Università degli Studi di Ancona
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COMPARING THE IMPULSE RESPONSE FUNCTIONS OF DIFFERENT MODELS

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ABSTRACT

The problem we want to solve in this paper is that of finding a statistical test that permits us to compare the impulse response function (IRF) of a linear model with that of a nonlinear one. We achieve our goal starting with a simple case where the comparison is between two VAR models of different order. Next, we briefly extend the results to VARs of the same order but with a different structuralization. A Monte Carlo simulation is performed to evaluate power and size of the test. We then give some insights for comparing VAR with multivariate SETAR IRFs. Finally, we present an alternative procedure (a variation of the encompassing test) for comparing linear and complicated nonlinear IRFs.

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COMPARING THE IMPULSE RESPONSE FUNCTIONS OF DIFFERENT MODELS[(*)]

Introduction

In this paper we try to derive a statistical framework that permits us to compare the impulse response functions (IRF) of different models. To achieve our goal, we make use of the well established asymptotic theory of IRF developed in several papers and textbooks[1]. We then extend this theory in order to derive statistical tests for various hypotheses concerning the IRFs and in particular we want to explore the possibility of testing the hypothesis that linear and nonlinear impulse response functions give the same response path.

1. Technical tools

In this paragraph we review some background literature, and in particular the passage from a VAR(p) model to its VMA(∞) representation.

Let $\{y_t\}_0^\infty$ be a stationary n-dimensional stochastic process that, we assume, has a VAR(p) representation:

[1.1] $\underline{y}_t = \underline{\mu} + A(L)\underline{y}_t + \underline{\varepsilon}_t$,

where: $A(L)=A_1 L+A_2L^2+....+A_pL^p$, $E(\underline{\varepsilon}_t\underline{\varepsilon}_t') = \Sigma_\varepsilon$, $E(\underline{\varepsilon}_t) = \underline{0}$, and $| I_n - A(z)| \neq 0$ for $| z | \leq 1$. Furthermore, consider the "structural" representations (SVAR) where in addition to [1.1] we have[2]:

(a) $\underline{u}_t = K\underline{\varepsilon}_t$,
(b) $C\underline{u}_t = \underline{\varepsilon}_t$,
(c) $A^*\underline{\varepsilon}_t = B\underline{u}_t$,

where K, C, A^*, and B are nonsingular matrices, $E(\underline{u}_t)=\underline{0}$ and $E(\underline{u}_t\underline{u}_t') = I_n$. For simplicity, we consider the K-model and that there is no constant term in model [1.1].

(*) I wish to thank Kenneth West and Kris Sachsenmeier for several comments and suggestions. Obviously, all remaining errors are mine.

1 Lütkepohl(1989,1990,1993), Baillie(1987), Giannini(1992).

2 In Giannini(1992), these representations are respectively called K-model, C-model and AB-model.

3 See also Baillie(1987), p.106.

Then, we can write [1.1] in a more compact way[3]:

$$[1.2]\ \underset{np\times 1}{\underline{Y}_t} = \underset{np\times np}{A}\,\underline{Y}_{t-1} + \underline{v}_t ,$$

where[4]:

$Y_t=[y'_t, y'_{t-1},....,y'_{t-p+1}]'$, $v_t=[\varepsilon_t', 0',....,0']'$, and:

$$A = \begin{bmatrix} A_1 & . & . & A_p \\ I_n & 0 & . & 0 \\ . & . & . & . \\ 0 & . & I_n & 0 \end{bmatrix}.$$

Being the polynomial matrix (I - A(L)) invertible, [1.1] has a VMA(∞) representation:

$$[1.3]\ Y_t = \sum_{i=0}^{\infty} A^i v_{t-i} .$$

Consider the n x (np) extraction matrix $J=[I_n ,0,...,0]'$, and noting that $J'Jv_t=v_t$, [1.3] can be manipulated as follows:

$$[1.3']\ JY_t = y_t = \sum_{i=0}^{\infty} JA^iJ'Jv_{t-i} = \sum_{i=0}^{\infty} JA^iJ'K^{-1}u_{t-i} ,$$

being J $v_t = \varepsilon_t$ and, denoting $\Phi_i = J A^i J' K^{-1}$ we obtain:

$$[1.4]\ y_t = \sum_{i=0}^{\infty} \Phi_i u_{t-i} .$$

The coefficient matrix Φ_i can also be calculated recursively by the use of the relation $\Phi_i = S_i K^{-1}$ and $S_i = \sum_{j=1}^{i} S_{i-j}A_j$, with $S_0 = I_n$ and i=1,....[5]. Furthermore, the autocovariance matrix between y_t and y_{t-s} is:

$$[1.5]\ \Omega(s) = \sum_{i=0}^{\infty} \Phi_{s+i}\Sigma_\varepsilon \Phi_i' .$$

Now define:

$\alpha = vec(A_1 ,...,A_p)$,
$\eta_i = vec(\Phi_i)$,

[4] Hereafter, vectors will not be underlined.
[5] For further details, cf. Lütkepohl(1993), pp.17-18.

$\eta^h = vec(\Phi_0 ,...,\Phi_h)$,
$\mu = vech(\Sigma_\varepsilon)$,

and:

L is the elimination matrix such that: $\mu = L\ vec(\Sigma_\varepsilon)$, dim(L)=(n(n+1)/2, n^2);
C is the commutation matrix such that: C $vec(\Phi_i) = vec(\Phi_i')$, dim(C)=$n^2$;
D is the duplication matrix such that: D $vech(\Sigma_\varepsilon) = vec(\Sigma_\varepsilon)$, dim(D)=($n^2$,n(n+1)/2)[6].

Using propositions 1 and 2 of Lütkepohl(1989), we obtain:

$$[1.6]\ \sqrt{T}(\hat{\mu} - \mu) \xrightarrow{d} N(0,\ \Sigma_\mu),$$

where:

$\Sigma_\mu = 2\,(D'D)^{-1}D'\,(\Sigma_\varepsilon \otimes \Sigma_\varepsilon)D\,(D'D)^{-1}$ is nonsingular. Furthermore:

$$[1.7]\ \sqrt{T}\ vec(\hat{\Phi}_0 - \Phi_0) \xrightarrow{d} N(0,\ \Sigma(0)),$$

where: $\Sigma(0) = L'V\Sigma_\mu V'L$, and $V = \{L(I_{n^2} + C)(K^{-1}\otimes I_n)L'\}^{-1}$. Now, suppose:

$$[1.8]\ \sqrt{T}\begin{bmatrix} \hat{\alpha} - \alpha \\ \hat{\mu} - \mu \end{bmatrix} \xrightarrow{d} N\left(0, \begin{bmatrix} \Sigma_\alpha & 0 \\ 0 & \Sigma_\mu \end{bmatrix}\right),$$

where: Σ_α and Σ_μ are nonsingular matrices with $\Sigma_\alpha = [E(Y_tY_t')]^{-1} \otimes \Sigma_\varepsilon$. Thus, from proposition 2 of Lütkepohl(1989):

$$[1.9]\ \sqrt{T}(\hat{\eta}^h - \eta^h) \xrightarrow{d} N(0,\ \underset{n^2(h+1)\times n^2(h+1)}{\Sigma(h)}),$$

with:

$$[1.10]\ \Sigma(h) = F\Sigma_\alpha F' + Q\Sigma(0)Q' ,$$

where:

$$F = [0\ F_1' \ ....F_h']' ,\ F_0 = 0, \text{and } F_i = \sum_{k=0}^{i-1}\{[(K')^{-1}J\,(A')^{i-1-k}] \otimes [JA^kJ']\} \text{for } i > 0.$$

Furthermore:

[6] For further details cf. Magnus(1988).

$$Q=\left[I_{n^2},(I_n\otimes JAJ')',.......,(I_n\otimes JA^hJ')'\right]',$$

thus:

$$\underset{n^2\times n^2}{\Sigma(h)_{ij}}=F_i\Sigma_\alpha F_j'+(I_n\otimes JA^iJ')\Sigma(0)(I_n\otimes JA^jJ')'.$$

2. Hypotheses testing on accumulated IRF

In this paragraph, we derive a chi-square test statistic to compare IRF of different models. To derive the test, we first need a theorem contained in Serfling(1980)[7]:

Theorem: suppose that $\sqrt{T}(\hat{x}-x)\xrightarrow{d}N(0,\Sigma_x)$, where $x=(x_1,....,x_n)'$ and consider the vector-valued function $g(x)=(g_1(x),.....,g_m(x))'$ which satisfies the condition $V=\frac{\partial g_i(x)}{\partial x'}\neq 0$ at x for i=1,...,m. Then: $\sqrt{T}(g(\hat{x})-g(x))\xrightarrow{d}N(0,V\Sigma_x V')$.

Consider now the i-th accumulated IRF:

$$[2.1]\ \Psi_i=\sum_{j=0}^{i}\Phi_j,$$

defining $\Psi^h=(\Psi_0',\Psi_1',.....,\Psi_h')'=(\Phi_0',\Phi_0'+\Phi_1',.........,\sum_{j=0}^{h}\Phi_j')'$, we have that:

$$[2.2]\ \Psi^h=\Xi\Phi^h,\ \text{where}\ \underset{n(h+1)\times n(h+1)}{\Xi}=\begin{bmatrix}I_n & 0 & . & 0\\ I_n & I_n & . & 0\\ . & . & . & .\\ I_n & I_n & . & I_n\end{bmatrix},\ \Phi^h=(\Phi_0',\Phi_1',...,\Phi_h')'.$$

Using some matrix algebra, it is easy to see that:

$$[2.3]\ \rho^h=vec(\Psi^h)=\Xi\eta^h.$$

Thus, being $\frac{\partial\,\Xi\eta^h}{\partial\,(\eta^h)'}=\Xi$:

$$[2.4]\ \sqrt{T}(\hat{\rho}^h-\rho^h)\xrightarrow{d}N(0,\Sigma_\rho),\ \text{where}\ \Sigma_\rho=\Xi\Sigma(h)\Xi'.$$

The first problem we want to solve is to find the asymptotic distribution of the "n" responses of length (h+1) due to an initial vector of shocks q (obviously of dimension n). Denoting with p^h the vector of dimension n(h+1)x1 containing the responses:

$$[2.5]\ p^h=\Xi\Phi^h q\ ,$$

thus, being $p^h=vec(p^h)=(q'\otimes\Xi)\,vec(\Phi^h)$:

$$[2.6]\ \sqrt{T}(\hat{p}^h-p^h)\xrightarrow{d}N(0,\Sigma_p),\ \text{where}\ \Sigma_p=(q'\otimes\Xi)\Sigma(h)(q'\otimes\Xi)'.$$

[7] Serfling(1980), p. 122.

3. Comparing IRF obtained from two different VAR models

The problem we want to solve now is the comparison of IRF from two different models in order to see if they can be considered statistically equivalent. Our goal is to find a statistical framework that enable us to compare linear and nonlinear IRF but for the moment we conduct our analysis on two VAR models of different order and then we try to extend our results for other situations.

Consider a VAR(p) and a VAR(q) model with $p\neq q$:

VAR(p):

$$[3.1]\ y_t=A^1(L)y_t+\varepsilon_t^1,\ A^1(L)=A_1^1L,\ A_2^1L^2,......,\ A_p^1L^p\ ,E(\varepsilon_t^1(\varepsilon_t^1)')=\Sigma_{\varepsilon^1}\ ,$$

with $u_t^1=K_1\varepsilon_t^1$;

VAR(q):

$$[3.2]\ y_t=A^2(L)y_t+\varepsilon_t^2,\ A^2(L)=A_1^2L,\ A_2^2L^2,......,\ A_q^1L^q\ ,E(\varepsilon_t^2(\varepsilon_t^2)')=\Sigma_{2_\varepsilon},$$

with $u_t^2=K_2\varepsilon_t^2$.

We can write [3.1] and [3.2] in a more compact way:

$$[3.3]\ y_{2,t}=\begin{bmatrix}y_t & 0\\ 0 & y_t\end{bmatrix}=A(L)y_{2,t}+\varepsilon_{2,t},\ \text{where: } A(L)=\begin{bmatrix}A^1(L) & 0\\ 0 & A^2(L)\end{bmatrix}\text{and}\ \varepsilon_{2,t}=\begin{bmatrix}\varepsilon_t^1 & 0\\ 0 & \varepsilon_t^2\end{bmatrix}.$$

Working on system [3.3] as we have done in paragraph 2, we define:

$$
\underset{n(p+q)\times 2}{Y_t} = \begin{bmatrix} y_t & 0 \\ . & . \\ . & . \\ y_{t-p+1} & 0 \\ 0 & y_t \\ . & . \\ . & . \\ . & . \\ . & . \\ 0 & y_{t-q+1} \end{bmatrix}, \quad \underset{(n(p+q))x(n(p+q))}{A} = \begin{bmatrix} A_1^1 & . & . & A_p^1 & 0 & . & . & 0 \\ I_n & 0 & . & 0 & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & . & I_n & 0 & 0 & . & . & 0 \\ 0 & . & . & 0 & A_1^2 & . & . & A_q^2 \\ . & . & . & . & I_n & 0 & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & . & . & 0 & . & . & I_n & 0 \end{bmatrix}, \quad v_t = \begin{bmatrix} \varepsilon_t^1 & 0 \\ . & . \\ . & . \\ . & 0 \\ 0 & \varepsilon_t^2 \\ . & 0 \\ . & . \\ . & . \\ . & . \\ 0 & 0 \end{bmatrix},
$$

and we obtain:

$$[3.4]\; Y_t = AY_{t-1} + v_t \;,$$

and thus:

$$[3.5]\; Y_t = \sum_{i=0}^{\infty} A^i v_{t-i} \;.$$

Consider now a J matrix of dimension 2n x n(p+q) having the following form:

$$
\begin{bmatrix} I_n & 0 & . & . & . & . & . & . & . & 0 \\ 0 & . & . & . & . & I_n & 0 & . & . & 0 \end{bmatrix}.
$$

Being again $JJ'v_t=v_t$, we can extend the results obtained in paragraph 2. In particular, pre-multiplying both sides of [3.5] by J and post-multiplying both sides by a vector of ones of dimension 2 x 1, we obtain:

$$
[3.6]\; y_{n,t} = \begin{bmatrix} y_t \\ y_t \end{bmatrix} = \sum_{i=0}^{\infty} JA^iJ' \; v_{n,t} = \sum_{i=0}^{\infty} JA^iJ'K^{-1} \; u_{n,t}, \text{ where } v_{n,t} = \begin{bmatrix} \varepsilon_t^1 \\ \varepsilon_t^2 \end{bmatrix}, u_{n,t} = \begin{bmatrix} u_t^1 \\ u_t^2 \end{bmatrix}, \text{and}
$$

$$
K^{-1} = \begin{bmatrix} K_1^{-1} & 0 \\ 0 & K_2^{-1} \end{bmatrix}.
$$

Then: $\Phi_i = JA^iJ'K^{-1}$, or recursively $\Phi_i = S_i K^{-1}$ and $S_i = \sum_{j=1}^{i} S_{i-j}A_j$, with $S_0 = I_{2n}$.

$E\left(v_{n,t}v_{n,t}'\right) = \begin{bmatrix} E(\varepsilon_t^1\varepsilon_t^{1'}) & E(\varepsilon_t^1\varepsilon_t^{2'}) \\ E(\varepsilon_t^2\varepsilon_t^{1'}) & E(\varepsilon_t^2\varepsilon_t^{2'}) \end{bmatrix}$ is our new Σ_ε (Σ_{ε^*}).

Similarly to paragraph 2, we define[8]:

[8] In this case our new "n" is equal to 2n and the new "p" is equal to p+q. Thus the dimension of the various matrices and vectors can be calculated as in paragraph 2.

$\alpha^* = vec(A_1^1,\ldots,A_p^1,0,\ldots,0, A_1^2,\ldots,A_q^2)$;
$\eta_i^* = vec(\Phi_i)$;
$\Phi^h = (\Phi_0,\ldots, \Phi_h)$;
$\eta^{*h} = vec(\Phi^h)$;
$\mu^* = vech(\Sigma_{\varepsilon^*})$;
$p^{*h} = \Xi\Phi^h q^*$, with $q^* = \begin{bmatrix} q \\ q \end{bmatrix}$,

and again, considering that in this case nothing is different from evaluating the IRF of a model with 2n variables, we obtain:

$$[3.7]\; \sqrt{T}\left(\hat{p}^{*h} - p^{*h}\right) \xrightarrow{d} N\left(0\,,\, \Sigma_{p^*}\right), \text{ where } \Sigma_{p^*} = \left(q^{*'} \otimes \Xi\right) \Sigma(h) \left(q^{*'} \otimes \Xi\right)' .$$

At this point, we can easily derive the test. Consider a full rank matrix R of order n(h+1) x 2n(h+1) which compares each IRF element (for each step) of one model with the corresponding element of the IRF of the other VAR model. Given a consistent estimator for $\Sigma(h)$ [$\hat{\Sigma}(h)$], under the null hypothesis that the two models consistently estimate the "true" IRF path[9], we have that:

$$[3.8]\; T\,(R\hat{p}^{*h})' \left[R\,(q^{*'} \otimes \Xi)\, \hat{\Sigma}(h)\, (q^{*'} \otimes \Xi)'\, R'\right]^{-1} (R\hat{p}^{*h}) \xrightarrow{d} \chi^2_{n(h+1)} \;.$$

4. Monte Carlo simulation

In what follows, we try to evaluate size and power of the test using Monte Carlo simulation. In our specific case, we have some limitations considering the small number of experiments we perform (1000), but our aim is just to get the idea of the test's performance.

We perform simulations using two different bivariate AR models that, henceforth, we call "model A" and "model B".

<u>Model A</u>:

$$[4.1]\; z_t = \mu_A + Bz_t + Az_{t-1} + \varepsilon_{A,t} \,,$$

where:

$$
z_t = \begin{bmatrix} y_t \\ x_t \end{bmatrix}, \; \varepsilon_{A,t} = \begin{bmatrix} \varepsilon_{y,t} \\ \varepsilon_{x,t} \end{bmatrix}, \; \mu_A = \begin{bmatrix} .3 \\ .1 \end{bmatrix}, \; A = \begin{bmatrix} .2 & .5 \\ -.1 & .3 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 0 \\ .3 & 0 \end{bmatrix},
$$

[9] This implies $R\,p^{*h} = 0$.

and $\varepsilon_{A,t} \sim N(0, I_2)$.

Model B:

[4.2] $z_t = \mu_B + Bz_t + Az_{t-1} + Cz_{t-2} + \varepsilon_{B,t}$,

where: μ_B, B and A are equal to those in [4.1]; $\varepsilon_{B,t} \sim N(0, I_2)$ and:

$$C = \begin{bmatrix} .6 & .3 \\ -.2 & .5 \end{bmatrix}.$$

Given these data generation processes (DGP), we evaluate size and power of the test in a nested and in a non-nested situation[10]. We evaluate the power of the test, given that the DGP is that of model A, considering the following non-nested models:

model A.1:

[4.3] $z_t = \mu_{A.1} + A_1 z_{t-1} + \varepsilon_{1,t}$,

where: $\varepsilon_{1,t} \sim N(0, \Sigma_1)$, $\Sigma_1 = K_1 K_1'$ and K_1 is a lower triangular matrix;
model A.2:

[4.4] $z_t = \mu_{A.2} + A_2 z_{t-2} + \varepsilon_{2,t}$,

where: $\varepsilon_{2,t} \sim N(0, \Sigma_2)$, $\Sigma_2 = K_2 K_2'$ again K_2 is a lower triangular matrix.
The assumptions underlined are that model [4.3] is correctly specified and the instantaneous causality is correctly specified for both [4.3] and [4.4][11]. We run the simulation for forecast horizon 2 and 3 without taking into account the first-period IRFs being, by construction, equal for both models[12]. Table A contains the results of our experiment and we can see that it seems to perform correctly in terms of power.

[10] To perform the analysis we use RATS procedures (version 4.00).
[11] K_1^{-1} and K_2^{-1} estimate (I-B).
[12] It is important to note that the test tends to reject H_0 (even in case it is true) if Σ_{p^*} is "close" to being singular. Therefore, before performing the test, it would be wise to check for possible singularity of Σ_{p^*}, let's say, by checking its determinant.

Table A: Power of the test in a non-nested situation(*)

	h=2 (DF=2)	h=3 (DF=4)
refused H_0 at 1%	100%	100%
refused H_0 at 2.5%	100%	100%
refused H_0 at 5%	**100%**	**100%**
refused H_0 at 10%	100%	100%
refused H_0 at 20%	100%	100%
1% empirical c.v.	28.59	336.75
2.5% emp. c.v.	27.64	302.04
5% emp. c.v.	**25.42**	**273.51**
10% emp. c.v.	23.94	246.67
20% emp. c.v.	22.23	212.68

Now, we turn to evaluate the power of the test when the two models are nested.

Model B.1:

[4.5] same specification as in [4.3];

model B.2:

[4.6] $z_t = \mu_{B.2} + B_1 z_{t-1} + B_2 z_{t-2} + \varepsilon_{2,t}$, (same assumptions as in [4.4]).

This time, the DGP is that of model B. The results of the simulation are reported in table B.

(*) T=200, number of experiments=1000, q=[1,1,1,1]'.

Table B: Power of the test in a nested situation(**)

	h=2 (DF=2)	h=3 (DF=4)
refused H_0 at 1%	99.7%	99.6%
refused H_0 at 2.5%	99.9%	99.8%
refused H_0 at 5%	**99.9%**	**99.8%**
refused H_0 at 10%	100%	99.9%
refused H_0 at 20%	100%	100%
1% empirical c.v.	80.04	551.11
2.5% emp. c.v.	63.37	472.22
5% emp. c.v.	**53.27**	**368.84**
10% emp. c.v.	45.08	297.46
20% emp. c.v.	37.65	215.55

As shown in table B, we can be satisfied with the test's performance.

Finally, we evaluate the size of the test considering the model A's DGP and specifications [4.5] and [4.6]. The results are contained in the following table.

Table C: Size of the test(***)

	h=2 (DF=2)	h=3 (DF=4)
refused H_0 at 1%	0%	6.2%
refused H_0 at 2.5%	0%	8.9%
refused H_0 at 5%	**0%**	**10.9%**
refused H_0 at 10%	0%	16.2%
refused H_0 at 20%	0%	23.9%
1% empirical c.v.	0.94	25.12
2.5% emp. c.v.	0.81	18.19
5% emp. c.v.	**0.67**	**14.08**
10% emp. c.v.	0.50	9.97
20% emp. c.v.	0.35	6.87

The performance is satisfactory for h=2 but not for h=3; in fact, if we consider the 5% significance level, for h=3 our experimental size is 10.9% which is outside the upper bounds of the 5% region. Even considering larger sample size, the situation does not change (see table D).

(**) T=200, number of experiments=1000, q=[1,1,1,1]'.
(***) T=200, number of experiments=1000, q=[1,1,1,1]'.

Table D: Size of the test for h=3 and T=400, T=600(+)

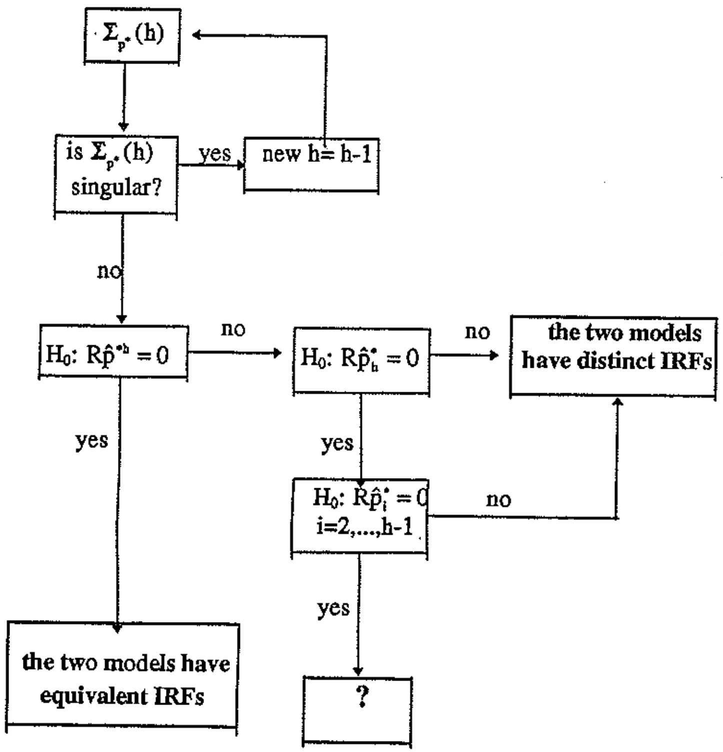
	h=3 T=400 (DF=4)	h=3 T=600 (DF=4)
refused H_0 at 1%	6.0%	4.6%
refused H_0 at 2.5%	8.4%	7.7%
refused H_0 at 5%	**11.0%**	**10.2%**
refused H_0 at 10%	16.5%	15.4%
refused H_0 at 20%	25.5%	23.1%
1% empirical c.v.	21.84	21.59
2.5% emp. c.v.	18.42	15.19
5% emp. c.v.	**14.25**	**12.61**
10% emp. c.v.	10.11	9.63
20% emp. c.v.	7.10	6.47

If we consider just the third IRF step, and not the entire path, the size of the test is inside the 5% region[13]. In view of those results, we suggest following the test strategy contained in diagram 1.

Obviously, our analysis is far from being exaustive; to improve it we should consider: different sample sizes, larger number of experiments, other DGPs, different VAR models and longer IRF horizons.

(+) Number of experiments=1000, q=[1,1,1,1]'.
13 Results not reported here.

Diagram 1: Test strategy



5. Some extensions

The test we obtained for two different VAR models (for the same set of variables), can be extended to different cases. Instead of considering the reduced forms as we did, we could perform the test on two models with p=q but a different structuralization (K) and thus we have to consider $y^*_{n,t} = Ky_{n,t}$. The analysis can also be extended to more than two models at once. Furthermore, we can compare linear structures with simple nonlinear models such as multivariate SETAR models. The only difference with the previous analysis consists of the fact that working with multivariate SETAR models is like comparing more than two models at once and that the variance-covariance matrix $\Sigma(h)$ is now a function of q^* (the vector of initial shocks) and of the past histories of the variables.

Finally, suppose that we want to test the equivalence of the path obtained using a VAR model (call it A model) with the path obtained with a complicated nonlinear model (call it B model). Using the statistical properties of the normal distribution and supposing that:

$$\sqrt{T}(\hat{p}^h_A - p^h) \xrightarrow{d} N(0, \Sigma_{A,p}), \text{where } \Sigma_{A,p} = (q' \otimes \Xi)\Sigma_A(h)(q' \otimes \Xi)',$$

under the null hypothesis that the nonlinear path is the "true" one[14] and that the linear IRF is a consistent estimator of it, we can test $\hat{p}^h_A = \hat{p}^h_B$ using a χ^2 statistic with n(h+1) degrees of freedom:

$$T(\hat{p}^h_A - \hat{p}^h_B)' \hat{\Sigma}^{-1}_{A,p} (\hat{p}^h_A - \hat{p}^h_B) \xrightarrow{d} \chi^2_{(n(h+1))} .$$

Relaxing the hypothesis $\hat{p}^h_B = p^h$ and supposing that $\hat{p}^h_A$ and $\hat{p}^h_B$ have a joint multivariate normal distribution with, again, $\hat{p}^h_A$ and $\hat{p}^h_B$ uncorrelated, we can conclude that if we accepted H_0 using the unconditional distribution we would obtain the same result using the joint distribution. Suppose that:

$$\sqrt{T}\begin{bmatrix} \hat{p}^h_A - p^h \\ \hat{p}^h_B - p^h \end{bmatrix} \xrightarrow{d} N(0, \Sigma_{p^*}), \text{ where } \Sigma_{p^*} = \begin{bmatrix} \Sigma_{A,p} & 0 \\ 0 & \Sigma_{B,p} \end{bmatrix} \text{(nonsingular)},$$

then:

$$\sqrt{T}(\hat{p}^h_A - \hat{p}^h_B) \xrightarrow{d} N(0, \Sigma_{A,p} + \Sigma_{B,p}) .$$

As we previously affirmed, in the case that H_0 is accepted using the unconditional distribution, it would be accepted anyway if we performed the test using the joint distribution. To prove this, what we need to show is that $\Sigma^{-1}_{A,p} - (\Sigma_{A,p} + \Sigma_{B,p})^{-1}$ is a negative definite matrix. In fact, if this is the case, then:

$$[5.1] \quad T(\hat{p}^h_A - \hat{p}^h_B)' \Sigma^{-1}_{A,p} (\hat{p}^h_A - \hat{p}^h_B) - T(\hat{p}^h_A - \hat{p}^h_B)' (\Sigma_{A,p} + \Sigma_{B,p})^{-1} (\hat{p}^h_A - \hat{p}^h_B) =$$

$$= T(\hat{p}^h_A - \hat{p}^h_B)' [\Sigma^{-1}_{A,p} - (\Sigma_{A,p} + \Sigma_{B,p})^{-1}] (\hat{p}^h_A - \hat{p}^h_B) > 0 ,$$

(see the appendix for the proof).

Obviously, we can perform tests on a subsample of $\hat{p}^h_A$ by verifying the usal condition $R\hat{p}^h_A = R\hat{p}^h_B$, where R is a full rank matrix of order r x n(h+1) and r is the number of restrictions.

The usefulness of this procedure is particularly clear when we want to test the equivalence between linear and nonlinear paths because we can avoid evaluating the asymptotic distribution of the nonlinear model.

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[14] The strong assumption underlying this is that we assume not only that plim $\hat{p}^h_B = p^h$, but we also have that $\hat{p}^h_B = p^h$.

Conclusions

In this paper, we developed a test for comparing IRF paths of different models using statistical tools originally developed by several authors. The simulation results of the test performance, for horizons equal to 2 and 3, are contained in tables A-D. Those results clearly show that the power of the test is acceptable both for nested and non-nested models while some problems arising for its size when we consider h=3. In view of this problem, we suggest a test strategy which is contained in diagram 1.

Paragraph 5 contains some extensions to VAR models, which have the same lag length, with a different structuralization of the variance-covariances matrix. Finally, we give some hints to compare linear and complicated nonlinear models considering the unconditional distribution of the linear IRF. Relaxing the hypothesis of non-stochasticity of $\hat{p}_B^h$, but mantaining its uncorrelation with $\hat{p}_A^h$, it is possible to verify that, using the unconditional distribution of $\hat{p}_A^h$, if we accept H_0 we would have accepted it anyway had we used the joint distribution.

APPENDIX

In this brief appendix, we want to prove that $\Sigma_{A,p}^{-1} - (\Sigma_{A,p} + \Sigma_{B,p})^{-1}$ is a negative definite (n.d.) matrix.

Consider the following rule involving positive definite matrices[15]:

<u>rule 1</u>: K is positive definite (p.d.) if and only if all its principle minors are positive;
<u>rule 2</u>: if K is p.d. then -K is n.d.;
<u>rule 3</u>: if K is p.d. then K^{-1} is p.d..
First, consider the following theorem:

<u>Theorem</u>: *if (a) A, B, A-B are p.d. matrices then (b) A^{-1}-B^{-1} is a negative matrix.*

<u>Proof</u>:

We know that A-B=Q is a p.d. matrix. Considering rule 1, $|Q|>0$. Now, suppose the theorem is false and thus:

[A .1] $A^{-1}-B^{-1}=P$ (p.d.),

pre and post-multiplying both sides of A.1 by A and B we obtain:

[A . 2] B-A=APB,

but, using rules 1 and 2, $|B-A|<0$ and $|A|$, $|B|>0$, we must have: $|P|<0$ (which contradict A.1). To ensure that A^{-1}-B^{-1} is negative definite, using rule 1, we need to obtain the same results for all of its principle minors. This is straighforward. Let's partition A and B in the following way:

$$\underset{n\times n}{A} = \begin{bmatrix} \underset{p\times p}{A_{11}} & \underset{p\times(n-p)}{A_{12}} \\ \underset{(n-p)\times p}{A_{21}} & \underset{(n-p)\times(n-p)}{A_{22}} \end{bmatrix}, \quad \underset{n\times n}{B} = \begin{bmatrix} \underset{p\times p}{B_{11}} & \underset{p\times(n-p)}{B_{12}} \\ \underset{(n-p)\times p}{B_{21}} & \underset{(n-p)\times(n-p)}{B_{22}} \end{bmatrix}$$

By assumption (a) and considering rule 1, we must have A_{11}-B_{11}=P_1 (p.d.) and thus A_{11}^{-1}-B_{11}^{-1} is n.d., then:

$$|B_{11}^{-1} - A_{11}^{-1}| = |A_{11}|\,|P_1|\,|B_{11}| > 0 \quad \forall\, p = 0,\ldots.n \quad \text{(Q.E.D.)}.$$

A= $\Sigma_{A,p} + \Sigma_{B,p}$ and B= $\Sigma_{A,p}$ ends our proof.

[15] See Lütkepohl(1993), p.459.

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