price to a nominal shock, respectively. In our setting we discard precisely this perfect information assumption.

so that it immediately follows:

Result 3: *Technological progress is not neutral, and it influences the aggregate price index in the long run more than one to one.*

Let us now move back, assuming absence of technological progress, in order to study the role played by the 'hard structure' parameters α (i.e. 'static' technology) and σ (i.e. preferences) on the neutrality of money. Notice that the impact multiplier of monetary innovations on the aggregate price index remains unaffected by this trivial modification, and it is given by:

$$\frac{\partial q^*(t)}{\partial m^*(t)} = \frac{\alpha - 1}{\alpha + \sigma(\alpha - 1)} = c_1 < 1 \quad (12)$$

It is straightforward to observe that:

$$\frac{\partial c_1}{\partial \alpha} = \frac{1}{[\alpha + \sigma(\alpha - 1)]^2} > 0 \quad (13)$$

and

$$\frac{\partial c_1}{\partial \sigma} = \frac{(-1)(\alpha - 1)^2}{[\alpha + \sigma(\alpha - 1)]^2} < 0 \quad (14)$$

so that we can state:

Result 4: *The higher are "static" decreasing returns (α) and the lower is the elasticity of substitution of demand (σ), the higher is the impact of a monetary shock on the aggregate price index.*

Now, notice that:

$$\frac{\partial c_2}{\partial \alpha} = \frac{-1}{[\alpha + \sigma(\alpha - 1)]^2} < 0 \quad (15)$$

$$\frac{\partial c_2}{\partial \sigma} = \frac{\alpha(\alpha - 1)}{[\alpha + \sigma(\alpha - 1)]^2} > 0 \quad (16)$$

that is to say:

Result 5: *The reverse of result 4 is true for the adjustment process towards the long run equilibrium.*

It follows that in the absence of technological progress, dropping the assumption of perfect foresight because of the introduction of heterogeneity implies that the lack of short-run neutrality of money depends on "static" decreasing returns and on the elasticity of substitution between couples of goods (i.e. on preferences). Furthermore, there exists a trade-off: the higher is the impact of a nominal shock, the lower is the speed of convergence towards the new long run equilibrium, and vice-versa.

3.3. Effective competition and market selection.

Let us now move our attention to a second capital issue deriving from the introduction of heterogeneity in a monopolistic competition model: the long-run structure of the market. As a first-order approximation, we assume that our monopolistic firms, endowed with different production costs, can not pursue any innovation activity.

Recall, from equation (1), that the demand function for a generic firm j is given by:

$$Y_j^d(t) = \left(\frac{p_j(t)}{Q(t)} \right)^{-\sigma} D(t, K) \quad (17)$$

where we have substituted for the term $\frac{Y(t)}{K}$ the expression $D(t, K)$. Notice

that the latter is equal to the market share of firm j along a symmetric long-run equilibrium, where $p_j = Q = 1$; its role is that of constituting a useful benchmark of comparison for the market share a firm can obtain along an heterogeneous equilibrium. $D(t, K)$ is clearly a function of the number of firms operating at time t .

The relative competitive position firm j occupies when adopting its peculiar technology ($1/e_j$) is then given by the ratio of its market share and the potential market share in the homogeneous case:

$$\frac{Y_j^d(t)}{D(t, K)} = \left(\frac{p_j(t)}{Q(t)} \right)^{-\sigma} \quad (17)$$

Let us define this competitive ratio as $X_j(t, K)$; hence, $\underline{X}(t, K)$ is the vector of relative competitive shares for each firm $j = 1, \dots, K$. This represents a complete configuration of a monopolistically competitive market composed of K firms at time t .

Taking logs, one obtains a sort of replicator-type function:

$$x_j(t, K) = -\sigma (\ln p_j(t) - q(t)) \quad j = 1, 2, \dots, K \quad (18)$$

where lower case letters represent the logarithms of the relative variables. The system of equations (18) identifies the equilibrium market share for every firm j , $j = 1, 2, \dots, K$, in a form which closely resembles the notion of 'fitness' largely employed by evolutionary economics⁴. In this literature the notion of fitness is a useful biological metaphor for the ability a firm has to survive in a competitive environment. In our framework, the lower the price a firm chooses, the higher its fitness is, i.e. its relative market share.

The Bertrand-Nash equilibrium in terms of relative market shares is defined as $\sum_{j=1}^K x_j = 0$. In the homogeneous case, the equilibrium can be immediately

derived as the condition $\underline{X}^h = \{0, 0, \dots, 0\}^T$. Suppose now that a set of heterogeneous firms might be ranked from the more to the less efficient, as $e_1 > e_2 > \dots > e_K$. When firms employ heterogeneous technologies, the non-homogeneous equilibrium is given by the vector $\underline{X}^n = \{x_1, x_2, \dots, x_K\}$, and there exists a technology e_j^* such that $x_j < 0$ holds if $e_j < e_j^*$, and $x_j > 0$ if $e_j > e_j^*$. This threshold value may be found noting that if one substitutes from the BRF, it follows that:

$$x_j(K) = -\sigma a_0 (a_1 - \ln e_j) \quad (19)$$

where $a_0 = 1/\delta$, and $a_1 = \ln A + (\alpha - 1) \ln D$. Thus, heterogeneity for itself does not affect sensibly the long-run equilibrium, given that the degree of

⁴In evolutionary economics, a relative fitness function is defined as the differential equation $\frac{ds_i}{dt} = v(E_i - \langle E \rangle)s_i$, where s is the relative market share of firm i , E_i its somehow determined competitiveness, and $\langle E \rangle$ the average competitiveness of all firms in the industry ($= \sum s_i E_i$). Notice that while the fitness function is a dynamical disequilibrium concept, the equation in the text (18) is an equilibrium object. For relevant applications of replicator dynamics, see Iwai (1984), Metcalfe (1992), Saviotti and Mani (1995) and Silberberg, Dosi and Orsenigo (1988); for interesting discussions of the relation between evolutionary equilibria and equilibrium concepts in economics, see Boylan (1994), Friedman (1991) and Joosten (1996).

polymorphism initially characterizing the market propagates to the long-run solution.

More interesting results emerge if we abandon this static representation and admit the influence of technological change. Just to have a taste of what is going on, suppose that each firm is engaged in learning-by-doing or on-the-job-training activities, which allow it to improve its productivity at a constant idiosyncratic rate. Prices are now allowed to change at each time, so that a proper replicator function can be obtained taking the derivative with respect to time of equation (18):

$$\dot{x}_j(t) = -\sigma \left[\frac{\dot{p}_j(t)}{p_j(t)} - \frac{\dot{q}(t)}{q(t)} \right] x_j(t) \quad j = 1, 2, \dots, m \quad (20)$$

i.e., the rate of change of the competitiveness of firm j as measured by the relative market share increases (decreases) as far as its price grows less (more) than the average price. The system of $K+1$ equations composed by the K equations in (20) and the relationship for the aggregate price index admits a unique solution under mild conditions (see Joosten (1996)). Firms whose share approaches zero exit, and the number of operating firms shrinks. Eventually, only the fittest firm will survive. The finding is at odds with the ones obtained in the evolutionary literature, where "... the only possible long-run equilibrium [...] without technological progress is the situation in which the least cost firms dominate the entire economy" (Iwai (1984), p.329). Whenever consumers have preferences which dictate that variety is better than uniformity, cost-decreasing technological progress is necessary in order to allow Schumpeterian competition to drive the market towards monopoly.

4. Transitional dynamics towards a new long-run equilibrium.

So far, technological change has proved to exert an influence of paramount importance on the long-run equilibrium price index. Furthermore, we have observed that when cost-decreasing technological change is at work, monopolistic competition evolves toward a monopolist market. In this section we want to go deeper in our investigation of how the new equilibrium could be attained when technological progress is at work.

The first step we take along this route consists in analyzing our stylized fact number 2. Simply stated, the idea we aim to explore is that the process of adaptation to a new technology or to a new organization in general takes time, so that its whole potentiality in terms of productivity can be reached

only after some time from its introduction has elapsed. To put it differently, the technological shock each firm receives, that in our case corresponds to a change of the production technology it employs, is likely to need a certain time to become fully operative, for example because the adoption of new capital equipments require the retraining of employees, because of sunk fixed costs due to old embodied technologies, or simply because people does not like novelties and adapts to them slowly. Hence, in this section we consider explicitly the effects of learning-by-doing and learning-by-using phenomena, showing that under rather plausible assumptions they may lead to interesting transitional dynamics.

A sensible amount of empirical evidence on technological change has demonstrated that the adoption of a new technology could be described by the mean of a logistic function: the gain in productivity allowed by the new technology at first grows very slowly with an increasing marginal rate, eventually reaching a point where its second derivative changes sign. Concavity implies that the intra-firm learning process converges in a finite time to its long-run potential value.

In order to formalize this idea we adapt to the behaviour of a single firm a piece of machinery typically employed in describing the adoption of a technology by a population of firms. The idea is that an individual firm is generally composed by a number $n > 1$ of employee, divided in $r > 1$ divisions, each division being responsible for a given activity (e.g. management, production, marketing, auditing and control, etc.). During the interval $t + \Delta t$ the expected increase of F , the relative number of members fully adopting the novelty, is given by $\lambda F_t(1-F_t)\Delta t$, where λ may be interpreted alternatively as a probability each non adopter has to adopt whenever matched with an adopter, or, in a deterministic fashion, simply as the speed of the logistic convergence process. The law of motion describing intra-firm adoption may therefore be written as $F_{t+\Delta t} - F_t = \lambda F_t(1-F_t)\Delta t$. Dividing both sides by Δt and letting Δt approaching zero, one obtains a Bernoulli differential equation $\dot{F}_t = \lambda F_t(1-F_t)$, whose solution gives rise to a logistic path. Applying this reasoning to our model, the transitional dynamic evolution of productivity for a firm after it has chosen to adopt a new technology may be expressed as:

$$e_j(t+h) = e_j(t) + \Delta e_j(t) \left[1 + \left(\frac{1}{F_0} - 1 \right) \exp(-\lambda h) \right]^{-1} \quad (21)$$

where $\Delta e_j(t)$ denotes the value of the productivity shock firm j receives at time t , F_0 is the number of employees or divisions immediately adopting (in

this case as close as possible to zero), and finally, as we said, λ rules linearly the speed of convergence of the logistic towards the value 1. To appreciate the latter point, recall that the lapse of time τ it takes to go from a percentage κ to a percentage κ' of the final after-shock productivity is given by:

$$\tau_j(\kappa \rightarrow \kappa') = \lambda_j \ln \left[\frac{\kappa'(1-\kappa)}{\kappa(1-\kappa')} \right] \quad (22)$$

i.e., a linear function of the parameter λ . The most interesting case is when the speed of convergence is inversely proportional to the magnitude of the positive shock. The assumption is likely to be a reasonable one, if one just notes that the bigger is the shock, the bigger is the distance between the old and the new technology adopted by the firm, and finally the longer is likely to be the adaptation process. If this is the case, firms receiving bigger shocks, i.e. firms incurring in a deeper technological innovation, are likely to loose competitiveness in the short-run, to be overcome by firms adopting less innovative technologies. Of course, in the long-run the right ranking is established. Hence, catching-up, forging ahead and falling behind phenomena are at works at the market level.

The idea can be better appreciated looking at the graph in figure 1, where the dynamic adjustment processes towards the long-run value of productivity are plotted for three firms, after they receive a shock which amounts to an increase of the 5% (B), of the 3.5% (M), and of the 0.5% (S) with respect to the ex-ante productivity, respectively. All three firms started from a common productivity level. Under the combined pressure of an idiosyncratic shock and of internal logistic adaptation activities inversely related to the magnitude of the shock itself, firm B falls behind with respect to both its competitors for 8 periods, it catches up and starts to forge ahead firm S at period 10 while continuing to loose competitiveness with respect to firm M; eventually it catches up and forges ahead firm M too starting from period 16. Notice that it takes roughly half of the whole convergence lapse of time for firm B to become the new technological leader.

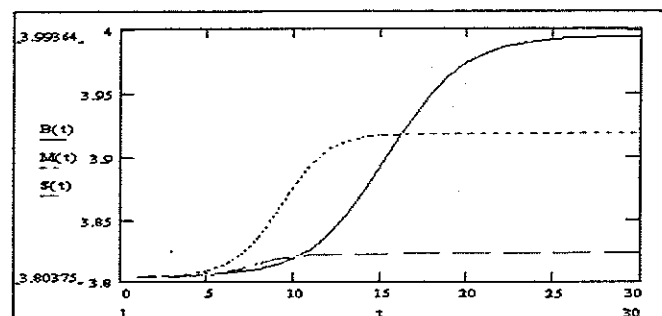


figure 1

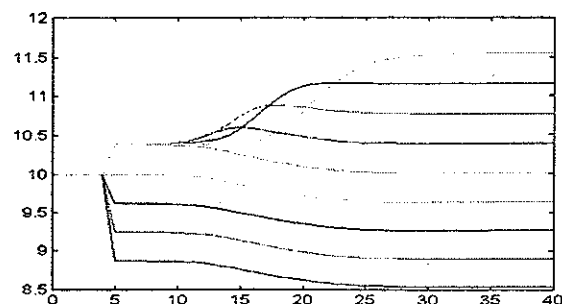


figure 2

What about negative shocks? In this framework, where shocks have been defined as technological changes which modify the production costs, as a matter of example we could think at the breaking of organizational equilibria implying a deterioration of the productivity of labour, or unexpected difficulties cropped up in its purchasing activities. Whatever nature the negative shock might have, it seems difficult to justify the presence of some 'retardation effect' for sudden decreases in productivity, however. Thus, when a negative shock hits the firms, its production costs are likely to increase immediately.

In figure 2 the transitional dynamics of the market shares correspondent to the long-run equilibrium for a market composed of ten firms is drawn. Starting from an homogeneous situation in which each firm produces adopting the same technology, we assume that idiosyncratic shocks hit our firms leaving the 'average' productivity unaffected on the long run. In other words, we are postulating a kind of 'mean-preserving' shock vector. In particular, firms ranked from #1 to #4 receive at period 5 a negative shock,

which decreases immediately their production costs; firms occupying the median position (#5,6) are unaffected, while firms from #7 to #10 experience positive technological shocks which follow logistic patterns. An interesting feature emerging from the transitional dynamics plotted in figure 2 is that a kind of reversed logistic pattern characterizes the adjustment path of 'unlucky' firms, i.e. firms receiving negative shocks, even if the shock they suffer display immediately its inter-firm effects. This non-linear transitional dynamics is completely induced by the evolution of the market. In the following section we will drop the possibility of negative shocks, the market structure's dynamics being driven primarily by 'Schumpeterian competition': firms can loose part of their market because of a relative deterioration of their competitiveness driven by its competitors' efforts in finding more efficient technologies, rather than by idiosyncratic negative shocks.

5. Growth driven by localized technological change.

The findings discussed so far have been obtained under two very special assumptions: 1) the aggregate demand remains constant, or it grows at an exogenous rate and 2) the profits gained during the dynamic process are not invested.

In this section we will drop the latter hypothesis, assuming that profits can be used to improve the relative competitiveness position through investments in R&D, whose outcome is inherently uncertain but whose expected value depends positively on the amount spent. Profits are therefore a necessary pre-condition for technological-driven growth.

Investments in R&D can be split into two different activities: innovation and imitation. The cornerstones in the literature are Nelson and Winter (1982) and Soete and Turner (1984); some interesting work in extending to a macroeconomic setting this issue has been recently done by Chiaromonte and Dosi (1993), Hutschenreiter, Kaniovski and Kryazhimskii (1995), and Ardeni and Gallegati (1996). In particular, the last two authors model both R&D activities by the mean of a single birth-and-death process. The technological situation at time t is given by a state s of a Markov chain; innovation is then represented as the exit of firms towards a new technological regime (death), and imitation as the entry of firms in the present one (birth).

It seems interesting to extend their results in two directions. First, convincing evidence has been furnished that R&D is characterized by localized spillovers. Antonelli (1995) discusses in details the topic, identifying the nature of localized technological change in two features. On the one hand, localization must be referred to the need of a sensible amount of learning in

fully acquiring a new superior technique, where learning is largely “... embedded in the techniques each firm has experienced” (Antonelli, (1995), pag. 10). Localization is then referred to the elasticity in technical substitution along an isoquant when the relative price of factor changes, or along an increasing productivity technological paradigm, its degree being measured by the amount of switching costs a firm must face when choosing a new technology. On the other hand, switching costs are a function of spatial proximity with firms adopting innovations. The argument runs on the notion of interactions among firms belonging to a common Marshallian industrial district. Finally, the notion of localized technological change seems to provide a consistent representation of the ‘*thrust and parry*’ process in innovations’ introduction described by Landes (1969)⁵.

Technological adoption ruled by some kind of localized spillovers has been recently theoretically treated by Allen (1982), Antonelli (1996), and An and Kiefer (1995). Furthermore, a large number of empirical studies have furnished us convincing evidence supporting the importance of spatial localization, or more generally of a neighbourhood structure, in technological change; among the others, see Jaffe, Traytenberg and Henderson (1992), and Henderson, Kuncoro and Turner (1995). This makes room for a description of the supply side of an economic system as a lattice, each firm occupying one single site and communicating only with a subset of neighbours on the whole lattice (Föllmer, (1974)). In particular, the link with the literature sprung from his seminal paper should become clearer in the following section.

Second, recall that knowledge is not a completely appropriable public good, implying that not only innovation, but imitation as well can be pursued only at the cost of investing in peculiar R&D (Cohen and Levinthal, (1989)); This issue gives rise to an interesting consideration. The profit space can be divided into three regions such that:

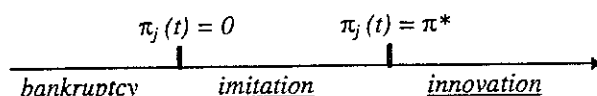


figure 3

⁵ The assumption of localized spillovers contributes furthermore to overcome a shortcoming of the Aghion and Howitt's ‘creative destruction’ story, according to which the arrival of a new invention replaces immediately all the old technologies available at that time. In reality, the process of substitution is likely to be smoother, with the closest substitute old technologies being the first ones to be discarded.

Under the assumptions that investments can be implemented only out of profits, and that innovation requires more effort in R&D than imitation, monopolists are driven out, imitate or innovate - that is they contribute to determine their relative future competitiveness - according to their present capacity of making profits, which depends ultimately on their competitive position one period before⁶.

The competitive selection process analyzed above constitutes the incentive to invest in R&D. While at the microeconomic level monopolists invest basically in order to conserving and expanding their competitiveness, at the aggregate level we experience ‘*competition-led*’ endogenous growth, given that successfully R&D allows an increase in the average productivity, thus an expansion in the aggregate productive capacity.

5.1. Technological change as a two-stage stochastic process.

A suitable way to express the argumentation above is the following. Suppose that a set of K monopolist firms follow fixed routines both in innovating and in imitating, which might be comprised under the heading: ‘*invest in R&D all the profits you gained in the last period and cross your fingers*’. Technological change is characterized by localized interactions among agents, on the one hand because information is assumed to flow locally among firms producing ‘close’ commodities, on the other one because learning to use a new technology is easier when the novelty is technologically closer to the old option. As a matter of example, we claim that a firm manufacturing shoes is ‘closest’ to a firm producing clothes than to firms producing cars or floppy disks.

At any time t , firms may be represented as spatially organized on a circle. Each firm has more or less close neighbours to the left and to the right as indicated by the number of edges connecting it to other firms on the circle. However, due to intertemporal spillovers both on innovation and imitation activities, the interaction structure of the economy is given by a two-dimensional cylinder.

For expository convenience, we assume that the set $N(j)$ of neighbours of firm j at time t is given by the two closest neighbours on the left and on the right, plus itself on period before. At any time t , only two levels of the cylinder, the level t and the level $t-1$, fully determine the structure of the economy. Hence, the system obeys the Markovian property.

⁶ The idea that firms may exclusively innovate or imitate lacks in some sense of realism. In fact, it seems more likely that a firm having high enough profits to afford innovation decides to spend a fraction of them imitating as well. In spite of this, in what follows we will continue to make use of the ‘exclusion’ assumption, given that discarding it would imply a considerable complication in the analysis without any appreciable gain.

R&D investments in innovative technologies are modelled as uncertain activities, whose outcome is ruled by a two-stage stochastic process⁷. In turn, according to the argumentations reported above, the probability firm j has to experience successful investments depend on the local environment surrounding it. Hence, provided that profits in period $(t-1)$ were higher than a threshold value π^* , the monopolist j enters in a two-stages lottery:

- a. Let $I_j(t)$ be a boolean random variable indicating firm j 's success in innovative R&D. It takes value 1 if a new technology has been found, 0 otherwise. The probability of $I_j(t)$ to be 1 is given by⁸:

$$\mu_j(t) = \Pr \{I_j(t) = 1\} = \frac{\exp \left[a_1 \pi_j(t-1) + a_2 \sum_{i \in N(j)} I_i(t-1) \right]}{1 + \exp \left[a_1 \pi_j(t-1) + a_2 \sum_{i \in N(j)} I_i(t-1) \right]}$$

The probability of success depends both on j 's profits at time $(t-1)$ - a measure of private efforts - and on the intellectual and technological environment in which j is plunged - a measure of localized knowledge spillovers, given by the number of period $(t-1)$ successfully innovating firms out of a quasi-fixed neighbourhood⁹.

- b. If $I_j(t) = 1$, j 's productivity is given by:

$$E(e_j(t)) = e_j(t-1)(1 + \lambda)$$

i.e. the monopolist experiences an average productivity increase by a percentage λ , that represents the mean value of a Poisson distributed stochastic process, with a variance equal to v^2 .

⁷ Applications of two-stage procedures very close in spirit to our may be found in Chiaromonte and Dosi (1993) and Nelson and Winter (1982). The main novelty in this version is that we consider explicitly local spillovers.

⁸ The choice of probabilities is not completely arbitrary. In fact, it naturally derives as a suitable conditional representation of a Markovian random field with a discrete state's domain (Haining (1990)). The convenience of this characterization will become clear in section 6 when we will deal with the mean-field approximation.

⁹ The concept of quasi-fixed neighbourhood will be defined in more detail later on.

Imitation is ruled by a slightly different set of procedures. Whenever profits are positive but not high enough to afford innovative research, monopolists invest in order to adopt the most competitive technology available at time $(t-1)$ in a 'spatial' neighbourhood of competitors. The temporal lag intends to capture the presence of patents or other legal and technological barriers to a sudden diffusion of innovations. The process is described again as a two-stage stochastic process:

- a. Let $R_j(t)$ be a boolean random variable indicating firm j 's efforts in imitative R&D. It takes the value 1 if the investment is successful. In this model, it means that the patents of technologies used by neighbour competitors at time $(t-1)$ have become available. Hence, the probability of $R_j(t)$ to be 1 is given by:

$$\chi_j(t) = \Pr \{R_j(t) = 1\} = \frac{\exp [b_1 \pi_j(t-1)]}{1 + \exp [b_1 \pi_j(t-1)]}$$

In this case the probability of success depends only on j 's profits at time $(t-1)$. Local spillovers come on the scene only in the second stage.

- b. If $R_j(t) = 1$, j 's productivity is given by:

$$e_j(t) = e_j(t-1) \{1 + \argmax [(e_{N(j)}^*(t-1) - e_j(t-1)), 0]\}$$

where $e_{N(j)}^*(t-1)$ is the most efficient technology locally available. Summarizing, monopolist j , when investing in imitative R&D, has not a perfect knowledge of the technologies employed by other monopolists, but in order to discover them he necessarily has to make an investment, whose outcome is inherently uncertain.

Two further issues need a qualification. The first one deals with the number of monopolists operating at each stage: indeed, while the mechanism of exit should be clear, nothing has been said about the entry of new firms. A suitable assumption is that the element of the commodity space turned empty after the correspondent non-efficient monopolist has been pushed out of the market at time $(t-1)$ remains empty one period, to be successively occupied by a new monopolist at time $(t+1)$. The entrant has at his disposal the most efficient available technology in the reference neighbourhood at time t . We

want to capture the fact that entrants seldom have at their disposal an absolute innovative technology, but at the same time they decide to enter only with the prospect to obtain positive profits. This assumption could be interpreted as the consequence of imperfections on financial markets: firms can borrow the funds they need only if creditors have a positive probability to see their money back home.

The final remark is centered on the threshold level of profits π^* . In our framework we have assumed it to be exogenous, even if we are well aware that a realistic model would require this critical level to become endogenous. Given that this issues would drive us too far, we leave it for future research.

5.2 Simulation results.

The structure of the composed stochastic process driving the dynamics of the model is too complex to be analyzed in a closed form. Nevertheless, some of its long-run properties can be inferred through simulations. In the figures below we present the result of a computational exercise for the market's shares of ten monopolistic firms, the path of growth of the average productivity, and the standard deviation of market's shares - i.e. a measure of the structure of the market -, respectively. In what follows, aggregate demand is assumed to be constant, and the average increase in productivity when innovating is equal to a jump of 2.3% from the old technology¹⁰. Firms which exit because their market share is not enough to allow them to break-even are replaced by new firms producing the same variety, with a technological level equal to the mean technology during the previous period. The threshold value for profits, π^* , changes with the distribution of profits among firms, and for computational convenience it is assumed to be equal to the average value of profits distribution at each time; all other parameters are set equal to the one employed for the simulations above.

The simulation has been run for 200 periods; however, only the last 150 periods are plotted for market's shares and productivity growth, respectively. This choice allows us to eliminate the influence that transient dynamics of the involved stochastic processes might cause on the system's dynamics.

The resulting series path seem to be consistent both with some of our proposed stylized facts and with the empirical evidence. First, a remarkable degree of heterogeneity characterizes the economy in the long-run, as we emphasized in our stylized fact # 1.

¹⁰This choice reflects estimations of annual productivity growth rate (for the period 1890-1979) as proposed by Maddison (1982) for the U.S. It constitutes a timid proposal of 'calibration' of the model; at the same time, we use it to suggest that further extensions of the model could allow a complete calibration, in order to replicate empirical time series.

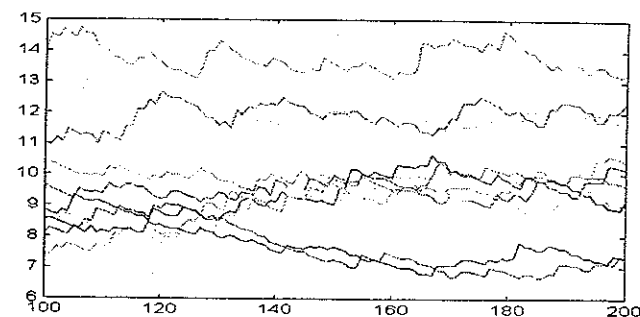


figure 4
market share dynamics

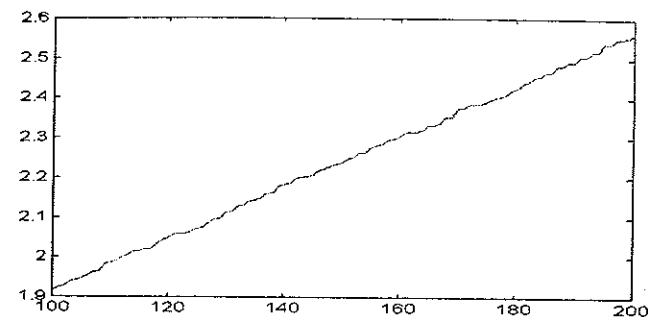


figure 5
logarithm of average productivity

Notably, starting from a situation very close to homogeneity, the market evolves towards a quasi-stable structure in the long-run, as one can appreciate from a joint consideration of figures 4 and 6. Second, catching-up and falling behind in market's shares indicates that the technological struggle pursued by firms plays a major role in shaping the complex intra-market dynamics, given the aggregate demand and preferences. Third, figure 5, in which the average technological time series is plotted, shows productivity as a fluctuating process around an increasing trend. Due to our assumptions of a constant aggregate demand and of constancy of the capital-output ratio, the time series in figure 5 approximates labour productivity as measured by output per man-hour.

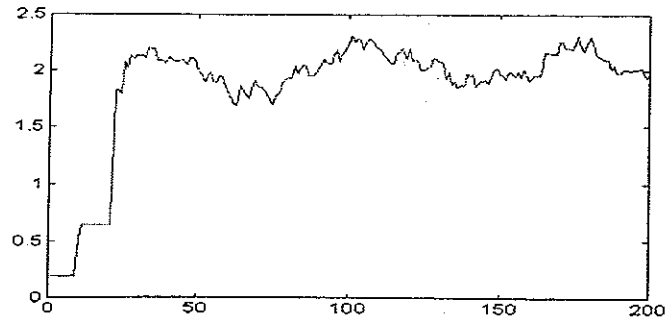


figure 6
standard deviations of market shares

As a final remark, recall that according to the story we told above, the adoption of a new technology is characterized by an intra-firm logistic learning process, whose length is likely to be directly proportional to the degree of its innovativeness. This implies that particularly innovative technologies may incur the risk of not being adopted by a large number of firms because the associated learning process is too long. In the presence of bounded capacities to forecast, the expected profitability derived from less productive techniques may therefore be higher. This consideration opens an alternatively route in explaining phenomena of inefficiency in technology adoption (Arthur, (1988)). In our case, given the process of cumulative increments in productivity due to innovation, we could better talk of inefficiency in the technological trajectory adoption.

6. A Mean Field analysis.

Thus far, technological change has been modelled as the complex composition of two double-stage stochastic processes, of which no analytical solutions are available, the only way to proceed being just through simulations. Nevertheless, introducing some drastic simplifying assumptions, a 'closed-form' mathematical expression for the average long-run rate of growth of productivity can be derived. In this section we look for a description of the system which does not rely on probabilistic short-run fluctuations, through the so-called 'mean-field' approximation. The rationale consists in focusing on the dynamics we obtain replacing the random dynamics of the stochastic variables at hand by their average paths. Ellis (1985) provides a comprehensive introduction to the topic, while for

applications to economics see Aoki (1994) and Weisbuch, Kirman and Herreiner (1996).

Let us suppose that at time $(t-1)$, n firms out of the total M obtained profits high enough to afford innovation, q were allowed simply to imitate, and finally the residual number $K-n-q$ incurred in a loss and went out. From the composition of this pair of two-stage lotteries it is easy to calculate the expected rate of growth of technology for a generic monopolist j to be equal to:

$$i_j(t) = \left(\frac{n}{K}\right) \lambda \mu_j(t) + \left(\frac{q}{K}\right) \frac{1}{2} \chi_j(t) (e_{\Delta_j} * (t-1) - e_j(t-1)) \quad (23)$$

The first addendum of the RHS of (23) is the probability to obtain an average percentage increase of productivity equal to λ , provided that profits were enough to belong to the fraction of firms allowed to innovate, and that the R&D activity succeeded. The second term, on the contrary, is the expected increment in productivity firm j can gain if profits were just sufficient to imitate, under the assumption that the ex-ante localized distribution of relative technological competitiveness is uniform, so that there exists an equal probability to be on the higher half or on the lower half of the distribution of technologies. To put it differently, there is a probability equal to $\frac{1}{2}$ to discover a more productive technology in the reference neighbourhood.

Equation (23) could be alternatively interpreted as follows: given n firm potentially innovating and q firms potentially imitating, let us draw randomly one firm out of the whole K market's components and find the expected rate

of change of its productivity. With probability $\frac{n}{K}$ we draw a firm belonging

to the innovative regime, which succeeds in obtaining an average productivity increase of λ with probability μ_j ; with probability $\frac{q}{K}$ the

extraction will result in a firm belonging to the imitation regime, which will have the opportunity to adopt a new technology with probability χ_j , the increase in productivity being given by the difference between the most efficient available technology in its neighbourhood and the technology

presently used; finally with probability $\left(1 - \frac{n+q}{K}\right)$ the firm extracted

experienced last period a loss, and can not enter in any technological advance process.

Notice finally that we can not treat each firm as an independent stochastic variable and apply the strong law of large numbers in 'averaging' the path of

productivity growth, because of the interaction among them. In this case one has to adopt the machinery developed in statistical mechanics in dealing with interacting particles, i.e. Gibbs fields, or equivalently their conditional counterpart, random fields. See Föllmer (1974), Allen (1982) and Durlauf (1993) for applications to economics of these concepts.

6.1 Approximating innovation.

As a first step, let us focus on the innovation process. In order to simplify the analytical treatment, a convenient representation consists in extracting at each time t from the whole set of operating firms only the firms which have gained at $(t-1)$ enough profits to enter in the innovation prospect, and to build with them an ordered set. We proceed therefore endowing the economy with the structure of a graph, and associating each firm with an index $a \in A^{11}$, representing a site on the lattice. Each site is characterized by a conditional probability law $r(\omega(a) = s | \eta)$, called the set of local characteristics, which is defined on all the possible states $s \in S$ the site can assume. The probability measure is conditioned to an environment, defined as the map $\eta: A - \{a\}$ specifying the state of agents other than a . The economy can now be represented by the model $(A, S, \mathfrak{R}, V)$, where $a \in A$ agents are embedded in a graph with a structure of social relations given by V^{12} , each one of them experiencing a state $s \in S$ with a probability $r_a \in \mathfrak{R}$. The fact that competition makes profits fluctuating with a positive probability between innovation and imitation regimes implies the graph has not a fixed structure. Nevertheless, given the postulated persistence of firms in the innovation subset due to the positive relation between profits and the probability to further increase their competitive position thanks to innovation, we will talk of 'quasi-fixed' structure.

Call ω the decision variable associated with the state s , with $\omega \in \Omega$. Hence, we define as a macroeconomic global phase of the economy any probability measure ψ belonging to the probability space $(\Omega, \mathcal{F}, \psi)$, which is compatible with $\mathfrak{R}$ in the sense that:

$$\psi[\omega(a) = s | \eta] = r_a(s | \eta) \quad \psi\text{-almost surely}$$

¹¹ Of course the total number of elements of A changes period by period, but it is comprised between 1 and K .

¹² What V measures is the extension of the relative neighbourhood influencing the decision of firm j . Obviously, when the number of members of the reference group of neighbours changes, the behaviour of the aggregate economy changes.

Intuitively, local characteristics are consistent whenever they admit one unique global phase. A well-known result in statistical mechanics is the equivalence between the conditional probability representation of a Markovian Ising type economy¹³ and the finite-volume Gibbs state representation. This latter can be written as:

$$\text{Prob}(\omega) = \exp(-\beta H) \frac{1}{Z} \quad (24)$$

where β , which in statistical mechanics represents the inverse of temperature, is a parameter that will be treated as a piece of machinery without any particular natural interpretation. H is called Hamiltonian or interaction energy function, which for the finite-volume case may be expressed as:

$$H = -b \sum_{j \in A} \omega_j - \frac{1}{2} \sum_{i, j \in A} J(i-j) \omega_i \omega_j \quad (25)$$

while the denominator of (24) Z is called partition function, its role being that of normalizing the expression for the Gibbs state:

$$Z = \sum_{\omega_i \in \Omega_A} \dots \sum_{\omega_a \in \Omega_{AA}} \exp(-\beta H). \quad (26)$$

The function J assumes a capital importance for the mechanics of the system. Indeed, on the one hand it characterizes the range of direct interactions, i.e. it defines the ray of the reference group; on the other one, J represents the strength of the influence exerted by neighbours on any generic firm j . Let us suppose that the strength of interaction is measurable and let us posit it equal to $\mathfrak{S}$, so that:

$$J(i-j) = \begin{cases} \mathfrak{S} & \text{if } \|i-j\| = 1 \\ 0 & \text{if } \|i-j\| \neq 1 \end{cases}$$

where $\|\cdot\|$ indicates the Euclidean norm on $\mathfrak{R}^D$. This characterization of J fully determines the well-known Ising (or auto-logistic) model. Its properties have been deeply studied in statistical mechanics, cluster analysis and

¹³ With this expression we summarized two distinct features: the Markov property of probabilistic local characteristics and the egalitarianism in initial endowments and in attitudes. The last assumption is in order if we want to deal with non-directed graphs.

pattern recognition, among the others. However, notice that other interaction strength structures may be considered using these framework without particular analytical difficulties.

Finally, b is an external influence on the probability statement, in our case approximating the influence exerted by profits on the probability of success. The value $b = 0$ represents a critical point for the Gibbs representation, in correspondence of which the system shows abruptly changes in its dynamics. Hence it seems natural to establish a transformation from π to b , $b(\pi)$, such that if $\pi = \pi^* \Rightarrow b(\pi) = 0$. For the sake of simplicity we do not go deeper in analyzing how the system's long-run trajectories change for different values of b .

The trick consists now in replacing the spatially distributed interaction with an average strength interaction, the so called 'mean field', defined as

$$\frac{S_0}{|A|} = \sum_{j \in A} \frac{J(j)}{|A|}. \text{ The Hamiltonian can be rewritten as:}$$

$$\tilde{H} = -b \sum_{j \in A} \omega_j - \frac{1}{2} \frac{S_0}{|A|} \sum_{i, j \in A} \omega_i \omega_j \quad (27)$$

Expression (27) characterizes what in statistical mechanics is known as the Curie-Weiss model. Its main peculiarity is that of allowing exact calculations of the expected per capita innovation rate, defined as d . In particular, the expected long-run growth rate of productivity due to innovation is given by the roots of the expression (see Ellis (1985), p.182; and for a related interpretation, see Brock and Durlauf (1995), p. 9):

$$d = \tanh [\beta(S_0 d + b)] \quad (28)$$

Notice that for $b \neq 0$ and $\beta S_0 > 1$ there exists a threshold Γ such that 1) $|\beta b| < \Gamma$ implies the existence of three roots; 2) $|\beta b| > \Gamma$ implies the existence of a unique root with the same sign of b (Brock and Durlauf (1995)). An alternative interpretation runs as follows: for any given β there exist a b^* , with $b^* > 0$, such that for $b > b^*$ the system becomes ergodic and converges towards a golden long-run growth rate; while for $b < b^*$ the economy is characterized by a multiplicity of equilibria.

Unfortunately, for the generalized case the mean field approximation through the Curie-Weiss model does not allow to obtain precise values of the long-run system's characteristics. The average per-capita rate of growth of technology due to innovation we can find through (28), which for convenience we redefine as d^{CW} , provides just an upper bound to the average

innovation rate, i.e. for $\beta > 0$ and $b \geq 0$ the following holds (Ellis (1985), theorem V.9.1, p.183):

$$d^{CW}(\beta, b) \geq d(\beta, b)$$

The correctness of this approximation grows as the dimensional degree of the system increases. For $D \geq 4$ the value of the mean field approximation of the generalized Gibbs state representation equals the Curie-Weiss model representation. In our case, with $D = 2$, what one can say when forecasting is that on the average the long-run growth rate of productivity due to the introduction of innovations will be no more than the value $[d^{CW}(\beta, b)]\lambda(n/K)$.

6.2 Approximating imitation.

Let us now make one step back in order, and consider the second moment of the technological change process, i.e. imitation. The only assumption we have made thus far is that the variance of the innovation stochastic process is uniformly distributed over the field describing the economy. Furthermore, for ease of exposition, let us assume that profits do not play any role in the probability of success in imitating; but for the fact that they have to be positive and below a given threshold.

In order to gain further insights for our analysis, recall that the diffusion/imitation process is related to the diffusion of innovations among the population which took place the previous period, and that how the neighbourhood structure is built matters. This considerations allow us to approximate the time $(t-1)$ average innovation rate as the result of a Spatio-Temporal Auto-Regressive model (STAR(1, I_1)):

$$\lambda_{t,t-1} = \sum \varsigma^s L^s \lambda_{t,t-2} + u_{t,t-1} \quad (29)$$

with L representing the lag operator and g indicating the order of the lag. How is it possible to justify the use of a lag operator for spatial distances? Let us follow Coli (1992), and define a matrix W , called connection matrix, where the generic element $w_{i,j}$ indicates the type of link between the i and j sites. This amounts to consider a delayed spatial operator L^g associated to the observed variable $\lambda_{i,t}$, $\forall i \in (A)$:

$$L^g \lambda_{t,t-1} = \sum_j w_{i,j} \lambda_{t,t-1} \quad (30)$$

This permits us to express the spatio-temporal autocovariance coefficient between two generic sites i and j as:

$$\gamma_{ij}(1) = E \left(\frac{\sum_i L^i \lambda_{i,t-1} L^j \lambda_{i,t-2}}{K} \right) \quad (31)$$

The sensible technical assumption we adopt is that the average rate of imitation could be suitably approximated as a function of the localized autocovariance coefficient referred to the innovation rate at time $t-1$. If this is the case, the neighbourhood structure on $\mathcal{R}^D$, with $D = 2$, of the Ising representation implies that:

$$e_{\Delta j}^*(t-1) - e_j(t-1) = 2 f(\gamma(1)) \quad (32)$$

where f is an increasing function.

6.3 Mean field approximation of the productivity growth rate.

Substituting (32) and the critical value(s) of (28) into the expression (23), the mean field approximation of the rate of growth of the technological change can be finally written as:

$$\tilde{\gamma}(t) \leq \frac{n}{K} \lambda d^* + \frac{q}{K} \chi f(\gamma(1)) \quad (33)$$

Expression (33) is particularly insightful, since it allows us to extract at least qualitative pieces of information about the average long-run rate of growth of productivity. In particular, the following consequences of the model and of its mean field approximation deserve to be stressed:

Result 6. *The average rate of growth of productivity is directly influenced by the structure of local interaction. Different regions or countries will experience different rates of growth whenever the institutional structure of interactions among agents dwelling therein differs.*

Result 7. *The distribution of profits among firms matters in two senses: a) the distribution of firms among the 3 regimes n , q , and $K-(n+q)$;*

b) the dispersion of profits in each single regime, provided that when profits are concentrate on the tales of the sub-distributions the autocovariance parameters $\gamma(1)$ increases as well.

Result 8. *The model may be characterized by the presence of multiple equilibria: whenever the nature of interactions implies the equation (20) has multiple roots, economies endowed with the same technology, profit distributions and the same relation structure may display different long-run behaviours. Some of them may be trapped in a low growth regime, while others experiencing high growth, the selection of the final outcome depending exclusively on historical accidents. The system displays therefore 'path-dependence' and, provided the long-run equilibrium are stable, it remains 'locked-in'.*

6. Concluding Remarks.

The paper intends to be a first attempt in extending the standard monopolistic competition model along two directions. First, we introduce heterogeneity, which amounts to discard the notion of a symmetric Nash long-run equilibrium solution. This amendment permits to appreciate the role played by preferences and technology in determining the degree of persistency of nominal shocks when the assumption of perfect foresight is discarded. Furthermore, a trade-off between short run and long run neutrality is assessed.

Second, we take expressly into account the dynamic selective pressure deriving from competition, which gives rise to interesting market dynamics whenever learning-by-doing phenomena are allowed to characterize the intra-firm technological adoption process.

Combining this two arguments with an explicit role for R&D investment, which in turn is characterized by localized technological spillovers, allows us to sketch the working of an evolutionary 'competition-led' endogenous growth model with heterogeneous agents. R&D investment may be split into two distinct activities: innovation and imitation. While characterized by different costs, they contribute in different ways in increasing the competitive position of the firm which adopts them. Finally, an alternative interpretation of phenomena of inefficiency in technological adoption (Arthur, (1988)) is provided.

References

- Abramovitz M. (1989) "Thinking about Growth", Cambridge University Press, Cambridge.
- Aghion P. and P. Howitt (1992) "A Model of Growth through Creative Destruction", *Econometrica*, 60:323-351.
- Allen B. (1982) "Some Stochastic Processes of Interdependent Demand and Technological Diffusion of an Innovation Exhibiting Externalities Among Adopters", *International Economic Review*, 23:595-608.
- An M.Y. and N.M. Kiefer (1995) "Local Externalities and Societal Adoption of Technologies", *Journal of Evolutionary Economics*, 5:103-117.
- Antonelli C. (1995) "The Economics of Localized Technological Change and Industrial Dynamics", Kluwer Academic Publishers, Boston.
- Antonelli C. (1996) "Localized Knowledge Percolation Processes and Information Networks", *Journal of Evolutionary Economics*, 6:281-295.
- Aoki M. (1994) "New Macroeconomic Modeling Approaches", *Journal of Economic Dynamics and Control*, 18:865-877.
- Ardeni P.G. and M. Gallegati (1996) "Fluctuations and Growth Due to Technological Innovation and Diffusion", mimeo, University of Bologna.
- Arthur B.W. (1988) "Self-Reinforcing Mechanisms in Economics", in Anderson P.W., Arrow K.J. and D. Pines (eds.) "The Economy as an Evolving Complex System", Santa Fe Institute Studies in the Sciences of Complexity, Addison Wesley, Redwood City, CA.
- Ball L. and D. Romer (1991) "Sticky Prices as Coordination Failure", *American Economic Review*, 81:539-552.
- Benassy J.-P. (1991) "Monopolistic Competition", in Hildebrand W. and H. Sonnenschein (eds.) "Handbook of Mathematical Economics", vol. IV, North-Holland, Amsterdam.
- Bils M. (1987) "The Cyclical Behavior of Marginal Cost and Price", *American Economic Review*, 77:838-855.
- Blanchard O.J. and N. Kiyotaki (1987) "Monopolistic Competition and the Effects of Aggregate Demand", *American Economic Review*, 77: 647-666.
- Boylan R.T. (1994) "Evolutionary Equilibria Resistant to Mutation", *Games and Economic Behaviour*, 7:10-34.
- Brock W.A. and S.N. Durlauf (1995) "Discrete Choice with Social Interactions I: Theory", NBER Working Paper, # 5291.
- Caplin A. and J. Leahy (1991) "State-Dependent Pricing and the Dynamics of Money and Output", *Quarterly Journal of Economics*, 106:683-708.
- Cheng L. and E. Dinopoulos (1992) "Schumpeterian Growth and International Business Cycles", *American Economic Review, Papers and Proceedings*, 82:409-414.
- Chiaromonte F. and G. Dosi (1993) "Heterogeneity, Competition, and Macroeconomic Dynamics", *Structural Change and Economic Dynamics*, 4:39-63.
- Cohen W.M. and D.A. Levinthal (1989) "Innovation and Learning: The Two Faces of R&D", *Economic Journal*, 99:569-596.
- Coli M. (1992) "On the Space-Time Interaction Models to Analyse Environmental Systems in Data and Frequencies Domains", *Statistica Applicata*, 4:703-717.
- Durlauf S.N. (1993) "Nonergodic Economic Growth", *Review of Economic Studies*, 60:349-366.
- Ellis R.S. (1985) "Entropy, Large Deviations, and Statistical Mechanics", Springer Verlag, Berlin.
- Föllmer H. (1974) "Random Economies with Many Interacting Agents", *Journal of Mathematical Economics*, 1:51-62.
- Friedman D. (1991) "Evolutionary Games in Economics", *Econometrica*, 59:637-666.
- Gali J. (1994) "Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand", *Journal of Economic Theory*, 63:73-96.
- Grossman G.M. and E. Helpman (1991) "Innovation and Growth in the Global Economy", MIT Press, Cambridge (MA).
- Haining R.P. (1990) "Spatial Data Analysis in the Social and Environmental Sciences", Cambridge University Press, Cambridge.
- Henderson V., Kuncoro A. and M. Turner (1995) "Industrial Development in Cities", *Journal of Political Economy*, 103:1067-1090.
- Henkin G.M. and V.M. Polterovich (1991) "Schumpeterian Dynamics as a Nonlinear Wave Theory", *Journal of Mathematical Economics*, 20:551-590.
- Hutschenreiter G., Kaniowski Y.M. and A.V. Kryazhimskii (1995) "Endogenous Growth, Absorptive Capacities, and International R&D Spillovers", mimeo, IIASA, Laxenburg.
- Iway K. (1984) "Schumpeterian Dynamics, Part I and II", *Journal of Economic Behavior and Organization*, 5:159-190 and 321-351.
- Jaffe A.B., Trajtenberg M. and R. Henderson (1992) "Geographic Localization of Knowledge Spillovers as Evidenced by Patent Citations", NBER Working Paper, #3993.
- Joosten R. (1996) "Deterministic Evolutionary Dynamics: A Unifying Approach", *Journal of Evolutionary Economics*, 6:313-324.

- Kiyotaki N. (1988) "Multiple Expectational Equilibria under Monopolistic Competition", *Quarterly Journal of Economics*, 103:695-713.
- Krugman P.R. (1991) "Geography and Trade", MIT Press, Cambridge, MA.
- Landes D.S. (1969) "The Unbound Prometheus", Cambridge University Press, Cambridge.
- Maddison A. (1982) "Phases of Capitalist Development", Oxford University Press, New York.
- Matsuyama K. (1995) "Complementarities and Cumulative Processes in Models of Monopolistic Competition", *Journal of Economic Literature*, 33:701-729.
- Metcalfe S. (1992) "Variety, Structure and Change: An Evolutionary Perspective on the Competitive Process", *Papers on Economics and Evolution*, edit. by the European Study Group for Evolutionary Economics, # 9201.
- Murphy K.M., Shleifer A. and R.W. Vishny (1989) "Industrialization and the Big Push", *Journal of Political Economy*, 97:1003-1026.
- Nelson R. and S. Winter (1982) "An Evolutionary Theory of Economic Change", Harvard University Press, Cambridge, MA.
- Romer P.M. (1990) "Endogenous Technological Change", *Journal of Political Economy*, 98:71-102.
- Rotemberg J.J. (1987) "The New Keynesian Microfoundations" in Fisher S. (Ed.) "NBER Macroeconomics Annual", 1:69-104, MIT Press, Cambridge, MA.
- Saviotti P.P. and G.S. Mani (1995) "Competition, Variety and Technological Evolution: A Replicator Dynamics Model", *Journal of Evolutionary Economics*, 5:369-392.
- Schumpeter J.A. (1934) "The Theory of Economic Development", Harvard University Press, Cambridge, MA.
- Schumpeter J.A. (1942) "Capitalism, Socialism, and Democracy", Harper, New York.
- Segerstrom P.S., Anant T. and E. Dinopoulos (1990) "A Schumpeterian Model of the Product Life Cycle", *American Economic Review*, 80:1077-1091.
- Silverberg G. (1984) "Embodied Technical Progress in a Dynamic Economic Model: the Self-Organization Paradigm", in Goodwin R., Krüger M. and A. Vercelli (eds.) "Nonlinear Models of Fluctuating Growth", Springer-Verlag, Berlin.
- Silverberg G., Dosi G. and L. Orsenigo (1988) "Innovation, Diversity and Diffusion: A Self-Organisation Model", *Economic Journal*, 98:1032-1054.

- Silvestre J. (1993) "The Market-Power Foundations of Macroeconomic Policy", *Journal of Economic Literature*, 31:105-141.
- Soete L. and R. Turner (1984) "Technology Diffusion and the Rate of Technical Change", *Economic Journal*, 94:612-623.
- Startz R. (1989) "Monopolistic Competition as a Foundation for Keynesian Macroeconomic Models", *Quarterly Journal of Economics*, 104:737-752.
- Weisbuch G., Kirman A. and D. Herreiner (1996) "Market Organization", mimeo, University of Marseille III.
- Winter S. (1984) "Schumpeterian Competition in Alternative Technological Regimes", *Journal of Economic Behavior and Organization*, 5:137-158.
- Witt U. (1992) "Evolutionary Concepts in Economics", *Eastern Economic Journal*, 18:405-419.



QUADERNI DEL DIPARTIMENTO DI ECONOMIA

- 1 **Marco CRIVELLINI**, *Vincoli organizzativi- imprenditoriali allo sviluppo: una stilizzazione all'approccio di Ancona*, aprile 1983.
- 2 **Paolo ERCOLANI**, *Prezzi relativi e sviluppo economico: un'analisi dell'evidenza empirica*, luglio 1983.
- 3 **Riccardo MAZZONI**, *Costi comparati e sviluppo regionale: un'analisi empirica*, maggio 1984.
- 4 **Paolo ERCOLANI**, *Sviluppo economico e mutamenti di struttura*, ottobre 1984.
- 5 **Valeriano BALLONI**, *Processi di integrazione nelle ristrutturazioni industriali*, ottobre 1984.
- 6 **Franco SOTTE, Luisa QUATTRINI, Simone RUSPOLI**, *Indagine sulle tipologie aziendali nell'agricoltura delle Marche*, maggio 1985.
- 7 **Geminello ALVI**, *Due scritti eterodossi sulla scienza in economia e la sua storia*, maggio 1985.
- 8 **Luca PAPI**, *Scelte e conseguenze della politica monetaria del primo dopoguerra*, giugno 1986.
- 9 **Massimo TAMBERI**, *Il modellaccio 2: analisi storica dei parametri diretti*, febbraio 1988.
- 10 **Luca PAPI**, *Dynamic specification in U.K. Demand for Money Studies*, marzo 1988.
- 11 **Enzo PESCIARELLI**, *Smith, Bentham and the Development of Contrasting Ideas on Entrepreneurship*, giugno 1988.
- 12 **Alessandro STERLACCHINI**, *Progresso tecnico, attività innovative e crescita della produttività: approcci teorici a livello inter-industriale*, ottobre 1988.
- 13 **Carlo GIANNINI**, *Cointegrazione, analisi di rango e stima consistente dello spazio di cointegrazione partendo dalle stime di un VAR in livelli*, marzo 1989.
- 14 **Carlo GIANNINI, Rocco MOSCONI**, *Non stazionarietà, integrazione, cointegrazione: analisi di alcuni aspetti della letteratura recente*, marzo 1989.
- 15 **Valeriano BALLONI**, *Strutture di mercato e comportamento strategico delle imprese. Il caso dell'industria americana degli elettrodomestici*, 1989.
- 16 **Mauro GALLEGATI, Massimo TAMBERI**, *Divergent Trajectories in Europe: An Analysis of the Recently Developed Countries*, ottobre 1989.
- 17 **Enrico SANTARELLI**, *R & D, Innovation, and the Signalling Properties of the firm's Financial Structure*, maggio 1990.
- 18 **Daniela FELIZIANI**, *Il dibattito internazionale sul tempo di lavoro: una nota introduttiva*, settembre 1990.
- 19 **Massimo TAMBERI**, *Pionieri, Imitatori e Processi di Catching-up*, novembre 1990.
- 20 **Antonio Giulio CALAFATI**, *Processo economico e ambiente naturale in K.W. Kapp*, dicembre 1990.
- 21 **Carlo GIANNINI**, *Topics in Structural Var Econometrics*, luglio 1991.
- 22 **Andrea RICCI**, *Il concetto di integrazione nella teoria economica: una breve rassegna critica*, agosto 1991.

- 23 **Claudio CASADIO TARABUSI, Stefano BRESCHI**, *A Selection Model of Economic Competition: The Role of Market Power and Technological Change*, dicembre 1991.
- 24 **Stefano STAFFOLANI**, *L'inserimento professionale dei giovani diplomati in Italia e in Francia*, dicembre 1991.
- 25 **Enrico SANTARELLI, Alessandro STERLACCHINI**, *Profili e determinanti settoriali della formazione di nuove imprese nell'industria italiana*, aprile 1992.
- 26 **Giorgio FUÀ**, *Appunti sulla crescita economica*, aprile 1992.
- 27 **Stefano STAFFOLANI**, *La flessibilità quantitativa nella gestione del fattore lavoro: una analisi teorica*, aprile 1992.
- 28 **Carlo GIANNINI, Antonio LANZAROTTI, Mario SEGHELINI**, *A Traditional Interpretation of Macroeconomic Fluctuations: the Case of Italy*, maggio 1992.
- 29 **Riccardo MAZZONI**, *I mercati locali del lavoro in Italia*, luglio 1992.
- 30 **Alessandro BARTOLA, Franco SOTTE, Giuseppe BUONCOMPAGNI**, *"L'AIMA"*, luglio 1992.
- 31 **Renato BALDUCCI**, *"Crescita in regime di rendimenti di scala costanti"*, febbraio 1993.
- 32 **Carlo A. FAVERO**, *"Ottimizzazione intertemporale e metodi econometrici in economia"*, maggio 1993.
- 33 **Alessandro BARTOLA, Franco SOTTE, Andrea FANTINI, Raffele ZANOLI**, *"L'agricoltura nelle Marche. Tendenze settoriali e politica agraria"*, maggio 1993.
- 34 **Riccardo MAZZONI**, *Sviluppo economico e localizzazione produttiva*, giugno 1993.
- 35 **Alberto ZAZZARO**, *Costi di liquidità e costi di solvibilità: il ruolo delle banche locali nello sviluppo economico regionale*, giugno 1993.
- 36 **Domenico MIGNACCA**, *An Application of a Structural VAR Technique to Interpret UK Macroeconomic Fluctuations*, giugno 1993.
- 37 **Stefano MANZOCCHI, Domenico MARINUCCI**, *Technology Adoption Under Strategic Complementarity: "Less" Information Can Do Better*, giugno 1993.
- 38 **Antonio G. CALAFATI**, *Scelta e Azione*, giugno 1993.
- 39 **Gabriele CATTAROZZI**, *Gestione delle obbligazioni e curva di inviluppo*, luglio 1993.
- 40 **Alberto ZAZZARO**, *Le banche in un'economia regionale: una rassegna della letteratura*, settembre 1993.
- 41 **Alessandro VAGLIO**, *A model of the audience for TV broadcasting implications for advertising competition and regulation*, ottobre 1993.
- 42 **Carlo A. FAVERO, Riccardo LUCCHETTI**, *Output, interest rates and the monetary transmission mechanism: some empirical evidence for Italy*, ottobre 1993.
- 43 **Riccardo LUCCHETTI**, *Modelli in differenze con errori di misura*, novembre 1993.
- 44 **Stefano STAFFOLANI**, *Lavoro a turni e durata di utilizzo del capitale*, novembre 1993.
- 45 **Alessandro STERLACCHINI**, *La ricerca universitaria in Italia*, novembre 1993.
- 46 **Daniela FELIZIANI**, *Organizzazione e regolamentazione degli orari di lavoro nei paesi industrializzati*, marzo 1994.
- 47 **Alessandro STERLACCHINI**, *The birth of new firms in Italian manufacturing*, marzo 1994.
- 48 **Franco SOTTE, Giuseppe BUONCOMPAGNI**, *An overview on public transfers in the Italian Agricultural Policy*, marzo 1994.
- 49 **Pietro ALESSANDRINI, Andrea RICCI**, *Squilibri demografici e scarsità di risparmio nell'economia mondiale*, marzo 1994.
- 50 **Alberto ZAZZARO**, *La specificità delle banche: teorie ortodosse e teorie eterodosse a confronto*, maggio 1994.
- 51 **Carlo MONTICELLI, Luca PAPI**, *La definizione di moneta in economie aperte e integrate: verso l'"eutanasia" del concetto di moneta nazionale?*, maggio 1994.
- 52 **Riccardo LUCCHETTI, Stefano STAFFOLANI**, *Orario di lavoro e occupazione: un approccio teorico con una applicazione alla grande industria italiana*, luglio 1994.
- 53 **Riccardo LUCCHETTI**, *Companion form representation of cointegrating VARs*, ottobre 1994.
- 54 **Paolo ERCOLANI**, *La terzianizzazione dell'occupazione. Analisi delle cause e dei problemi aperti*, dicembre 1994.
- 55 **Rossano BRUSCHI**, *La teoria delle aspettative razionali e la curva dei tassi di interesse a scadenza: un'applicazione al mercato monetario tedesco*, dicembre 1994.
- 56 **Roberto ESPOSTI, Pierpaolo PIERANI, Franco SOTTE**, *Fattori quasi fissi e produttività totale dei fattori in agricoltura. Teoria e applicazione ad una impresa marchigiana ex-mezzadrile*, gennaio 1995.
- 57 **Michela VECCHI**, *Human capital and excess labour*, febbraio 1995.
- 58 **Alberto BAGNAI, Stefano MANZOCCHI**, *Un'indagine empirica sulla mobilità dei capitali nei paesi in via di sviluppo*, marzo 1995.
- 59 **Domenico MIGNACCA**, *Comparing the impulse response functions of different models*, marzo 1995.
- 60 **Manuela VICONI**, *L'Unione Europea e gli shock esogeni ai sistemi economici*, marzo 1995.
- 61 **Paolo Emilio MISTRULLI, Roberto TORRINI**, *Salari di efficienza, costi di controllo e decentramento produttivo*, marzo 1995.
- 62 **Alessandro VAGLIO**, *Potere di mercato, consumi e crescita*, dicembre 1994.
- 63 **Luca PAPI, Carlo MONTICELLI**, *EU-Wide money demand: An assessment of competing approaches*, maggio 1995.
- 64 **Antonio G. CALAFATI**, *Mercati e gerarchie nel processo di apprendimento degli agenti collettivi*, giugno 1995.
- 65 **Marco ACCORRONI, Luca PAPI**, *La valutazione dell'efficienza degli sportelli bancari*, ottobre 1995.
- 66 **Stefano STAFFOLANI**, *Interazioni nelle scelte e norme sociali*, ottobre 1995.
- 67 **Tommaso PEREZ**, *Multinational enterprises and technological spillovers: An evolutionary model*, ottobre 1995.
- 68 **Edoardo GAFFEO**, *Macroeconomics without the representative agent: Difficulties and new perspectives*, dicembre 1995.

- 69 **Pietro ALESSANDRINI, Alessandro STERLACCHINI**, *Ricerca, formazione e rapporti con l'industria: I problemi irrisolti dell'università italiana*, dicembre 1995.
- 70 **Paolo GUERRIERI, Stefano MANZOCCHI**, *Patterns of Trade and Foreign Direct Investment in European Manufacturing: "Convergence" or "Polarization"?*, marzo 1996.
- 71 **Laura CHIES, Francesco TROMBETTA**, *Riduzione dell'orario di lavoro e disoccupazione: il dibattito tedesco*, marzo 1996.
- 72 **Stefano FIORI**, *Ordine visibile e ordine invisibile. Il difficile rapporto fra natura e società nell'economia politica smithiana e presmithiana (1690-1790)*, marzo 1996.
- 73 **Paolo Emilio MISTRULLI**, *Rendita informativa, intermediazione finanziaria e scelte di portafoglio*, marzo 1996.
- 74 **Fabio FIORILLO**, *Il problema dell'isteresi in economia: confronto tra isteresi fisica e passeggiate aleatorie, significato e applicazioni economiche*, aprile 1996.
- 75 **Tommaso LUZZATI**, *Una testimonianza sull'ipotesi di piena razionalità*, maggio 1996.
- 76 **Nicola BOARI**, *Law and Economics in Action: An Efficiency Analysis of Italian Penal Procedures after 1989*, maggio 1996.
- 77 **Roberto GIORGI, Franco SOTTE**, *Riuscirà il mondo a sfamare se stesso nel 2025?*, maggio 1996.
- 78 **Alessandro STERLACCHINI**, *Inputs and Outputs of Innovative Activities in Italian Manufacturing*, giugno 1996.
- 79 **Marco GALLEGATI**, *Firm's optimal capital accumulation path with asymmetric informations and debt instead of equity finance*, giugno 1996.
- 80 **Erica SEGHETTI, Massimo TAMBERI**, *Competitività, crescita e localizzazione in un settore tradizionale*, giugno 1996.
- 81 **Pietro ALESSANDRINI**, *I sistemi locali del credito in regioni a diverso stadio di sviluppo*, settembre 1996.
- 82 **Aldo FEMIA**, *Input-Output Analysis of Material Flows: an application to the German Economic System for the year 1990*, settembre 1996.
- 83 **Michela VECCHI**, *Increasing Returns versus Externalities: Pro-Cyclical Productivity in US and Japan*, ottobre 1996.
- 84 **Stefano FIORI**, *Conoscenza e informazione in F.A. von Hayek*, ottobre 1996.
- 85 **Cecilia BENVENUTO**, *Le opzioni esotiche: Problemi di pricing e copertura*, ottobre 1996.
- 86 **Laura CHIES, Riccardo LUCCHETTI, Stefano STAFFOLANI**, *Occupazione, Disoccupazione, Intattività: determinanti della mobilità tra stati in Italia*, marzo 1997.
- 87 **Marco CUCCULELLI**, *Struttura finanziaria, seniority rules del debito e decisioni di investimento delle imprese*, marzo 1997.
- 88 **Edeardo GAFFEO**, *Multilevel Interactions with a Keynesian Flavour in a Stochastic Macroeconomic Model*, maggio 1997.
- 89 **Antonio G. CALAFATI**, *Labour Supply and Unemployment*, maggio 1997.
- 90 **Roberto ESPOSTI**, *Progresso Technico Multioutput e Ruolo di R&S e Assistenza Tecnica. Applicazione dell'Analisi Nonparametrica all'Agricoltura Italiana*, giugno 1997.
- 91 **Edoardo GAFFEO**, *Competition-led Endogenous Growth with Localized Technological Change*, giugno 1997.